NCERT Class 12 Maths

Solutions

Exercise 9.2

In each of the Exercises 1 to 6 verify that the given functions (explicit) is a solution of the corresponding differential equation:

1. $y = e^x + 1 : y'' - y' = 0$

Sol. Given: $y = e^x + 1$ ...(*i*)

To prove: y given, by (*i*) is a solution of the D.E. $y'' - y' = 0$ ...(*ii*)

From (*i*), $y' = e^x + 0 = e^x$ and $y'' = e^x$

$\therefore$ L.H.S. of D.E. (*ii*) $= y'' - y' = e^x - e^x = 0 =$ R.H.S. of D.E. (*ii*)

$\therefore$ y given by (*i*) is a solution of D.E. (*ii*).

2. $y = x^2 + 2x + C : y' - 2x - 2 = 0$

Sol. Given: $y = x^2 + 2x + C$ ...(*i*)

To prove: y given by (*i*) is a solution of the D.E.

$y' - 2x - 2 = 0$ ...(*ii*)

From (*i*), $y' = 2x + 2$

$\therefore$ L.H.S. of D.E. (*ii*) $= y' - 2x - 2$

$= (2x + 2) - 2x - 2 = 2x + 2 - 2x - 2 = 0 =$ R.H.S. of D.E. (*ii*)

$\therefore$ y given by (*i*) is a solution of D.E. (*ii*).

3. $y = \cos x + C : y' + \sin x = 0$

Sol. Given: $y = \cos x + C$ ...(*i*)

To prove: y given by (*i*) is a solution of D.E. $y' + \sin x = 0$ ...(*ii*)

From (*i*), $y' = -\sin x$

$\therefore$ L.H.S. of D.E. (*ii*) $= y' + \sin x = -\sin x + \sin x$

$= 0 =$ R.H.S. of D.E. (*ii*)

$\therefore$ y given by (i) is a solution of D.E. (ii).

4. $\boldsymbol{y = \sqrt{1+x^2} \;:\; y' = \dfrac{xy}{1+x^2}}$

Sol. Given: $y = \sqrt{1+x^2}$ ...(i)

To prove: y given by (i) is a solution of D.E. $y' = \dfrac{xy}{1+x^2}$ ...(ii)

From (i), $y' = \dfrac{d}{dx} \sqrt{1+x^2} = \dfrac{d}{dx} (1 + x^2)^{1/2}$

$= \dfrac{1}{2} (1 + x^2)^{-1/2} \dfrac{d}{dx} (1 + x^2) = \dfrac{1}{2} (1 + x^2)^{-1/2} . 2x = \dfrac{x}{\sqrt{1+x^2}}$ ...(iii)

R.H.S. of D.E. (ii) $= \dfrac{xy}{1+x^2} = \dfrac{x}{1+x^2} \sqrt{1+x^2}$ (By (i))

$= \dfrac{x}{\sqrt{1+x^2}}$ $\left[\because \dfrac{\sqrt{t}}{t} = \dfrac{\sqrt{t}}{\sqrt{t}\,\sqrt{t}} = \dfrac{1}{\sqrt{t}}\right]$

$= y'$ [By (iii)] = L.H.S. of D.E. (ii)

$\therefore$ y given by (i) is a solution of D.E. (ii).

5. $\boldsymbol{y = Ax : xy' = y \; (x \neq 0)}$

Sol. Given: $y = Ax$ ...(i)

To prove: y given by (i) is a solution of the D.E. $xy' = y$ $(x \neq 0)$ ...(ii)

From (i), $y' = A(1) = A$

L.H.S. of D.E. (ii) $= xy' = xA$

$= Ax = y$ [By (i)] = R.H.S. of D.E. (ii)

$\therefore$ y given by (i) is a solution of D.E. (ii).

6. $\boldsymbol{y = x \sin x : xy' = y + x\sqrt{x^2 - y^2} \; (x \neq 0 \text{ and } x > y \text{ or } x < -y)}$

Sol. Given: $y = x \sin x$ ...(i)

To prove: y given by (i) is a solution of D.E.

$xy' = y + x \sqrt{x^2 - y^2}$ ...(ii) $(x \neq 0$ and $x > y$ or $x < -y)$

From (i), $\dfrac{dy}{dx}$ $(= y') = x \dfrac{d}{dx} (\sin x) + \sin x \dfrac{d}{dx} x = x \cos x + \sin x$

L.H.S. of D.E. (ii) $= xy' = x (x \cos x + \sin x)$

$= x^2 \cos x + x \sin x$ ...(iii)

R.H.S. of D.E. (ii) $= y + x \sqrt{x^2 - y^2}$

Putting $y = x \sin x$ from (i),

$= x \sin x + x \sqrt{x^2 - x^2 \sin^2 x} = x \sin x + x \sqrt{x^2 (1 - \sin^2 x)}$

$= x \sin x + x \sqrt{x^2 \cos^2 x} = x \sin x + x . x \cos x$

$= x \sin x + x^2 \cos x = x^2 \cos x + x \sin x$ ...(iv)

From (iii) and (iv), L.H.S. of D.E. (ii) = R.H.S. of D.E. (ii)

$\therefore$ y given by (i) is a solution of D.E. (ii).

In each of the Exercises 7 to 10, verify that the given functions (Explicit or Implicit) is a solution of the corresponding differential equation:

7. $xy = \log y + C : y' = \dfrac{y^2}{1 - xy}$ $(xy \neq 1)$

Sol. Given: $xy = \log y + C$ ...(*i*)

To prove that Implicit function given by (*i*) is a solution of the

D.E. $$y' = \frac{y^2}{1 - xy} \quad ...(ii)$$

Differentiating both sides of (*i*) w.r.t. x, we have

$$xy' + y(1) = \frac{1}{y}\, y' + 0$$

$$\Rightarrow \quad xy' - \frac{y'}{y} = -y \qquad \Rightarrow \quad y'\left(x - \frac{1}{y}\right) = -y$$

$$\Rightarrow \quad y'\left(\frac{xy - 1}{y}\right) = -y \qquad \Rightarrow \quad y'(xy - 1) = -y^2$$

$$\Rightarrow \quad y' = \frac{-y^2}{xy - 1} = \frac{-y^2}{-(1 - xy)} = \frac{y^2}{1 - xy}$$

which is same as differential equation (*ii*), *i.e.*, Eqn. (*ii*) is proved.

$\therefore$ Function (Implicit) given by (*i*) is a solution of D.E. (*ii*).

8. $y - \cos y = x : (y \sin y + \cos y + x)\, y' = y$

Sol. Given: $y - \cos y = x$ ...(*i*)

To prove that function given by (*i*) is a solution of D.E.

$$(y \sin y + \cos y + x)\, y' = y \quad ...(ii)$$

Differentiating both sides of (*i*) w.r.t. x, we have

$$y' + (\sin y)\, y' = 1 \qquad \Rightarrow \quad y'(1 + \sin y) = 1$$

$$\Rightarrow \quad y' = \frac{1}{1 + \sin y} \quad ...(iii)$$

Putting the value of x from (*i*) and value of y' from (*iii*) in L.H.S. of (*ii*), we have

L.H.S. $= (y \sin y + \cos y + x)\, y'$

$$= (y \sin y + \cos y + y - \cos y)\frac{1}{1 + \sin y} = (y \sin y + y)\frac{1}{1 + \sin y}$$

$$= y(\sin y + 1)\frac{1}{(1 + \sin y)} = y = \text{R.H.S. of } (ii).$$

$\therefore$ The function given by (*i*) is a solution of D.E. (*ii*).

9. $x + y = \tan^{-1} y : y^2 y' + y^2 + 1 = 0$

Sol. Given: $x + y = \tan^{-1} y$ ...(*i*)

To prove that function given by (*i*) is a solution of D.E.

$$y^2 y' + y^2 + 1 = 0 \quad ...(ii)$$

Differentiating both sides of (*i*), w.r.t. x, $1 + y' = \dfrac{1}{1 + y^2}\, y'$

Cross-multiplying

$$(1 + y')(1 + y^2) = y' \quad \Rightarrow \quad 1 + y^2 + y' + y'y^2 = y'$$

$\Rightarrow$ $y^2y' + y^2 + 1 = 0$ which is same as D.E. (*ii*).

$\therefore$ Function given by (*i*) is a solution of D.E. (*ii*).

10. $\boldsymbol{y = \sqrt{a^2 - x^2}, x \in (-a, a) : x + y\dfrac{dy}{dx} = 0 \; (y \neq 0)}$

Sol. Given: $y = \sqrt{a^2 - x^2}$, $x \in (-a, a)$ ...(*i*)

To prove that function given by (*i*) is a solution of D.E.

$$x + y\frac{dy}{dx} = 0 \quad \text{...(ii)}$$

From (*i*), $\dfrac{dy}{dx} = \dfrac{1}{2}(a^2 - x^2)^{-1/2}\dfrac{d}{dx}(a^2 - x^2)$

$$= \frac{1}{2\sqrt{a^2 - x^2}}(-2x) = \frac{-x}{\sqrt{a^2 - x^2}} \quad \text{...(iii)}$$

Putting these values of y and $\dfrac{dy}{dx}$ from (*i*) and (*iii*) in L.H.S. of (*ii*),

$$\text{L.H.S.} = x + y\frac{dy}{dx} = x + \sqrt{a^2 - x^2}\left(\frac{-x}{\sqrt{a^2 - x^2}}\right)$$

$= x - x = 0 =$ R.H.S. of D.E. (*ii*).

$\therefore$ Function given by (*i*) is a solution of D.E. (*ii*).

11. Choose the correct answer:

The number of arbitrary constants in the general solution of a differential equation of fourth order are:

(A) 0 (B) 2 (C) 3 (D) 4.

Sol. Option (D) 4 is the correct answer.

Result. The number of arbitrary constants (c_1, c_2, c_3 etc.) in the general solution of a differential equation of nth order is n.

12. The number of arbitrary constants in the particular solution of a differential equation of third order are

(A) 3 (B) 2 (C) 1 (D) 0.

Sol. The number of arbitrary constants in a particular solution of a differential equation of any order is zero (0).

[$\because$ By definition, a particular solution is a solution which contains no arbitrary constant.]

$\therefore$ Option (D) is the correct answer.