Exercise 9.1

Question 1:

Write the first five terms of the sequences whose n^{th} term is $a_n = n(n+2)$.

Solution 1:

$a_n = n(n+2)$

Substituting $n = 1, 2, 3, 4$ and 5, we obtain

$a_1 = 1(1+2) = 3$

$a_2 = 2(2+2) = 8$

$a_3 = 3(3+2) = 15$

$a_4 = 4(4+2) = 24$

$a_5 = 5(5+2) = 35$

Therefore, the required terms are 3, 8, 15, 24 and 35.

Question 2:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{n}{n+1}$

Solution 2:

$a_n = \frac{n}{n+1}$

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$a_1 = \frac{1}{1+1} = \frac{1}{2}$, $a_2 = \frac{2}{2+1} = \frac{2}{3}$, $a_3 = \frac{3}{3+1} = \frac{3}{4}$, $a_4 = \frac{4}{4+1} = \frac{4}{5}$, $a_5 = \frac{5}{5+1} = \frac{5}{6}$

Therefore, the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ and $\frac{5}{6}$

Question 3:

Write the first five terms of the sequences whose n^{th} term is $a_n = 2^n$

Solution 3:

$a_n = 2^n$

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$a_1 = 2^1 = 2$

$a_2 = 2^2 = 4$

$a_3 = 2^3 = 8$

$a_4 = 2^4 = 16$

$a_5 = 2^5 = 32$

Therefore, the required terms are 2, 4, 8, 16 and 32.

Question 4:

Write the first five terms of the sequences whose n^{th} term is $a_n = \dfrac{2n-3}{6}$

Solution 4:

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$

$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$

$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$

$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$

$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Therefore, the required terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$ and $\frac{7}{6}$.

Question 5:

Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$

Solution 5:

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$

$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$

$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$

$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$

$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, $-125, 625, -3125$ and 15625.

Question 6:

Write the first five terms of the sequences whose n^{th} term is $a_n = n\frac{n^2+5}{4}$

Solution 6:

Substituting $n=1,2,3,4,5$, we obtain

$$a_1 = 1\cdot\frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$

$$a_2 = 2\cdot\frac{2^2+5}{4} = 2.\frac{9}{4} = \frac{9}{2}$$

$$a_3 = 3\cdot\frac{3^2+5}{4} = 3\cdot\frac{14}{4} = \frac{21}{2}$$

$$a_4 = 4\cdot\frac{4^2+5}{4} = 21$$

$$a_5 = 5\cdot\frac{5^2+5}{4} = 5\cdot\frac{30}{4} = \frac{75}{2}$$

Therefore, the required terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}, 21$ and $\frac{75}{2}$.

Question 7:

Find the 17th and 24th term in the following sequence whose n^{th} term is $a_n = 4n-3$

Solution 7:

Substituting $n=17$, we obtain

$$a_{17} = 4(17)-3 = 68-3 = 65$$

Substituting $n=24$, we obtain

$$a_{24} = 4(24)-3 = 96-3 = 93.$$

Question 8:

Find the 7th term in the following sequence whose n^{th} term is $a_n = \frac{n^2}{2^n}$

Solution 8:

Substituting $n=7$, we obtain

$$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$$

Question 9:

Find the 9th term in the following sequence whose n^{th} term is $a_n = (-1)^{n-1} n^3$

Solution 9:

Substituting $n = 7$, we obtain

$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$

Question 10:

Find the 20th term in the following sequence whose n^{th} term is $a_n = \dfrac{n(n-2)}{n+3}$

Solution 10:

Substituting $n = 20$, we obtain

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Question 11:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = 3,\ a_n = 3a_{n-1} + 2$ for all $n > 1$

Solution 11:

$a_1 = 3,\ a_n = 3a_{n-1} + 2$ for $n > 1$

$\Rightarrow a_2 = 3a_1 + 2 = 3(3) + 2 = 11$

$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$

$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$

$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$

Hence, the first five terms of the sequence are 3, 11, 35, 107 and 323.

The corresponding series is $3 + 11 + 35 + 107 + 323 + \ldots$

Question 12:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = -1,\ a_n = \dfrac{a_{n-1}}{n},\ n \geq 2$

Solution 12:

$a_1 = -1,\ a_n = \dfrac{a_{n-1}}{n},\ n \geq 2$

$\Rightarrow a_2 = \frac{a_1}{2} = \frac{-1}{2}$

$a_3 = \frac{a_2}{3} = \frac{-1}{6}$

$a_4 = \frac{a_3}{4} = \frac{-1}{24}$

$a_5 = \frac{a_4}{5} = \frac{-1}{120}$

Hence, the first five terms of the sequence are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}$ and $\frac{-1}{120}$.

The corresponding series is $(-1)+\left(\frac{-1}{2}\right)+\left(\frac{-1}{6}\right)+\left(\frac{-1}{24}\right)+\left(\frac{-1}{120}\right)+\ldots$

Question 13:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = a_2 = 2,\ a_n = a_{n-1} - 1,\ n > 2$

Solution 13:

$a_1 = a_2 = 2,\ a_n = a_{n-1} - 1,\ n > 2$

$\Rightarrow a_3 = a_2 - 1 = 2 - 1 = 1$

$a_4 = a_3 - 1 = 1 - 1 = 0$

$a_5 = a_4 - 1 = 0 - 1 = -1$

Hence, the first five terms of the sequence are 2, 2, 1, 0 and -1.

The corresponding series is $2+2+1+0(-1)+\ldots$

Question 14:

The Fibonacci sequence is defined by $1 = a_1 = a_2$ and $a_n = a_{n-1} + a_{n-2},\ n > 2$

Find $\frac{a_{n+1}}{a_n}$, for $n = 1, 2, 3, 4, 5$

Solution 14:

$1 = a_1 = a_2$

$a_n = a_{n-1} + a_{n-2},\ n > 2$

$\therefore a_3 = a_2 + a_1 = 1 + 1 = 2$

$a_4 = a_3 + a_2 = 2 + 1 = 3$

$a_5 = a_4 + a_3 = 3 + 2 = 5$

$a_6 = a_5 + a_4 = 5 + 3 = 8$

$\therefore$ For $n=1$, $\dfrac{a_{n+1}}{a_n}=\dfrac{a_2}{a_1}=\dfrac{1}{1}=1$

For $n=2$, $\dfrac{a_{n+1}}{a_n}=\dfrac{a_3}{a_2}=\dfrac{2}{1}=2$

For $n=3$, $\dfrac{a_{n+1}}{a_n}=\dfrac{a_4}{a_3}=\dfrac{3}{2}$

For $n=4$, $\dfrac{a_{n+1}}{a_n}=\dfrac{a_5}{a_4}=\dfrac{5}{3}$

For $n=5$, $\dfrac{a_{n+1}}{a_n}=\dfrac{a_6}{a_5}=\dfrac{8}{5}$.