## Exercise 1.6

**Question 1:**

If X and Y are two sets such that $n(X)=17, n(Y)=23$ and $n(X\cup Y)=38$, find $n(X\cap Y)$.

**Solution 1:**

It is given that:

$$n(X)=17, n(Y)=23, n(X\cup Y)=38$$

We know that:

$$n(X\cup Y)=n(X)+n(Y)-n(X\cap Y)$$

$$\therefore 38=17+23-n(X\cap Y)$$

$$\Rightarrow n(X\cap Y)=40-38=2$$

$\therefore n(X \cap Y) = 2$

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**Question 2:**

If X and Y are two sets such that $X \cup Y$ has 18 elements, X has 8 elements and Y has 15 elements; how many elements does $X \cap Y$ have?

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

**Solution 2:**

It is given that:

$n(X \cup Y) = 18, n(X) = 8, n(Y) = 15$

$n(X \cap Y) = ?$

We know that:

$\therefore 18 = 8 + 15 - n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 23 - 18 = 5$

$\therefore n(X \cap Y) = 5$

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**Question 3:**

In a group of 400 people, 250 can speak Hindi and 200 can speak English. How many people can speak both Hindi and English?

**Solution 3:**

Let H be the set of people who speak Hindi, and E be the set of people who speak English
$\therefore n(H \cup E) = 400, n(H) = 250, n(E) = 200$

$n(H \cap E) = ?$

We know that:

$n(H \cup E) = n(H) + n(\mathrm{E}) - n(H \cap E)$

$\therefore 400 = 250 + 200 - n(H \cap E)$

$\Rightarrow 400 = 450 - n(H \cap E)$

$\Rightarrow n(H \cap E) = 450 - 400$

$\therefore n(H \cap E) = 50$

Thus, 50 people can speak both Hindi and English.

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## Question 4:

If S and T are two sets such that S has 21 elements, T has 32 elements, and $S \cap T$ has 11 elements, how many elements does $S \cup T$ have?

## Solution 4:

It is given that:

$$n(S) = 21, n(T) = 32, n(S \cap T) = 11$$

We know that:

$$n(S \cup T) = n(S) + n(T) - n(S \cap T)$$

$$\therefore n(S \cup T) = 21 + 32 - 11 = 42$$

Thus, the set $(S \cup T)$ has 42 elements.

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## Question 5:

If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

## Solution 5:

It is given that:

$$n(X) = 40, n(X \cup Y) = 60, n(X \cap Y) = 10$$

We know that:

$$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$$

$$\therefore 60 = 40 + n(Y) - 10$$

$$\therefore n(Y) = 60 - (40 - 10) = 30$$

Thus, the set Y has 30 elements.

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## Question 6:

In a group of 70 people, 37 like coffee, 52 like tea, and each person likes at least one of the two drinks. How many people like both coffee and tea?

## Solution 6:

Let C denote the set of people who like coffee, and T denote the set of people who like tea

$$n(C \cup T) = 70, n(C) = 37, n(T) = 52$$

We know that:

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$\therefore 70 = 37 + 52 - n(C \cap T)$$

$$\Rightarrow 70 = 89 - n(C \cap T)$$

$$\Rightarrow (C \cap T) = 89 - 70 = 19$$

Thus, 19 people like both coffee and tea.

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**Question 7:**

In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?

**Solution 7:**

Let C denote the set of people who like cricket, and T denote the set of people who like tennis

$$\therefore n(C \cup T) = 65, n(C) = 40, n(C \cap T) = 10$$

We know that:

$$n(C \cup T) = n(C) + n(T) - n(C \cap T)$$

$$\therefore 65 = 40 + n(T) - 10$$

$$\Rightarrow 65 = 30 + n(T)$$

$$\Rightarrow n(T) = 65 - 30 = 35$$

Therefore, 35 people like tennis.

Now,

$$(T - C) \cup (T \cap C) = T$$

Also,

$$(T - C) \cap (T \cap C) = \varnothing$$

$$\therefore n(T) = n(T - C) + n(T \cap C)$$

$$\Rightarrow 35 = n(T - C) + 10$$

$$\Rightarrow n(T - C) = 35 - 10 = 25$$

Thus, 25 people like only tennis.

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**Question 8:**

In a committee, 50 people speak French, 20 speak Spanish and 10 speak both Spanish and French. How many speak at least one of these two languages?

**Solution 8:**

Let F be the set of people in the committee who speak French, and S be the set of people in the committee who speak Spanish

$\therefore n(F)=50, n(S)=20, n(S\cap F)=10$

We know that:

$n(S\cup F)=n(S)+n(F)-n(S\cap F)$

$=20+50-10$

$=70-10=60$

Thus, 60 people in the committee speak at least one of the two languages.

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## Miscellaneous Exercise 1

**Question 1:**

Decide, among the following sets, which sets are subsets of one and another:

$A=\{x : x\in R \text{ and } x \text{ satisfy } x^2-8x+12=0\}$,

$B=\{2,4,6\}, C=\{2,4,6,8\ldots\}, D=\{6\}$.

**Solution 1:**

$A=\{x : x\in R \text{ and } x \text{ satisfy } x^2-8x+12=0\}$

2 and 6 are the only solutions of $x^2-8x+12=0$.

$\therefore A=\{2,6\}$

$B=\{2,4,6\}, C=\{2,4,6,8\ldots\}, D=\{6\}$

$\therefore D\subset A\subset B\subset C$

Hence, $A\subset B, A\subset C, B\subset C, D\subset A, D\subset B, D\subset C$

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**Question 2:**