Exercise 7.4

Question 1:

If ${}^nC_8 = {}^nC_2$, find ${}^nC_2$

Solution 1:

It is known that, ${}^nC_a = {}^nC_b \Rightarrow a = b$ or $m = a + b$

Therefore,

${}^nC_8 = {}^nC_2 \Rightarrow n = 8 + 2 = 10$

$$\therefore {}^nC_2 = {}^{10}C_2 = \frac{10!}{2!(10-2)!} = \frac{10!}{2!8!} = \frac{10\times 9\times 8!}{2\times 1\times 8!} = 45$$

Question 2:

Determine n if

(i) ${}^{2n}C_3 : {}^{n}C_3 = 12:1$ (ii) ${}^{2n}C_3 : {}^{n}C_3 = 11:1$

Solution 2:

(i) $\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{12}{1}$

$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = \frac{12}{1}$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!} = 12$

$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 12$

$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 12$

$\Rightarrow \frac{(2n-1)}{(n-2)} = 3$

$\Rightarrow 2n-1 = 3(n-2)$

$\Rightarrow 2n-1 = 3n-6$

$\Rightarrow 3n-2n = -1+6$

$\Rightarrow n = 5$

(ii) $\frac{{}^{2n}C_3}{{}^{n}C_3} = \frac{11}{1}$

$\Rightarrow \frac{(2n)!}{3!(2n-3)!} \times \frac{3!(n-3)!}{n!} = 11$

$\Rightarrow \frac{(2n)(2n-1)(2n-2)(2n-3)!}{(2n-3)!} \times \frac{(n-3)!}{n(n-1)(n-2)(n-3)!}$

$\Rightarrow \frac{2(2n-1)(2n-2)}{(n-1)(n-2)} = 11$

$\Rightarrow \frac{4(2n-1)(n-1)}{(n-1)(n-2)} = 11$

$\Rightarrow \frac{4(2n-1)}{n-2} = 11$

$\Rightarrow 4(2n-1) = 11(n-2)$

$\Rightarrow 8n-4 = 11n-22$

$\Rightarrow 11n-8n = -4+22$

$\Rightarrow 3n = 18$
$\Rightarrow n = 6$

Question 3:
How many chords can be drawn through 21 points on a circle?

Solution 3:
For drawing one chord a circle, only 2 points are required.
To know the number of chords that can be drawn through the given 21 points on a circle, the number of combinations have to be counted.
Therefore, there will be as many chords as there are combinations of 21 points taken 2 at a time.
Thus, required number of chords $= {}^{21}C_2 = \frac{21!}{2!(21-2)!} = \frac{21!}{2!19!} = \frac{21 \times 20}{2} = 210$

Question 4:
In how many ways can a team of 3 boys and 3 girls be selected from 5 boys and 4 girls?

Solution 4:
A team of 3 boys and 3 girls is to be selected from 5 boys and 4 girls.
3 boys can be selected from 5 boys in ${}^5C_3$ ways.
3 girls can be selected from 4 girls in ${}^4C_3$ ways.
Therefore, by multiplication principle, number of ways in which a team of 3 boys and 3 girls can be selected $= {}^5C_3 \times {}^4C_3 = \frac{5!}{3!2!} \times \frac{4!}{3!1!}$

$= \frac{5 \times 4 \times 3!}{3! \times 2} \times \frac{4 \times 3!}{3!}$
$= 10 \times 4 = 40$

Question 5:
Find the number of ways of selecting 9 balls from 6 red balls, 5 white balls and 5 blue balls if each selection consists of 3 balls each colour.

Solution 5:
There are a total of 6 red balls, 5 white balls, and 4 blue balls.
9 balls have to be selected in such a way that each selection consists of 3 balls of each colour.
Here,
3 balls can be selected from 6 red balls in ${}^6C_3$ ways.
3 balls can be selected from 5 white balls in ${}^5C_3$ ways.

3 balls can be selected from 5 blue balls in ${}^5C_3$ ways.

Thus, by multiplication principle, required number of ways of selecting 9 balls.

$$= {}^6C_3 \times {}^5C_3 \times {}^5C_3 = \frac{6!}{3!3!} \times \frac{5!}{3!2!} \times \frac{5!}{3!2!}$$

$$= \frac{6\times5\times4\times3!}{3!\times3\times2} \times \frac{5\times4\times3!}{3!\times2\times1} \times \frac{5\times4\times3!}{3!\times2\times1}$$

$$= 20\times10\times10 = 2000$$

Question 6:

Determine the number of 5 card combinations out of a deck of 52 cards if there is exactly one ace in each combination.

Solution 6:

In a deck of 52 cards, there are 4 aces. A combinations of 5 cards have to be made in which there is exactly one ace.

Then, one ace can be selected in ${}^4C_3$ ways and the remaining 4 cards can be selected out of the 48 cards in ${}^{48}C_4$ ways.

Thus, by multiplication principle, required number of 5 card combinations

$$= {}^{48}C_4 \times {}^4C_1 = \frac{48!}{4!44!} \times \frac{4!}{1!3!}$$

$$= \frac{48\times47\times46\times45}{4!\times3\times2\times1} \times 4!$$

$$= 778320$$

Question 7:

In how many ways can one select a cricket team of eleven from 17 players in which only 5 players can bowl if each cricket team of 11 must include exactly 4 bowlers?

Solution 7:

Out of 17 players, 5 players are bowlers.

A cricket team of 11 players is to be selected in such a way that there are exactly 4 bowlers.

4 bowlers can be selected in ${}^5C_4$ ways and the remaining 7 players can be selected out of the 12 players in ${}^{12}C_7$ ways.

Thus, by multiplication principle, required number of ways of selecting cricket team

$$= {}^5C_4 \times {}^{12}C_7 = \frac{5!}{4!1!} \times \frac{12!}{7!5!} = 5!\times\frac{12\times11\times10\times9\times8}{5!\times4\times3\times2\times1} = 3960$$

Question 8:

A bag contains 5 black and 6 red balls. Determine the number of ways in which 2 black and 3 red balls can be selected.

Solution 8:

There are 5 black and 6 red balls in the bag.

2 black balls can be selected out of 5 black balls in ${}^{5}C_{2}$ ways and 3 red balls can be selected out of 6 red balls in ${}^{6}C_{3}$ ways.

Thus, by multiplication principle, required number of ways of selecting 2 black and 3 red balls

$$= {}^{5}C_{2} \times {}^{6}C_{3} = \frac{5!}{2!3!} \times \frac{6!}{3!3!} = \frac{5 \times 4}{2} \times \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 10 \times 20 = 200.$$

Question 9:

In how many ways can a student choose a programme of 5 courses if 9 courses are available and 2 specific courses are compulsory for every student?

Solution 9:

There are 9 courses available out of which, 2 specific courses are compulsory for every student. Therefore, every student has to choose 3 courses out of the remaining 7 courses. This can be chosen in ${}^{7}C_{3}$ ways.

Thus, required number of ways of choosing the programme

$$= {}^{7}C_{3} = \frac{7!}{3!4!} = \frac{7 \times 6 \times 5 \times 4!}{3 \times 2 \times 1 \times 4!} = 35.$$