Exercise 5.2

Question 1:

Find the modulus and the argument of the complex number $z=-1-i\sqrt{3}$

Solution 1:

$z=-1-i\sqrt{3}$

Let $r\cos\theta=-1$ and $r\sin\theta=-\sqrt{3}$

On squaring and adding, we obtain

$(r\cos\theta)^2+(r\sin\theta)^2=(-1)^2+(-\sqrt{3})^2$

$\Rightarrow r^2(\cos^2\theta+\sin^2\theta)=1+3$

$\Rightarrow r^2=4$ $\quad [\cos^2\theta+\sin^2\theta=1]$

$\Rightarrow r=\sqrt{4}=2$ $\quad$ [Conventionally, $r>0$]

$\therefore$ Modulus = 2

$\therefore 2\cos\theta=-1$ and $2\sin\theta=-\sqrt{3}$

$\Rightarrow \cos\theta=\frac{-1}{2}$ and $\sin\theta=\frac{-\sqrt{3}}{2}$

Since both the values of $\sin\theta$ and $\cos\theta$ negative and $\sin\theta$ and $\cos\theta$ are negative in III quadrant,

Argument $=-\left(\pi-\frac{\pi}{3}\right)=\frac{-2\pi}{3}$

Thus, the modulus and argument of the complex number $-1-\sqrt{3}i$ are 2 and $-\frac{2\pi}{3}$ respectively.

Question 2:

Find the modulus and the argument of the complex number $z=-\sqrt{3}+i$

Solution 2:

$z=-\sqrt{3}+i$

Let $r\cos\theta=-\sqrt{3}$ and $r\sin\theta=1$

On squaring and adding, we obtain

$r^2\cos^2\theta+r^2\sin^2\theta=(-\sqrt{3})^2+1^2$

$\Rightarrow r^2=3+1=4$ $\quad [\cos^2\theta+\sin^2\theta=1]$

$\Rightarrow r=\sqrt{4}=2$ $\quad$ [Conventionally, $r>0$]

$\therefore$ Modulus = 2

$\therefore 2\cos\theta=-\sqrt{3}$ and $2\sin\theta=1$

$\Rightarrow \cos\theta = \frac{-\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$ [As θ lies in the II quadrant]

Thus, the modulus an argument of the complex number $-\sqrt{3}+i$ are 2 and $\frac{5\pi}{6}$ respectively.

Question 3:

Convert the given complex number in polar form: $1-i$

Solution 3:

$1-i$

Let $r\cos\theta = 1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = 1^2 + (-1)^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$ $[\text{Conventionally, } r > 0]$

$\therefore \sqrt{2}\cos\theta = 1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta = -\frac{\pi}{4}$ $[\text{As } \theta \text{ lies in the IV quadrant}]$

$\therefore 1-i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\left(-\frac{\pi}{4}\right) + i\sqrt{2}\sin\left(-\frac{\pi}{4}\right) = \sqrt{2}\left[\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right]$

This is the required polar form.

Question 4:

Convert the given complex number in polar form: $-1+i$

Solution 4:

$-1+i$

Let $r\cos\theta = -1$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + 1^2$

$\Rightarrow r^2(\cos^2\theta + \sin^2\theta) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$ [Conventionally, r > 0]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = 1$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$ and $\sqrt{2}\sin\theta = 1$

$\therefore\ \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$ [As θ lies in the II quadrant]

It can be written,

$\therefore\ -1+i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\frac{3\pi}{4} + i\sqrt{2}\sin\frac{3\pi}{4} = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

This is the required polar form.

Question 5:

Convert the given complex number in polar form: $-1-i$

Solution 5:

$-1-i$

Let $r\cos\theta = -1$ and $r\sin\theta = -1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-1)^2 + (-1)^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1+1$

$\Rightarrow r^2 = 2$

$\Rightarrow r = \sqrt{2}$ [Conventionally, $r > 0$]

$\therefore \sqrt{2}\cos\theta = -1$ and $\sqrt{2}\sin\theta = -1$

$\Rightarrow \cos\theta = -\frac{1}{\sqrt{2}}$ and $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore\ \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4}$ [As θ lies in the III quadrant]

$\therefore\ -1-i = r\cos\theta + i r\sin\theta = \sqrt{2}\cos\frac{-3\pi}{4} + i\sqrt{2}\sin\frac{-3\pi}{4}$

$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$

This is the required polar form.

Question 6:

Convert the given complex number in polar form: -3

Solution 6:

-3

Let $r\cos\theta = -3$ and $r\sin\theta = 0$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = (-3)^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 9$

$\Rightarrow r^2 = 9$

$\Rightarrow r = \sqrt{9} = 3$ [Conventionally, r > 0]

$\therefore\ 3\cos\theta = -3$ and $3\sin\theta = 0$

$\Rightarrow \cos\theta = -1$ and $\sin = 0$

$\therefore \theta = \pi$

$\therefore\ -3 = r\cos\theta + ir\sin\theta = 3\cos\pi + i3\sin\pi = 3(\cos\pi + i\sin\pi)$

This is the required polar form.

Question 7:

Convert the given complex number in polar form: $\sqrt{3}+i$

Solution 7:

$\sqrt{3}+i$

Let $r\cos\theta = \sqrt{3}$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = \left(\sqrt{3}\right)^2 + 1^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 3+1$

$\Rightarrow r^2 = 4$

$\Rightarrow r = \sqrt{4} = 2$ [Conventionally, $r > 0$]

$\therefore 2\cos\theta = \sqrt{3}$ and $2\sin\theta = 1$

$\Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\therefore \theta = \frac{\pi}{6}$ [As θ lies in the I quadrant]

$\therefore\ \sqrt{3}+i = r\cos\theta + ir\sin\theta = 2\cos\frac{\pi}{6} + i2\sin\frac{\pi}{6} = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

This is the required polar form.

Question 8:

Convert the given complex number in polar form: i

Solution 8:

i

Let $r\cos\theta = 0$ and $r\sin\theta = 1$

On squaring and adding, we obtain

$r^2\cos^2\theta + r^2\sin^2\theta = 0^2 + 1^2$

$\Rightarrow r^2\left(\cos^2\theta + \sin^2\theta\right) = 1$

$\Rightarrow r^2 = 1$

$\Rightarrow r = \sqrt{1} = 1$ [Conventionally, $r > 0$]

$\therefore \cos\theta = 0$ and $\sin\theta = 1$

$\therefore \theta = \frac{\pi}{2}$

$\therefore i = r\cos\theta + i\,r\sin\theta = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$

This is the required polar form.
