NCERT Class 12 Maths

Solutions

Chapter - 13

Exercise 13.5

1. A die is thrown 6 times. If 'getting an odd number' is a success, what is the probability of
(*i*) 5 successes? (*ii*) at least 5 successes?
(*iii*) at most 5 successes?

Sol. (By Definition 7 Page 818), We know that the repeated throws of a die are Bernoulli trials. Let X denote the number of successes when a die is thrown 6 times. Here $n = 6$.

p = P(a success) = P(an odd number on a die)

$= \frac{3}{6} = \frac{1}{2}$ ($\because$ 1, 3, 5 are the only odd outcomes out of a total of 6 namely {1, 2, 3, 4, 5, 6})

$\therefore \quad q = \text{P(a failure)} = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$

We know that **P(x successes in n trials) = $^nC_x\, q^{n-x}\, p^x$** ...(i)

$= {}^6C_x\, q^{6-x}\, p^x$ ($\because$ $n = 6$ here)

(i) Putting $x = 5$ in (i), P(5 successes) = P(X = 5) = $^6C_5 q^1 p^5$

$$= {}^6C_1 \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5 = 6 \times \frac{1}{64} = \frac{3}{32}.$$

[Using $^nC_r = {}^nC_{n-r}$; $^6C_5 = {}^6C_1$]

(ii) P(at least 5 successes) = P(X ≥ 5) = P(X = 5) + P(X = 6)

Putting $x = 5$ and $x = 6$ in (i)

$= {}^6C_5 q^1 p^5 + {}^6C_6 p^6$

$$= {}^6C_1\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^5 + 1\left(\frac{1}{2}\right)^6 \qquad [\because \ {}^nC_n = {}^nC_0 = 1]$$

$$= 6 \times \frac{1}{64} + \frac{1}{64} = \frac{7}{64}.$$

(*iii*) P(at most 5 successes) = P(X ≤ 5)

= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)

= 1 – P(X = 6)[∵ We know that P(X = 0) + P(X = 1) + ... + P(X = 5) + P(X = 6 = n) = 1]

$$= 1 - {}^6C_6 p^6 \ \text{[By (i)]} \quad = 1 - 1 \times \left(\frac{1}{2}\right)^6 = 1 - \frac{1}{64} = \frac{63}{64}.$$

Most Important Note. At least *k* successes ⇒ *k* or more than *k* successes. At most *k* successes ⇒ *k* or less than *k* successes upto *x* = 0]

2. A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability of two successes.

Sol. (By definition given at no. 7 in "Lesson at a glance" Page 818), We know that the repeated throws of a pair of dice are Bernoulli trials. Let X denote the number of successes when a pair of dice is thrown 4 times.

Here, $n = 4$

p = P(a success) = P(a doublet in a single throw of a pair of dice)

$$= \frac{6}{36} = \frac{1}{6}$$ [∵ (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) are the only doublets out of a total of 6 × 6 = 36 pairs]

$$\therefore \quad q = \text{P(a failure)} = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

We know that probability of x successes in n trials

$$= {}^nC_x \, q^{n-x} \, p^x = {}^4C_x \, q^{4-x} \, p^x$$

Putting $x = 2$ in (*i*); P(two successes) = P(X = 2)

$$= {}^4C_2 q^2 p^2 = \frac{4 \times 3}{2 \times 1}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right)^2 = \frac{25}{216}.$$

3. There are 5% defective items in a large bulk of items. What is the probability that a sample of 10 items will include not more than one defective item?

Sol. Let X denote the number of defective items in a sample of n = 10 items.

p = Probability of an item being **defective** (*i.e.*, probability of a success)

$$= \frac{5}{100} = \frac{1}{20}$$ is same for each trial (draw of an item)

$\therefore \quad q = 1 - p = 1 - \dfrac{1}{20} = \dfrac{19}{20}$. Also $n = 10 > 3$

$\therefore$ X has Binomial Distribution with $n = 10$ and $p = \dfrac{1}{20}$.

We know that

$$P(X = x \text{ defective items}) = {}^nC_x\, p^x\, q^{n-x}$$

$$= {}^{10}C_x \left(\frac{1}{20}\right)^x \left(\frac{19}{20}\right)^{10-x} \quad ...(i)$$

$\therefore$ Required probability that a sample of 10 items will not include more than one defective item

= P(either one defective item or no defective item)

= P(X = 0) + P(X = 1)

Putting $x = 0$ and $x = 1$ in eqn. (i),

= P(X = 0) + P(X = 1)

$$= {}^{10}C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^{10} + {}^{10}C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^9$$

$$= \left(\frac{19}{20}\right)^{10} + 10 \cdot \frac{1}{20} \cdot \left(\frac{19}{20}\right)^9 = \left(\frac{19}{20}\right)^9 \left[\frac{19}{20} + \frac{10}{20}\right]$$

$$= \frac{29}{20} \times \left(\frac{19}{20}\right)^9.$$

4. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. What is the probability that

(*i*) all the five cards are spades?

(*ii*) only 3 cards are spades?

(*iii*) none is a spade?

Sol. Let X denote the number of spades in the 5 cards drawn. Since the drawing is done with replacement, the trials are Bernoulli trials.

(By Definition 7 Page 818)

Here $n = 5$ (> 3).

p = P(a success) = P(a spade card is drawn) = $\dfrac{13}{52} = \dfrac{1}{4}$

[$\because$ We know that there are 13 spade cards in a pack of 52 cards]

$\therefore \quad q$ = P(a failure) = $1 - p = 1 - \dfrac{1}{4} = \dfrac{3}{4}$

We know that P(x successes in n trials)

$$= {}^nC_x\, q^{n-x}\, p^x = {}^5C_x\, q^{5-x}\, p^x \qquad (\because \ n = 5) \quad ...(i)$$

(*i*) Putting $x = 5$ in (i),

P(all the five cards are spades) = P(X = 5) = ${}^5C_5 p^5$

$$= \left(\frac{1}{4}\right)^5 = \frac{1}{1024}.$$

(*ii*) Putting $x = 3$ in (*i*),

P(only three cards are spades) = $P(X = 3) = {}^5C_3 q^2 p^3$

$$= {}^5C_2 \left(\frac{3}{4}\right)^2 \left(\frac{1}{4}\right)^3 \qquad [\because \ {}^nC_r = {}^nC_{n-r}]$$

$$= \frac{5 \times 4}{2 \times 1} \times \frac{9}{16} \times \frac{1}{64} = \frac{45}{512}.$$

(*iii*) Putting $x = 0$ in (*i*),

$$\text{P(none is a spade)} = P(X = 0) = {}^5C_0 q^5 = 1\left(\frac{3}{4}\right)^5 = \frac{243}{1024}.$$

5. The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

(*i*) none **(*ii*) not more than one**

(*iii*) more than one **(*iv*) at least one**

will fuse after 150 days of use.

Sol. Let X denote the number of bulbs fused after 150 days of use.

$n = 5$ (such bulbs) is finite.

Given: p = Probability that a bulb will fuse after 150 days of use (*i.e.,* prob. of a success)

$$= \frac{5}{100}$$

$$= \frac{1}{20} \text{ is same for each bulb being fused after 150 days}$$

$\therefore \quad q = 1 - p = 1 - \frac{1}{20} = \frac{19}{20}$. Also $n = 5 > 3$

$\therefore$ X has Binomial Distribution with $n = 5$ and $p = \frac{1}{20}$

P(X = x fused bulbs)

$$= {}^nC_x \, p^x \, q^{n-x} = {}^5C_x \left(\frac{1}{20}\right)^x \left(\frac{19}{50}\right)^{5-x} \qquad ...(i)$$

(*i*) Putting $x = 0$ in eqn. (*i*),

P(X = 0) *i.e.,* P(No bulb is fused)

$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^5 = \left(\frac{19}{20}\right)^5.$$

(*ii*) P(not more than one fused bulb)

$= P(X = 0 \text{ or } X = 1)$

$= P(X = 0) + P(X = 1)$

$$= {}^5C_0 \left(\frac{1}{20}\right)^0 \left(\frac{19}{20}\right)^5 + {}^5C_1 \left(\frac{1}{20}\right)^1 \left(\frac{19}{20}\right)^4$$

[By putting $x = 0$ and $x = 1$ in eqn. (*i*)]

$$= \left(\frac{19}{20}\right)^5 + 5 \cdot \frac{1}{20} \left(\frac{19}{20}\right)^4$$

$$= \left(\frac{19}{20}\right)^4 \left[\frac{19}{20} + \frac{5}{20}\right] = \frac{6}{5}\left(\frac{19}{20}\right)^4. \quad \left[\because \frac{19}{20} + \frac{5}{20} = \frac{24}{20} = \frac{6}{5}\right]$$

(*iii*) P(More than one fused bulb out of 5)

$= P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5)$

$= P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3)$
$+ P(X = 4) + P(X = 5) - [P(X = 0) + P(X = 1)]$

$= 1 - [P(X = 0) + P(X = 1)]$

$\left[\because \text{By Definition of P.D., } \sum_{x=0}^{5} P(X = x) = 1\right]$

$$= 1 - \frac{6}{5}\left(\frac{19}{20}\right)^4. \qquad \text{[By (ii) part]}$$

(*iv*) **P(at least one fused bulb) = 1 – P(no fused bulb)**

$$= 1 - \left(\frac{19}{20}\right)^5. \qquad \text{[By (i) part]}$$

6. A bag consists of 10 balls each marked with one of the digits 0 to 9. If four balls are drawn successively with replacement from the bag, what is the probability that none is marked with the digit 0?

Sol. Let X denote the number of balls drawn with the digit 0 marked on it out of the four balls drawn successively with replacement. Because the balls are being drawn with replacement, the trials are Bernoulli Trials.

p = Probability that ball is marked with digit 0 = $\frac{1}{10}$ is same for each trial (draw).

[∵ The balls are drawn one by one with replacement.]

$\left[p = \frac{1}{10}\right.$ because 0 is one of the 10 digits from 0 to 9]

$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$. Also $n = 4$.

$\therefore$ X has Binomial Distribution with $n = 4$ and $p = \frac{1}{10}$.

$\therefore$ P(none is marked with the digit 0) = **P(X = 0) = q^n** $= \left(\frac{9}{10}\right)^4$.

7. In an examination, 20 questions of true-false type are asked. Suppose a student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true'; if it falls tails, he answers 'false'. Find the probability that he answers at least 12 questions correctly.

Sol. Let X denote the number of questions answered correctly. The trials are Bernoulli trials (By Definition 7) with $n = 20$. (given)

We know that p = P(a success)

$$= \text{P (a coin falls heads } i.e., \text{ he answers 'true')} = \frac{1}{2}$$

$$\therefore \qquad q = \text{P(a failure)} = 1 - p = 1 - \frac{1}{2} = \frac{1}{2}$$

Please note here p is not necessarily the probability that he "answers correctly". It can be q also. Since here $p = q = \frac{1}{2}$. So it does not affect whether we take p or q as probability of "answers correctly".

We know that P(x correct answers) = ${}^nC_x\, q^{n-x}\, p^x$

$$= {}^{20}C_x \left(\frac{1}{2}\right)^{20-x} \left(\frac{1}{2}\right)^x = {}^{20}C_x \left(\frac{1}{2}\right)^{20-x+x}$$

$$= {}^{20}C_x \left(\frac{1}{2}\right)^{20} \qquad ...(i)$$

P(at least 12 questions are answered correctly)

$$= P(X \ge 12)$$

$$= P(X = 12) + P(X = 13) + P(X = 14) + ... + P(X = 20)$$

Putting x = 12, 13, ... 20 in (i);

$$= {}^{20}C_{12}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{13}\left(\frac{1}{2}\right)^{20} + {}^{20}C_{14}\left(\frac{1}{2}\right)^{20} + ... + {}^{20}C_{20}\left(\frac{1}{2}\right)^{20}$$

$$= \left(\frac{1}{2}\right)^{20} [{}^{20}C_{12} + {}^{20}C_{13} + {}^{20}C_{14} + ... + {}^{20}C_{20}].$$

8. Suppose X has a binomial distribution B$\left(6, \frac{1}{2}\right)$. Show that X = 3 is the most likely outcome.

Sol. X has a binomial distribution B $\left(6, \frac{1}{2}\right)$ = B(n, p)

$$\Rightarrow \quad n = 6, \quad p = \frac{1}{2} \qquad \therefore \quad q = 1 - p = \frac{1}{2}$$

We know that P(x successes in n trials)

$$= {}^nC_x\, q^{n-x}\, p^x = {}^6C_x \left(\frac{1}{2}\right)^{6-x} \left(\frac{1}{2}\right)^x$$

$$= {}^6C_x \left(\frac{1}{2}\right)^{6-x+x} = {}^6C_x \left(\frac{1}{2}\right)^6 \qquad ...(i)$$

Putting x = 0, 1, 2, 3, 4, 5, 6 in (i), we have

$$P(X = 0) = {}^6C_0\left(\frac{1}{2}\right)^6 = 1\left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

$$P(X = 1) = {}^6C_1\left(\frac{1}{2}\right)^6 = 6 \cdot \frac{1}{64} = \frac{6}{64}$$

$$P(X = 2) = {}^6C_2\left(\frac{1}{2}\right)^6 = \frac{6.5}{2.1} \cdot \frac{1}{64} = \frac{15}{64}$$

$$P(X = 3) = {}^6C_3\left(\frac{1}{2}\right)^6 = \frac{6.5.4}{3.2.1} \cdot \frac{1}{64} = \frac{20}{64}$$

$$P(X = 4) = {}^6C_4\left(\frac{1}{2}\right)^6 = {}^6C_2\left(\frac{1}{2}\right)^6 = \frac{6 \times 5}{2 \times 1} \times \frac{1}{64} = \frac{15}{64}$$

[Using, ${}^6C_4 = {}^6C_2$ as ${}^nC_r = {}^nC_{n-r}$]

$$P(X = 5) = {}^6C_5\left(\frac{1}{2}\right)^6 = {}^6C_1 \frac{1}{64} = \frac{6}{64}$$

$$P(X = 6) = {}^6C_6\left(\frac{1}{2}\right)^6 = 1\left(\frac{1}{2}\right)^6 = \frac{1}{64}$$

Clearly, P(X = 3) is the maximum among all $P(X = x_i)$ where

$$x_i = 0, 1, 2, 3, 4, 5, 6$$

$\Rightarrow$ X = 3 is the most likely outcome.

9. On a multiple choice examination with three possible answers for each of the five questions, what is the probability that a candidate would get four or more correct answers just by guessing?

Sol. Let X denote the number of correct answers just by guessing.

Here $n = 5$.

Since every multiple choice question has 3 options (answer) (given)

$\therefore$ p = P(a correct answer) = $\frac{1}{3}$

$\therefore$ $q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$

We know that P(x correct answers out of n questions)

${}^nC_x\, q^{n-x}\, p^x = {}^5C_x\, q^{5-x}\, p^x$ ($\because$ $n = 5$) ...(*i*)

Required probability of getting four or more correct answers

$$= P(X \geq 4) = P(X = 4) + P(X = 5)$$

Putting $x = 4$ and $x = 5$ in (*i*), $= {}^5C_4 qp^4 + {}^5C_5 p^5$

$$= {}^5C_1\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^4 + 1\left(\frac{1}{3}\right)^5$$

$$= 5 \times \frac{2}{243} + \frac{1}{243} = \frac{10}{243} + \frac{1}{243} = \frac{11}{243}.$$

10. A person buys a lottery ticket in 50 lotteries, in each of which his chance of winning a prize is $\frac{1}{100}$. What is the probability that he will win a prize

(*a*) at least once (*b*) exactly once (*c*) at least twice?

Sol. Let X denote the number of times a person wins a prize.

Here, $n = 50$ (given)

Given: $p = \text{P(person wins a prize)} = \dfrac{1}{100}$

$\therefore \quad q = 1 - p = 1 - \dfrac{1}{100} = \dfrac{99}{100}$

$\therefore$ P(x prizes in $n = 50$ lotteries) $= {}^nC_x\, q^{n-x}\, p^x = {}^{50}C_x\, q^{50-x}\, p^x$

(*a*) **P(person wins a prize at least once)**

$= P(X \geq 1) = 1 - P(X = 0)$

Putting $x = 0$ in (*i*), $= 1 - {}^{50}C_0\, q^{50} = 1 - \left(\dfrac{99}{100}\right)^{50}$.

(*b*) P(person wins a prize exactly once) = P(X = 1)

Putting $x = 1$ in (*i*),

$$= {}^{50}C_1 q^{49} p = 50\left(\frac{99}{100}\right)^{49}\left(\frac{1}{100}\right) = \frac{1}{2}\left(\frac{99}{100}\right)^{49}$$

(*c*) P(person wins a prize at least twice)

$= P(X \geq 2) = 1 - [P(X = 0) + P(X = 1)]$

Putting $x = 0$ and $x = 1$ in (*i*), $= 1 - [{}^{50}C_0 q^{50} + {}^{50}C_1 q^{49} p]$

$$= 1 - \left[1\left(\frac{99}{100}\right)^{50} + 50\left(\frac{99}{100}\right)^{49}\left(\frac{1}{100}\right)\right]$$

$$= 1 - \left(\frac{99}{100}\right)^{49}\left[\frac{99}{100} + \frac{50}{100}\right] = 1 - \left(\frac{99}{100}\right)^{49}\left(\frac{149}{100}\right).$$

11. Find the probability of getting 5 exactly twice in 7 throws of a die.

Sol. Let X denote the number of times 5 is thrown with a die.

Here, $n = 7$. (given)

$$p = \text{P(5 is thrown with a die)} = \frac{1}{6}$$

$\therefore \quad q = 1 - p = 1 - \dfrac{1}{6} = \dfrac{5}{6}$

We know that P(x successes, here 5's in $n = 7$ throws)

$= {}^7C_x\, q^{7-x}\, p^x$ ...(*i*)

Putting $x = 2$ in (*i*), we have required probability of getting 5 exactly twice

$$= {}^7C_2 q^5 p^2 = \frac{7 \times 6}{2 \times 1}\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right)^2 = 21\left(\frac{5}{6}\right)^5 \cdot \frac{1}{36} = \frac{7}{12}\left(\frac{5}{6}\right)^5.$$

Note. It may be noted that probability of throwing any of the six numbers with a single dice $= \dfrac{1}{6}$.

12. Find the probability of throwing at most 2 sixes in 6 throws of a single die.

Sol. Let X denote the number of times 6 is thrown with a single die. Here, $n = 6$

$$p = \text{P(6 is thrown with a single die)} = \frac{1}{6}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$$

We know that P(x successes, here 6's in $n = 6$ throws of a single dice) $= {}^6C_x\, q^{6-x}\, p^x$ ...(*i*)

$\therefore$ Required probability of throwing at most 2 sixes

$$= P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

Putting $x = 0, 1, 2$ in (*i*),

$$= {}^6C_0 q^6 + {}^6C_1 q^5 p + {}^6C_2 q^4 p^2$$

$$= 1\left(\frac{5}{6}\right)^6 + 6\left(\frac{5}{6}\right)^5\left(\frac{1}{6}\right) + \frac{6\times5}{2\times1}\left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)^2$$

$$= \left(\frac{5}{6}\right)^4\left[\left(\frac{5}{6}\right)^2 + \frac{5}{6} + \frac{15}{36}\right] = \frac{25+30+15}{36}\left(\frac{5}{6}\right)^4 = \frac{70}{36}\left(\frac{5}{6}\right)^4 = \frac{35}{18}\left(\frac{5}{6}\right)^4$$

13. It is known that 10% of certain articles manufactured are defective. What is the probability that in a random sample of 12 such articles, 9 are defective?

Sol. Let X denote the number of defective articles. Here, $n = 12$.

$$p = \text{P(an article is defective)} = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

We know that P(x defective articles in a sample of 12 articles)

$$= {}^{12}C_x\, q^{12-x}\, p^x \qquad | \ {}^nC_x\, q^{n-x}\, p^x$$

Putting $x = 9$; $\quad P(X = 9) = {}^{12}C_9 q^3 p^9 = {}^{12}C_3\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^9$

$$[\because \ {}^{12}C_9 = {}^{12}C_{12-9} = {}^{12}C_3]$$

$$= \frac{12\times11\times10}{3\times2\times1} \times \frac{9^3}{10^3} \times \frac{1}{10^9} = \frac{22\times9^3}{10^{11}}.$$

In each of the Exercises 14 and 15, choose the correct answer:

14. In a box containing 100 bulbs, 10 are defective. The probability that out of a sample of 5 bulbs, none is defective is

(A) 10^{-1} **(B)** $\left(\frac{1}{2}\right)^5$ **(C)** $\left(\frac{9}{10}\right)^5$ **(D)** $\frac{9}{10}$.

Sol. Let X denote the number of defective bulbs. Here, $n = 5$.

$$p = \text{P(a defective bulb)} = \frac{n(\text{E})}{n(\text{S})} = \frac{10}{100} = \frac{1}{10}$$

$$\therefore \quad q = 1 - p = 1 - \frac{1}{10} = \frac{9}{10}$$

We know that $\text{P}(\text{X} = 0) = q^n = \left(\frac{9}{10}\right)^5$

$\Rightarrow$ The correct option is (C).

15. The probability that a student is not a swimmer is $\frac{1}{5}$. Then the probability that out of five students, four are swimmers is

(A) $^5C_4 \left(\frac{4}{5}\right)^4 \frac{1}{5}$

(B) $\left(\frac{4}{5}\right)^4 \frac{1}{5}$

(C) $^5C_4 \frac{1}{5}\left(\frac{4}{5}\right)^5$

(D) None of these.

Sol. Let X denote the number of swimmers. Here, $n = 5$.

Given: The probability that a student is not a swimmer $= \frac{1}{5}$.

$$\therefore \quad p = \text{P(a student can swim)} = 1 - \frac{1}{5} = \frac{4}{5}, \; q = \frac{1}{5} \quad \text{(given)}$$

$\therefore$ $\text{P}(\text{X} = 4)$ (= value of $^nC_x\, q^{n-x}\, p^x$, when $n = 5$, $x = 4$)

$$= {}^5C_4 q p^4 = {}^5C_4 \left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$$

$\Rightarrow$ The correct option is (A).