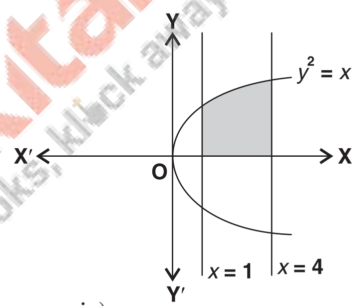
NCERT Class 12 Maths Solutions

Chapter - 8

EXERCISE 8.1

1. Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis.



Sol. Equation of the curve (rightward parabola) is $y^2 = x$

$\therefore \quad y = \sqrt{x} \quad ...(i)$

(For branch of the parabola above x-axis)

$\therefore$ Required area (as shown shaded in the figure)

$$= \left| \int_1^4 y\,dx \right| = \left| \int_1^4 \sqrt{x}\,dx \right| \qquad \left(\because \text{ From } (i)\ y = \sqrt{x}\right)$$

$$= \left| \int_1^4 x^{1/2}\,dx \right| = \left| \frac{\left(x^{\frac{3}{2}}\right)_1^4}{\frac{3}{2}} \right| = \left| \frac{2}{3}\left(4^{\frac{3}{2}} - 1^{\frac{3}{2}}\right) \right|$$

$$= \left| \frac{2}{3}(4\sqrt{4} - 1\sqrt{1}) \right| \left[\because\ x^{\frac{3}{2}} = x^{\frac{2+1}{2}} = x^{\frac{2}{2}+\frac{1}{2}} = x^{1+\frac{1}{2}} = x^1 \cdot x^{\frac{1}{2}} = x\sqrt{x}\ \right]$$

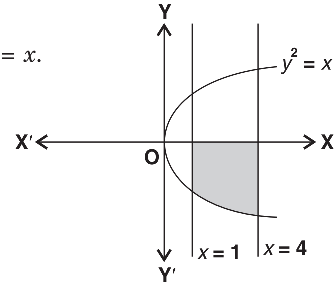
$$= \left| \frac{2}{3}(4(2) - 1(1)) \right| = \frac{2}{3}(8 - 1) = \frac{2}{3} \times 7 = \frac{14}{3} \text{ sq. units.}$$

Note. $x^{\frac{3}{2}} = x\sqrt{x}$.

Remark. Equation of the curve is $y^2 = x$.

$\therefore \quad y = -\sqrt{x}$ for branch of the parabola below the x-axis.

The reader is within his or her rights to find the required area as shown shaded in the figure in the remark as $\left| \int_{x=1}^{x=4} y\, dx \right|$ taking $y = -\sqrt{x}$.



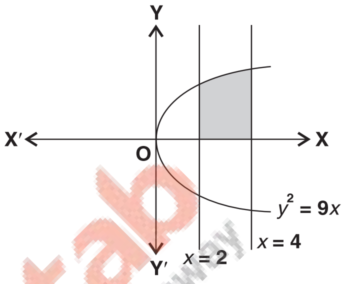
2. Find the area of the region bounded by $y^2 = 9x$, $x = 2$, $x = 4$ and the x-axis in the first quadrant.

Sol. Equation of (rightward parabola) curve is $y^2 = 9x$

$\therefore \quad y = \sqrt{9x} = 3\sqrt{x} \qquad ...(i)$

for branch of curve in first quadrant.

$\therefore$ Required (shaded) area bounded by curve $y^2 = 9x$, (vertical lines $x = 2$, $x = 4$), and x-axis in first quadrant



$= \left| \int_2^4 y\, dx \right| = \left| \int_2^4 3\sqrt{x}\, dx \right|$ (By (i))

$$= \left| 3\int_2^4 x^{\frac{1}{2}}\, dx \right| = 3\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_2^4 = 3\left(\frac{2}{3}\right)[4^{\frac{3}{2}} - 2^{\frac{3}{2}}]$$

$= 2(4\sqrt{4} - 2\sqrt{2})$ $\qquad [\because \ x^{\frac{3}{2}} = x\sqrt{x}\,]$

$= 2(8 - 2\sqrt{2}) = (16 - 4\sqrt{2})$ sq. units.

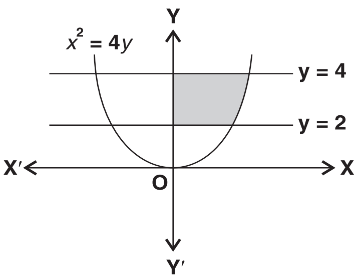
3. Find the area of the region bounded by $x^2 = 4y$, $y = 2$, $y = 4$ and the y-axis in the first quadrant.

Sol. Equation of (upward parabola) curve is $x^2 = 4y$

$\therefore \quad x = \sqrt{4y} = 2\sqrt{y} \qquad ...(i)$

for branch of curve in first quadrant.

$\therefore$ Required (shaded) area bounded by curve $x^2 = 4y$, (Horizontal lines $y = 2$, $y = 4$) and y-axis in first quadrant



$$= \left| \int_2^4 \boldsymbol{x\,dy} \right| = \left| \int_2^4 2\sqrt{y}\,dy \right| \qquad \text{(By } (i))$$

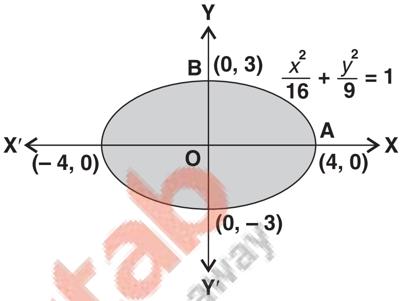
$$= \left| 2\int_2^4 y^{\frac{1}{2}}\,dy \right| = \left| 2\,\frac{\left(y^{\frac{3}{2}}\right)_2^4}{\frac{3}{2}} \right|$$

$$= \frac{4}{3}\left| (4^{\frac{3}{2}} - 2^{\frac{3}{2}}) \right| = \frac{4}{3}(4\sqrt{4} - 2\sqrt{2}) \qquad \left(\because\ x^{\frac{3}{2}} = x\sqrt{x}\right)$$

$$= \frac{4}{3}(4(2) - 2\sqrt{2}) = \left(\frac{32 - 8\sqrt{2}}{3}\right) \text{ sq. units.}$$

4. Find the area of the region bounded by the ellipse

$$\boldsymbol{\frac{x^2}{16} + \frac{y^2}{9} = 1.}$$



Sol. Equation of ellipse is

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \qquad ...(i)$$

Here $a^2(= 16) > b^2(= 9)$

From (i), $\frac{y^2}{9} = 1 - \frac{x^2}{16}$

$$= \frac{16 - x^2}{16}$$

$$\Rightarrow \qquad y^2 = \frac{9}{16}(16 - x^2)$$

$$\Rightarrow \qquad y = \frac{3}{4}\sqrt{16 - x^2} \qquad ...(ii)$$

for arc of ellipse in first quadrant.

Ellipse (i) is symmetrical about x-axis.

($\because$ On changing $y \to -y$ in (i), it remains unchanged).

Ellipse (i) is symmetrical about y-axis.

($\because$ On changing $x \to -x$ in (i), it remains unchanged)

Intersections of ellipse *(i)* with *x*-axis $(y = 0)$

Putting $y = 0$ in (i), $\frac{x^2}{16} = 1 \Rightarrow x^2 = 16 \Rightarrow x = \pm 4$

$\therefore$ Intersections of ellipse (i) with x-axis are $(4, 0)$ and $(-4, 0)$.

Intersections of ellipse *(i)* with *y*-axis $(x = 0)$

Putting $x = 0$ in (i), $\frac{y^2}{9} = 1 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$.

$\therefore$ Intersections of ellipse (i) with y-axis are $(0, 3)$ and $(0, -3)$.

$\therefore$ Area of region bounded by ellipse (i)

= Total shaded area

= **4** $\times$ Area OAB of ellipse in first quadrant

$= 4 \left| \int_0^4 y\, dx \right|$ ($\because$ At end B of arc AB of ellipse; $x = 0$ and at end A of arc AB; $x = 4$)

$= 4 \left| \int_0^4 \frac{3}{4} \sqrt{16 - x^2}\, dx \right|$ [By (ii)]

$= 3 \left| \int_0^4 \sqrt{4^2 - x^2}\, dx \right| = 3 \left[\frac{x}{2} \sqrt{4^2 - x^2} + \frac{4^2}{2} \sin^{-1} \frac{x}{4} \right]_0^4$

$\left[\because \int \sqrt{a^2 - x^2}\, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]$

$= 3 \left[\frac{4}{2} \sqrt{16 - 16} + 8 \sin^{-1} 1 - (0 + 8 \sin^{-1} 0) \right] = 3 \left[0 + \frac{8\pi}{2} \right]$

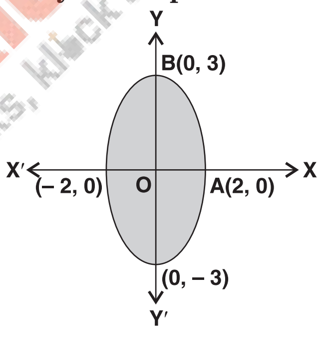
$\left(\because \sin \frac{\pi}{2} = 1 \Rightarrow \sin^{-1} 1 = \frac{\pi}{2} \text{ and } \sin 0 = 0 \Rightarrow \sin^{-1} 0 = 0 \right)$

$= 3(4\pi) = 12\pi$ sq. units.

Remark. We can also find area of this ellipse as $4 \left| \int_0^3 \boldsymbol{x\, dy} \right|$

5. Find the area of the region bounded by the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$



Sol. Equation of the ellipse is

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 \quad ...(i)$$

Here $a^2(= 4) < b^2(= 9)$

From (i), $\frac{y^2}{9} = 1 - \frac{x^2}{4} = \frac{4 - x^2}{4}$

$\Rightarrow \quad y^2 = \frac{9}{4}(4 - x^2) \quad \Rightarrow \quad y = \frac{3}{2} \sqrt{4 - x^2} \quad ...(ii)$

for arc of ellipse in first quadrant. Clearly ellipse (i) is symmetrical about x-axis and y-axis both.

[$\because$ On changing y to $-y$ in (i) or x to $-x$ in (i) keep it unchanged]

Intersections of ellipse (i) with x-axis ($y = 0$)

Putting $y = 0$ in (i), $\frac{x^2}{4} = 1 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

∴ Intersections of ellipse (*i*) with *x*-axis are (2, 0) and (– 2, 0)

Intersections of ellipse (*i*) with *y*-axis (*x* = 0)

Putting $x = 0$ in (*i*), $\frac{y^2}{9} = 1 \Rightarrow y^2 = 9 \Rightarrow y = \pm 3$

∴ Intersections of ellipse (*i*) with *y*-axis are (0, 3) and (0, – 3).

∴ Area of region bounded by ellipse (*i*)

= Total shaded area

= 4 × area OAB of ellipse in first quadrant

$= 4 \left| \int_0^2 y\, dx \right|$ (∵ At end B of arc AB of ellipse $x = 0$ and at end A of arc AB, $x = 2$)

$= 4 \left| \int_0^2 \frac{3}{2}\sqrt{4 - x^2}\, dx \right|$ (By (*ii*))

$= 4 \cdot \frac{3}{2} \left| \int_0^2 \sqrt{2^2 - x^2}\, dx \right| = 6 \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} \right]_0^2$

$\left[\because \int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]$

$= 6 \left[\frac{2}{2}\sqrt{4-4} + 2\sin^{-1} 1 - 0 - 2\sin^{-1} 0 \right]$

$= 6 \left[0 + 2 \cdot \frac{\pi}{2} - 0 \right] = 6\pi$ sq. units.

6. Find the area of the region in the first quadrant enclosed by *x*-axis, line $x = \sqrt{3}\, y$ and the circle $x^2 + y^2 = 4$.

Sol. Step I. To draw the graphs and shade the region whose area we are to find.

Equation of the circle is

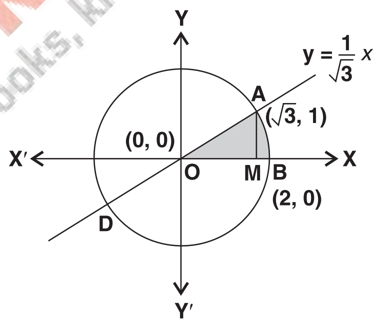
$x^2 + y^2 = 4 = 2^2$ ...(*i*)

We know that eqn. (*i*) represents a circle whose centre is (0, 0) and radius is 2.

Equation of the given line is

$x = \sqrt{3}\, y$

$\Rightarrow \quad y = \frac{1}{\sqrt{3}} x$ ...(*ii*)



We know that equation (*ii*) being of the form $y = mx$ where $m = \frac{1}{\sqrt{3}} = \tan 30° = \tan \theta \Rightarrow \theta = 30°$ represents a straight line passing through the origin and making angle of 30° with *x*-axis.

We are to find area of shaded region OAB in first quadrant (only).

Step II. Let us solve (*i*) and (*ii*) for *x* and *y* to find their points of intersection.

Putting $y = \frac{x}{\sqrt{3}}$ from (*ii*) in (*i*), $x^2 + \frac{x^2}{3} = 4$

$\Rightarrow \quad 3x^2 + x^2 = 12 \quad \Rightarrow \quad 4x^2 = 12 \quad \Rightarrow \quad x^2 = 3$

$\Rightarrow \quad x = \pm \sqrt{3}$

For $x = \sqrt{3}$, from (*ii*), $y = \frac{1}{\sqrt{3}} \sqrt{3} = 1$

For $x = -\sqrt{3}$, from (*ii*), $y = \frac{1}{\sqrt{3}}(-\sqrt{3}) = -1$

$\therefore$ The two points of intersections of circle (*i*) and line (*ii*) are A($\sqrt{3}$, 1) and D($-\sqrt{3}$, $-$ 1).

Step III. Now shaded area OAM between segment OA of line (*ii*) and *x*-axis

$= \left| \int_0^{\sqrt{3}} y\, dx \right|$ $\quad$ ($\because$ At O, $x = 0$ and at A, $x = \sqrt{3}$)

$= \left| \int_0^{\sqrt{3}} \frac{1}{\sqrt{3}} x\, dx \right|$ $\quad$ [By (*ii*)]

$= \frac{1}{\sqrt{3}} \left(\frac{x^2}{2} \right)_0^{\sqrt{3}} = \frac{1}{\sqrt{3}} \left(\frac{3}{2} - 0 \right) = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$ sq. units $\quad$ ...(*iii*)

Step IV. Now shaded area AMB between arc AB of circle and *x*-axis

$= \left| \int_{\sqrt{3}}^{2} y\, dx \right|$ $\quad$ ($\because$ at A, $x = \sqrt{3}$ and at B, $x = 2$)

$= \left| \int_{\sqrt{3}}^{2} \sqrt{2^2 - x^2}\, dx \right|$ $\quad$ (From (*ii*), $y^2 = 2^2 - x^2 \Rightarrow y = \sqrt{2^2 - x^2}$)

$= \left(\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2} \sin^{-1} \frac{x}{2} \right)_{\sqrt{3}}^{2}$

$\left[\because \int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]$

$= \left[\frac{2}{2}\sqrt{4-4} + 2\sin^{-1} 1 - \left(\frac{\sqrt{3}}{2}\sqrt{4-3} + 2\sin^{-1}\frac{\sqrt{3}}{2} \right) \right]$

$= 0 + 2 \cdot \frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2 \cdot \frac{\pi}{3}$ $\quad$ $\left[\because \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \frac{\pi}{3} = \sin^{-1}\frac{\sqrt{3}}{2} \right]$

$= \pi - \frac{\sqrt{3}}{2} - \frac{2\pi}{3} = \pi - \frac{2\pi}{3} - \frac{\sqrt{3}}{2} = \frac{3\pi - 2\pi}{3} - \frac{\sqrt{3}}{2}$

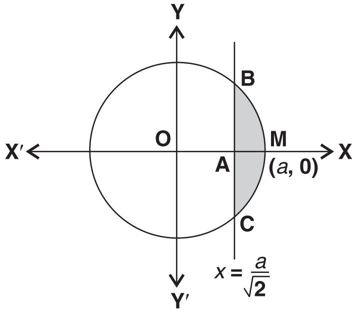
$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)$ sq. units. $\quad$ ...(*iv*)

Step V. Required shaded area OAB

$$= \text{Area OAM} + \text{Area AMB}$$

$$= \frac{\sqrt{3}}{2} + \left(\frac{\pi}{3} - \frac{\sqrt{3}}{2}\right) = \frac{\pi}{3} \text{ sq. units.} \qquad \text{[By } (iii) \text{ and } (iv)]$$

7. Find the area of the smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$.



Sol. Given: Equation of the circle is

$x^2 + y^2 = a^2$ ...(i)

$\therefore \quad y^2 = a^2 - x^2$

$\therefore \quad y = \sqrt{a^2 - x^2}$ ...(ii)

for arc BM of circle in Ist quadrant.

We know that equation (i) represents a circle whose centre is origin (0, 0) and radius a.

Clearly, circle (i) is symmetrical both about x-axis and y-axis.

We also know that graph of (vertical) line $x = \frac{a}{\sqrt{2}}$ is parallel to y-axis at a distance $\frac{a}{\sqrt{2}}$ ($< a$) to the right of origin.

$\therefore$ Area of smaller part of the circle $x^2 + y^2 = a^2$ cut off by the line $x = \frac{a}{\sqrt{2}}$ = Area ABMC = **2** × Area ABM

$$= 2\left|\int_{\frac{a}{\sqrt{2}}}^{a} y\,dx\right|$$

[$\because$ At point B (point of vertical line BC) $x = \frac{a}{\sqrt{2}}$ and at point M, x = radius a)

$$= 2\left|\int_{\frac{a}{\sqrt{2}}}^{a} \sqrt{a^2 - x^2}\,dx\right| \qquad \text{(By } (ii))$$

$$= 2\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]_{\frac{a}{\sqrt{2}}}^{a}$$

$$= 2\left[\frac{a}{2}\sqrt{a^2 - a^2} + \frac{a^2}{2}\sin^{-1}1 - \left(\frac{\frac{a}{\sqrt{2}}}{2}\sqrt{a^2 - \frac{a^2}{2}} + \frac{a^2}{2}\sin^{-1}\frac{\frac{a}{\sqrt{2}}}{a}\right)\right]$$

$$= 2\left[0 + \frac{a^2}{2}\frac{\pi}{2} - \frac{a}{2\sqrt{2}}\sqrt{\frac{a^2}{2}} - \frac{a^2}{2}\sin^{-1}\frac{1}{\sqrt{2}}\right]$$

$$= 2\left[\frac{\pi a^2}{4} - \frac{a}{2\sqrt{2}}\,\frac{a}{\sqrt{2}} - \frac{a^2}{2}\,\frac{\pi}{4}\right] \quad \left(\because\ \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} \Rightarrow \sin^{-1}\frac{1}{\sqrt{2}} = \frac{\pi}{4}\right)$$

$$= 2\left[\frac{\pi a^2}{4} - \frac{\pi a^2}{8} - \frac{a^2}{4}\right] \qquad [\because\ \sqrt{2}\,\sqrt{2} = (\sqrt{2})^2 = 2]$$

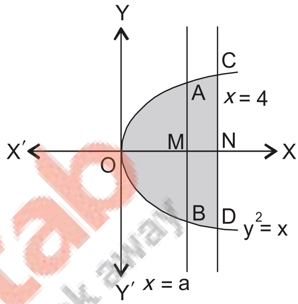
$$= 2a^2\left(\frac{\pi}{4} - \frac{\pi}{8} - \frac{1}{4}\right) = 2a^2\left(\frac{2\pi - \pi - 2}{8}\right)$$

$$= \frac{a^2}{4}(\pi - 2) = \frac{a^2}{2}\left(\frac{\pi}{2} - 1\right) \text{ sq. units.}$$

Note. It may be clearly noted that in this question No. 7 we were not to find only area AMB or only area AMC because x-axis is not given to be a boundary of the region in question whose area is required.

We have drawn x-axis here only as a line of reference because without drawing x-axis and y-axis as lines of reference, we can't draw any graph.

8. The area between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$, find the value of a.



Sol. Equation of the curve (rightward parabola) is

$$x = y^2 \quad i.e., \quad y^2 = x \qquad ...(i)$$

From (i), $y = \sqrt{x}$ ...(ii)

for arc OAC of parabola in first quadrant.

We know that equation (i) represents a right-ward parabola with symmetry about x-axis.

($\because$ Changing y to $-y$ in (i) keeps it unchanged)

Given: Area bounded by parabola (i) and vertical line $x = 4$ is divided into two equal parts by the vertical line $x = a$.

$\Rightarrow$ Area OAMB = Area AMBDNC.

$$\Rightarrow \quad 2\left|\int_0^a y\,dx\right| = 2\left|\int_a^4 y\,dx\right|$$

(For multipliction by 2 on each side, see **Note** above after solution of Q. No. 7)

Dividing by 2 and putting $y = \sqrt{x} = x^{\frac{1}{2}}$ from (ii),

$$\left|\int_0^a x^{\frac{1}{2}}\,dx\right| = \left|\int_a^4 x^{\frac{1}{2}}\,dx\right|$$

$$\Rightarrow \quad \frac{\left(x^{\frac{3}{2}}\right)_0^a}{\frac{3}{2}} = \frac{\left(x^{\frac{3}{2}}\right)_a^4}{\frac{3}{2}} \quad \Rightarrow \quad \frac{2}{3}\,[a^{\frac{3}{2}} - 0] = \frac{2}{3}\,[4^{\frac{3}{2}} - a^{\frac{3}{2}}]$$

Dividing both sides by $\frac{2}{3}$, $a^{\frac{-3}{2}} = 4\sqrt{4} - a^{\frac{-3}{2}}$

Transposing, $2a^{\frac{-3}{2}} = 8 \Rightarrow a^{\frac{-3}{2}} = 4 \Rightarrow a = 4^{\frac{-2}{3}}$.

9. Find the area of the region bounded by the parabola $y = x^2$ and $y = |x|$.

Sol. The required area is the area included between the parabola $y = x^2$ and the modulus function

$$y = |x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$$

We know that, the graph of the modulus function consists of two rays (*i.e.,* half lines $y = x$ for $x \geq 0$ and $y = -x$ for $x \leq 0$) passing through the origin and at right angles to each other. The half line $y = x$ if $x \geq 0$ has slope 1 and hence makes an angle of 45° with positive x-axis.

$y = x^2$ represents an upward parabola with vertex at origin.

The graphs of the two functions $y = x^2$ and $y = |x|$ are symmetrical about the y-axis.

[$\because$ Both equations remain unchanged on changing x to $-x$ as $|-x| = |x|$]

Let us first find the area between the parabola

$$y = x^2 \quad ...(i)$$

and the ray $\quad y = x$ for $x \geq 0 \quad ...(ii)$

To find limits of integration, let us solve (*i*) and (*ii*) for *x*.

Putting $y = x^2$ from (*i*) in (*ii*), we have $x^2 = x$

or $x^2 - x = 0$ or $x(x - 1) = 0 \therefore x = 0$ or $x = 1$

For $y = |x|$

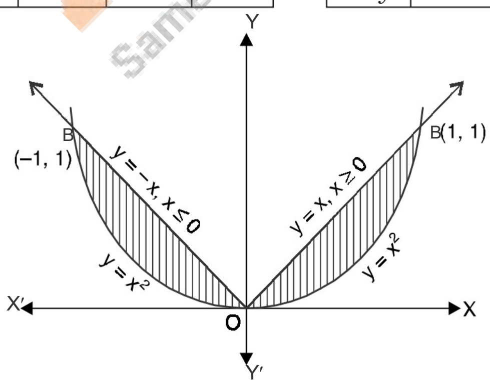
Table of values

$y = x$ if $x \geq 0$

x	0	1	2
y	0	1	2

$y = -x$ if $x \leq 0$

x	0	-1	-2
y	0	1	2



Area between parabola (i) and x-axis between limits

$x = 0$ and $x = 1$

$$= \int_0^1 y\,dx = \int_0^1 x^2\,dx = \left(\frac{x^3}{3}\right)_0^1 = \frac{1}{3} \qquad ...(iii)$$

Area between ray (ii) and x-axis,

$$= \int_0^1 y\,dx = \int_0^1 x\,dx = \left(\frac{x^2}{2}\right)_0^1 = \frac{1}{2} \qquad ...(iv)$$

$\therefore$ Required shaded area in first quadrant

= Area between ray $y = x$ for $x \geq 0$ and x-axis

– Area between parabola (i) and x-axis in first quadrant

= Area given by (iv) – Area given by (iii)

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units}$$

Similarly, shaded area in second quadrant = $\frac{1}{6}$ sq. units.

$\therefore$ Total area of shaded region in the above figure

$$= \frac{1}{6} + \frac{1}{6} = 2 \times \frac{1}{6} = \frac{1}{3} \text{ sq. units.}$$

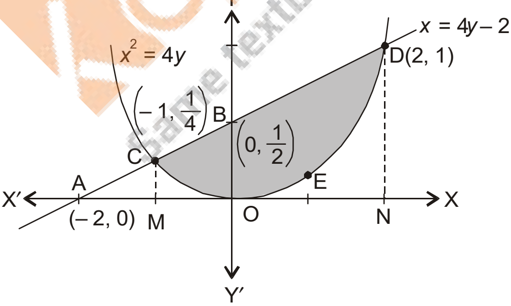
10. Find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Sol. Step I. Graphs and region of Integration.

Equation of the given curve is $x^2 = 4y$ ...(i)

We know that eqn. (i) reprsents an upward parabola symmetrical about y-axis

[$\because$ on changing x to $-x$ in (i), eqn. (i) remains unchanged]



Equation of the given line is

$$x = 4y - 2 \qquad ...(ii)$$

$\Rightarrow \quad x + 2 = 4y \qquad \Rightarrow \quad y = \frac{x+2}{4}$

Table of values for $x = 4y - 2$

x	0	-2
y	$\frac{1}{2}$	0

We are to find the area of the shaded region shown in the adjoining figure.

Step II. To find points of intersections of curve (*i*) and line (*ii*), let us solve (*i*) and (*ii*) for *x* and *y*.

Putting $y = \frac{x^2}{4}$ from (*i*) in (*ii*),

$$x = 4 \cdot \frac{x^2}{4} - 2 \Rightarrow x = x^2 - 2 \Rightarrow -x^2 + x + 2 = 0$$

or $\quad x^2 - x - 2 = 0$

$\Rightarrow \quad x^2 - 2x + x - 2 = 0$ or $x(x - 2) + (x - 2) = 0$

or $\quad (x - 2)(x + 1) = 0$

$\therefore \quad$ Either $x - 2 = 0$ or $x + 1 = 0$

i.e., $\quad x = 2$ or $x = -1$

For $x = 2$, from (*i*), $\quad y = \frac{x^2}{4} = \frac{4}{4} = 1 \quad \therefore \quad (2, 1)$

For $x = -1$, from (*i*), $\quad y = \frac{x^2}{4} = \frac{1}{4} \quad \therefore \left(-1, \frac{1}{4}\right)$.

$\therefore$ The two points of intersection of parabola (*i*) and line (*ii*) are $C\left(-1, \frac{1}{4}\right)$ and D(2, 1).

Step III. Area CMOEDN between parabola (*i*) and *x*-axis

$$= \left|\int_{-1}^{2} y\, dx\right| = \left|\int_{-1}^{2} \frac{x^2}{4}\, dx\right| \qquad \left(\because \text{ From } (i)\ y = \frac{x^2}{4}\right)$$

$$= \left|\frac{\left(x^3\right)_{-1}^{2}}{12}\right| = \left|\frac{1}{12}(2^3 - (-1)^3)\right| = \frac{1}{12}(8 - (-1))$$

$$= \frac{1}{12}(8 + 1) = \frac{9}{12} = \frac{3}{4} \text{ sq. units} \qquad ...(iii)$$

Step IV. Area of trapezium CMND between line (*ii*) and *x*-axis

$$= \left|\int_{-1}^{2} y\, dx\right| = \left|\int_{-1}^{2} \frac{x+2}{4}\, dx\right| = \left|\frac{1}{4}\int_{-1}^{2} (x+2)\, dx\right|$$

$$= \frac{1}{4}\left|\left(\frac{x^2}{2} + 2x\right)_{-1}^{2}\right| = \frac{1}{4}\left|\left(\frac{4}{2} + 4\right) - \left(\frac{1}{2} - 2\right)\right|$$

$$= \frac{1}{4}\left|2 + 4 - \frac{1}{2} + 2\right| = \frac{1}{4}\left|8 - \frac{1}{2}\right|$$

$$= \frac{1}{4}\left|\frac{16-1}{2}\right| = \frac{1}{4}\left(\frac{15}{2}\right) = \frac{15}{8} \text{ sq. units.} \quad ...(iv)$$

$\therefore$ Required shaded area

= Area given by (*iv*) – Area given by (*iii*)

= Area of trapezium CMND – Area (CMOEDN)

$$= \frac{15}{8} - \frac{3}{4} = \frac{15-6}{8} = \frac{9}{8} \text{ sq. units.}$$

11. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$.

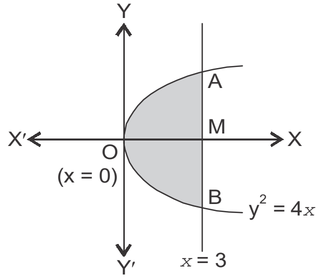
Sol. Equation of the (parabola) curve is

$y^2 = 4x$ ...(*i*)

$\therefore y = \sqrt{4x} = 2x^{\frac{1}{2}}$ ...(*ii*)

for arc OA of parabola in first quadrant.

We know that equation (*i*) represents a rightward parabola with symmetry about *x*-axis.



($\because$ Changing y to $-y$ in (*i*), keeps it unchanged)

$\therefore$ Required shaded area OAMB.

(See Note after solution of example 7)

= 2(Area OAM)

$$= 2\left|\int_0^3 y\,dx\right| = 2\left|\int_0^3 2x^{\frac{1}{2}}\,dx\right| \quad \text{(By } (ii))$$

$$= 4\left|\frac{\left(x^{\frac{3}{2}}\right)_0^3}{\frac{3}{2}}\right| = 4 \cdot \frac{2}{3}\left[3^{\frac{3}{2}} - 0\right] = \frac{8}{3}\,3\sqrt{3} = 8\sqrt{3} \text{ sq. units.}$$

12. Choose the correct answer:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is

(A) π **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi}{3}$ **(D)** $\frac{\pi}{4}$.

Sol. Equation of the circle is

$x^2 + y^2 = 4 = 2^2$ ...(*i*)

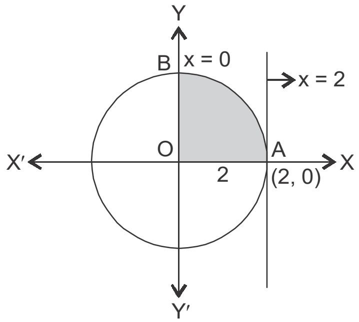
We know that equation (*i*) represents a circle whose centre is origin and radius is 2.

$\therefore \quad y^2 = 2^2 - x^2$

$\therefore \quad y = \sqrt{2^2 - x^2}$ ...(*ii*)

for arc AB of the circle in first quadrant.

$\therefore$ Required area lying in the first quadrant bounded by the



circle $x^2 + y^2 = 4$ and the (vertical) lines $x = 0$ and (tangent line) $x = 2$.

$$= \left|\int_0^2 y\,dx\right| = \left|\int_0^2 \sqrt{2^2 - x^2}\,dx\right| \qquad \text{By } (ii))$$

$$= \left|\left(\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right)_0^2\right|$$

$$\left[\because \int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$= \frac{2}{2}\sqrt{4 - 4} + 2\sin^{-1} 1 - (0 + 2\sin^{-1} 0)$$

$$= 0 + 2 \cdot \frac{\pi}{2} - 0 - 0 = \pi \text{ sq. units.}$$

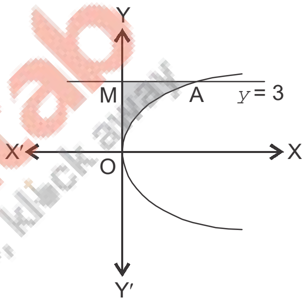
$[\because \ \sin 0 = 0 \ \Rightarrow \ \sin^{-1} 0 = 0]$

$\therefore$ Option (A) is the correct answer.

13. Choose the correct answer:
Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

(A) 2 **(B) $\frac{9}{4}$**

(C) $\frac{9}{3}$ **(D) $\frac{9}{2}$.**



Sol. Equation of the curve (rightward parabola) is

$$y^2 = 4x \qquad ...(i)$$

$\therefore$ Required area of the region bounded by parabola (i), y-axis and the (horizontal) line $y = 3$

= Area OAM

$$= \left|\int_0^3 x\,dy\right| \qquad ...(ii)$$

[$\because$ For arc OA of the parabola (i), at point O, $y = 0$ and at point A, $y = 3$]

Putting $x = \frac{y^2}{4}$ from (i) in (ii), required area

$$= \left|\int_0^3 \frac{y^2}{4}\,dy\right|$$

$$= \frac{1}{4}\left|\left(\frac{y^3}{3}\right)_0^3\right| = \frac{1}{4}\left|\frac{27}{3} - 0\right| = \frac{9}{4} \text{ sq. units}$$

$\therefore$ Option (B) is correct answer.