NCERT Class 12 Maths Solutions

Exercise 9.1

Determine order and degree (if defined) of differential equations given in Exercise 1 to 10:

1. $\frac{d^4y}{dx^4} + \sin(y''') = 0$

Sol. The given D.E. is $\frac{d^4y}{dx^4} + \sin y''' = 0$

The highest order derivative present in the differential equation is $\frac{d^4y}{dx^4}$ and its order is 4.

The given differential equation is not a polynomial equation in derivatives ($\because$ The term $\sin y'''$ is a T-function of derivative y'''). Therefore degree of this D.E. is not defined.

Ans. Order 4 and degree not defined.

2. $y' + 5y = 0$

Sol. The given D.E. is $y' + 5y = 0$.

The highest order derivative present in the D.E. is y' $\left(= \frac{dy}{dx}\right)$ and so its order is one. The given D.E. is a polynomial equation in derivatives (y' here) and the highest power raised to highest order derivative y' is one, so its degree is one.

Ans. Order 1 and **degree 1.**

3. $\left(\frac{ds}{dt}\right)^4 + 3s\,\frac{d^2s}{dt^2} = 0$

Sol. The given D.E. is $\left(\frac{ds}{dt}\right)^4 + 3s\,\frac{d^2s}{dt^2} = 0.$

The highest order derivative present in the D.E. is $\frac{d^2s}{dt^2}$ and its order is 2. The given D.E is a polynomial equation in derivatives and the highest power raised to highest order derivative $\frac{d^2s}{dt^2}$ is one. Therefore degree of D.E. is 1.

Ans. Order 2 and degree 1.

4. $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\,\frac{dy}{dx} = 0$

Sol. The given D.E. is $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0.$

The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$ and its order is 2.

The given D.E. is not a polynomial equation in derivatives ($\because$ The term $\cos\,\frac{dy}{dx}$ is a T-function of derivative $\frac{dy}{dx}$).

Therefore degree of this D.E. is not defined.

Ans. Order 2 and degree not defined.

5. $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$

Sol. The given D.E. is $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x.$

The highest order derivative present in the D.E. is $\frac{d^2y}{dx^2}$ and its order is 2.

The given D.E. is a polynomial equation in derivatives and the highest power raised to highest order $\frac{d^2y}{dx^2} = \left(\frac{d^2y}{dx^2}\right)^1$ is one, so its degree is 1.

Ans. Order 2 and degree 1.

Remark. It may be remarked that the terms $\cos 3x$ and $\sin 3x$ present in the given D.E. are trigonometrical functions (but not T-functions of derivatives).

It may be noted that $\left(\cos 3\,\frac{dy}{dx}\right)$ is not a polynomial function of derivatives.

6. $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$

Sol. The given D.E. is $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0.$ ...(*i*)

The highest order derivative present in the D.E. is y''' and its order is 3.

The given D.E. is a polynomial equation in derivatives y''', y'' and y' and the highest power raised to highest order derivative y''' is two, so its degree is 2.

Ans. Order 3 and degree 2.

7. $\boldsymbol{y''' + 2y'' + y' = 0}$

Sol. The given D.E. is $y''' + 2y'' + y' = 0$. ...(i)

The highest order derivative present in the D.E. is y''' and its order is 3.

The given D.E. is a polynomial equation in derivatives y''', y'' and y' and the highest power raised to highest order derivative y''' is one, so its degree is 1.

Ans. Order 3 and degree 1.

8. $\boldsymbol{y' + y = e^x}$

Sol. The given D.E. is $y' + y = e^x$. ...(i)

The highest order derivative present in the D.E. is y' and its order is 1.

The given D.E. is a polynomial equation in derivative y'. (It may be noted that e^x is an exponential function and not a polynomial function but is not an exponential function of derivatives) and the highest power raised to highest order derivative y' is one, so its degree is 1.

Ans. Order 1 and degree 1.

9. $\boldsymbol{y'' + (y')^2 + 2y = 0}$

Sol. The given D.E. is $y'' + (y')^2 + 2y = 0$. ...(i)

The highest order derivative present in the D.E. is y'' and its order is 2.

The given D.E. is a polynomial equation in derivatives y'' and y' and the highest power raised to highest order derivative y'' is one, so its degree is 1.

Ans. Order 2 and degree 1.

10. $\boldsymbol{y'' + 2y' + \sin y = 0}$

Sol. The given D.E. is $y'' + 2y' + \sin y = 0$. ...(i)

The highest order derivative present in the D.E. is y'' and its order is 2.

The given D.E. is a polynomial equation in derivatives y'' and y'. (It may be noted that $\sin y$ is not a polynomial function of y, it is a T-function of y but is not a T-function of derivatives) and the highest power raised to highest order derivative y'' is one, so its degree is one.

Ans. Order 2 and degree 1.

11. The degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \text{ is}$$

(A) 3 **(B) 2** **(C) 1** **(D) Not defined.**

Sol. The given D.E. is

$$\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0 \qquad ...(i)$$

This D.E. (i) is not a polynomial equation in derivatives.

$$\left[\because \sin\left(\frac{dy}{dx}\right) \text{is a T-function of derivative } \frac{dy}{dx}\right]$$

$\therefore$ Degree of D.E. (i) is not defined.

Answer. Option (D) is the correct answer.

12. The order of the differential equation

$$\mathbf{2x^2\,\frac{d^2y}{dx^2} - 3\,\frac{dy}{dx} + y = 0 \text{ is}}$$

(A) 2 (B) 1 (C) 0 (D) Not defined

Sol. The given D.E. is $2x^2\,\frac{d^2y}{dx^2} - 3\,\frac{dy}{dx} + y = 0$

The highest order derivative present in the differential equation is $\frac{d^2y}{dx^2}$ and its order is 2.

Answer. Order of the given D.E. is 2.