## Exercise 12.2

1. **Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs ₹ 60/kg and Food Q costs ₹ 80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.**

**Sol.** **Step I. Mathematical formulation of L.P.P.**

Suppose Reshma mixes $x$ kg of food P and $y$ kg of food Q. The given data is condensed in the following table:

| Type of Food | Quantity (kg) | Cost (₹/kg) | Vitamin A (units/kg) | Vitamin B (units/kg) |
|---|---|---|---|---|
| P | $x$ | 60 | 3 | 5 |
| Q | $y$ | 80 | 4 | 2 |

Cost of mixture (in ₹) $= 60x + 80y$

Let $Z = 60x + 80y$

We have the following mathematical model for the given problem:

Minimise $Z = 60x + 80y$ ...(*i*)

subject to the constraints:

$3x + 4y \geq 8$ (Vitamin A constraint) ...(*ii*)

[**Given:** Vitamin A content of foods X and Y is at least (*i.e.*, ≥) 8 units]

$5x + 2y \geq 11$ (Vitamin B constraint) ...(*iii*)

[**Given:** Vitamin B content of foods X and Y is at least (*i.e.*, ≥) 11 units]

$x, y \geq 0$ [∵ Quantities of food can't be negative] ...(*iv*)

**Step II.** The constraint (*iv*), $x, y \geq 0$.

⇒ Feasible region is in first quadrant.

**Table of values for the line $3x + 4y = 8$ of constraint (*ii*)**

| | | |
|---|---|---|
| $x$ | 0 | $\frac{8}{3}$ |
| $y$ | 2 | 0 |

Let us draw the line joining the points (0, 2) and $\left(\frac{8}{3}, 0\right)$.

Let us test for origin ($x = 0$, $y = 0$) in constraint (*ii*) $3x + 4y \geq 8$, we have $0 \geq 8$ which is not true.

$\therefore$ The region for constraint (*ii*) is the half plane on non-origin side of the line $3x + 4y = 8$ *i.e.,* the region does not contain the origin.

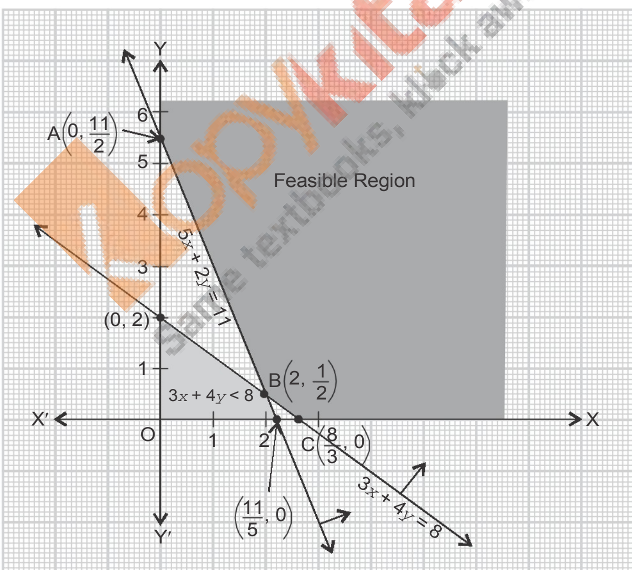
**Now table of values for the line $5x + 2y = 11$ of constraint (*iii*).**

| | | |
|---|---|---|
| $x$ | 0 | $\frac{11}{5}$ |
| $y$ | $\frac{11}{2}$ | 0 |

Let us draw the line joining the points $\left(0, \frac{11}{2}\right)$ and $\left(\frac{11}{5}, 0\right)$.

Let us test for origin ($x = 0$, $y = 0$) in constraint (*iii*) $5x + 2y \geq 11$, we have $0 \geq 11$ which is not true.

$\therefore$ Region for constraint (*iii*) is on the non-origin side of the line *i.e.,* does not contain the origin.




The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is unbounded.

**Step III.** The coordinates of the corner points A and C are A$\left(0, \frac{11}{2}\right)$ and C$\left(\frac{8}{3}, 0\right)$ respectively.

**Corner point B;** is the point of intersection of the lines

$$3x + 4y = 8 \quad \text{and} \quad 5x + 2y = 11$$

Solve for $x$ and $y$: First equation $-$ 2 $\times$ second equation gives $3x + 4y - 10x - 4y = 8 - 22$

$\Rightarrow \quad -7x = -14 \quad \Rightarrow \quad x = 2$

Putting $x = 2$ in $3x + 4y = 8$, we have, $6 + 4y = 8 \Rightarrow 4y = 2$

$\Rightarrow \quad y = \dfrac{2}{4} = \dfrac{1}{2}$. Therefore vertex $B\left(2, \dfrac{1}{2}\right)$.

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 60x + 80y$ | |
|---|---|---|
| $A\left(0, \dfrac{11}{2}\right)$ | 440 | |
| $B\left(2, \dfrac{1}{2}\right)$ | 160 | $= m$ ← Minimum |
| $C\left(\dfrac{8}{3}, 0\right)$ | 160 | |

From this table, we find that 160 is the minimum value of Z at each of the two corner points $B\left(2, \dfrac{1}{2}\right)$ and $C\left(\dfrac{8}{3}, 0\right)$.

**Step V.** Since the feasible region is unbounded, 160 may or may not be the minimum value of Z. To decide this, we graph the inequality $Z < m$

*i.e.,* $60x + 80y < 160$ or $3x + 4y < 8$

**Table of values for the line $3x + 4y = 8$ for this constraint $Z < m$.**

| | | |
|---|---|---|
| $x$ | 0 | $\dfrac{8}{3}$ |
| $y$ | 2 | 0 |

The line joining these two points (0, 2) and $\left(\dfrac{8}{3}, 0\right)$ has already been drawn for the line of constraint (*ii*).

Let us test for origin ($x = 0$, $y = 0$) in constraint $Z < m$

*i.e.,* $3x + 4y < 8$, we have $0 < 8$ which is true.

$\therefore$ Region for constraint $Z < m$ in the origin side of the line $3x + 4y = 8$.

Of course points on the line segment BC are included in the feasible region ($\because$ of constraint (*ii*)) and not in the half-plane determined by $Z < m$ *i.e.,* $3x + 4y < 8$. We observe that the half-plane determined by $Z < m$ has no point in common with the feasible region. Hence $m = 160$ is the minimum value of Z attained at each of the points $B\left(2, \dfrac{1}{2}\right)$ and $C\left(\dfrac{8}{3}, 0\right)$. Therefore, minimum cost = ₹ 160 at all points lying on the segment joining $\left(2, \dfrac{1}{2}\right)$ and $\left(\dfrac{8}{3}, 0\right)$.

2. **One kind of cake requires 200 g of flour and 25 g of fat, and another kind of cake requires 100 g of flour and 50 g of fat. Find the maximum number of cakes which can be made from 5 kg of flour and 1 kg of fat assuming that there is no shortage of the other ingredients used in making the cakes.**

**Sol. Step I. Mathematical Formulation of L.P.P.**

Let $x$ be the number of cakes of first kind and $y$, the number of cakes of other kind. The given data is condensed in the following table:

| Kind of cake | Number of cakes | Flour (gm/cake) | Fat (gm/cake) |
|---|---|---|---|
| I | $x$ | 200 | 25 |
| II | $y$ | 100 | 50 |

Total number of cakes = $x + y$ Let Z = $x + y$

We have the following mathematical model for the given problem:

Maximise Z = $x + y$ ...(*i*)

subject to the constraints:

$200x + 100y \le 5000$

(**Given:** (Maximum) amount of flour available for both types of cakes is 5 kg = 5000 gm)

Dividing by 100,

or $2x + y \le 50$ (Flour constraint) ...(*ii*)

$25x + 50y \le 1000$

(Fat constraint)

(**Given:** (Maximum) amount of fat available for both types of cakes is 1 kg = 1000 gm)

Dividing by 25,

or $x + 2y \le 40$ (Fat constraint) ...(*iii*)

$x, y \ge 0$ ...(*iv*)

($\because$ Number of cakes can't be negative)

**Step II.** The constraint (*iv*) $x, y \ge 0$.

$\Rightarrow$ Feasible region is in first quadrant.

**Table of values for the line $2x + y = 50$ of constraint (*ii*)**

| | | |
|---|---|---|
| $x$ | 0 | 25 |
| $y$ | 50 | 0 |

Let us draw the line joining the points (0, 50) and (25, 0).

Let us test for origin (0, 0) ($x$ = 0 and $y$ = 0) in constraint (*ii*) $2x + y \le 50$, we have $0 \le 50$ which is true.

$\therefore$ Region for constraint (*ii*) is towards the origin side of the line.

**Table of values for the line $x + 2y = 40$ of constraint (*iii*)**

| | | |
|---|---|---|
| $x$ | 0 | 40 |
| $y$ | 20 | 0 |

Let us draw the line joining the points (0, 20) and (40, 0).

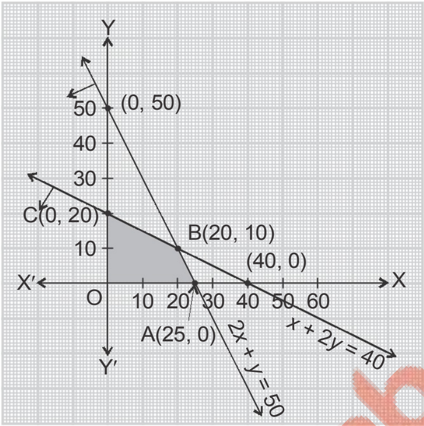
Let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $x + 2y \le 40$, we have $0 \le 40$ which is true.

$\therefore$Region for constraint (*iii*) is also towards the origin side of the line.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (25, 0) and (0, 20) respectively.

**Corner point B:** It is the point of intersection of the boundary lines $2x + y = 50$ and $x + 2y = 40$




Let us solve them for $x$ and $y$.

First equation – 2 × second equation gives

$2x + y - 2x - 4y = 50 - 80 \Rightarrow -3y = -30 \Rightarrow y = 10.$

Putting $y = 10$ in $2x + y = 50$

$\Rightarrow 2x + 10 = 50 \Rightarrow 2x = 40 \Rightarrow x = 20$

Therefore corner point B is (20, 10).

**Step IV.** Now we evaluate Z at each corner point.

| Corner Point | $Z = x + y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(25, 0) | 25 | |
| B(20, 10) | 30 = M | ← Maximum |
| C(0, 20) | 20 | |

By Corner Point Method, the maximum value of Z is 30 attained at the point B(20, 10).

Hence, maximum number of cakes = 30, 20 of first kind and 10 of second kind.

**3. A factory makes tennis rackets and cricket bats. A tennis racket takes 1.5 hours of machine time and 3 hours of craftman's time in its making while a cricket bat takes 3 hours of machine time and 1 hour of craftman's time. In a day, the factory has the availability of not more than 42 hours of machine time and 24 hours of craftsman's time.**

**(*i*) What number of rackets and bats must be made if the factory is to work at full capacity?**

**(*ii*) If the profit on a racket and on a bat is ₹ 20 and ₹ 10 respectively, find the maximum profit of the factory when it works at full capacity.**

**Sol. Step I. Mathematical Formulation of L.P.P.**

Suppose $x$ is the number of tennis rackets and $y$ is the number of cricket bats to be made in a day. The given data is condensed in the following table:

| Item | Number | Machine Time (hours/item) | Craftman's Time (hours/item) | Profit (₹) |
|---|---|---|---|---|
| Tennis Racket | $x$ | 1.5 | 3 | 20 |
| Cricket Bat | $y$ | 3 | 1 | 10 |

Total number of items = $x + y$ and total profit = $20x + 10y$

Let Z = $x + y$ and P = $20x + 10y$

We have the following mathematical model for the given problem:

Maximise Z = $x + y$ and P = $20x + 10y$ ...(*i*)

subject to the constraints:

$$1.5x + 3y \le 42 \quad \text{or} \quad \frac{3}{2}x + 3y \le 42$$

[**Given:** Number of machine hours available is not more than 42 hours *i.e.*, $\le 42$]

Dividing by 3 and multiplying by 2,

$x + 2y \le 28$ (Machine time constraint) ..(*ii*)

$3x + y \le 24$ (Craftman's time constraint) ...(*iii*)

[**Given:** Number of craftman's hours is not more than 24 hours *i.e.*, $\le 24$]

$x, y \ge 0$

($\because$ Number of tennis rackets and cricket bats can't be negative) ...(*iv*)

**Step II.** The constraint (*iv*) $x, y \ge 0$ $\Rightarrow$ Feasible region is in first quadrant.

**Table of values of equation $x + 2y = 28$ of constraint (*ii*)**

| | | |
|---|---|---|
| $x$ | 0 | 28 |
| $y$ | 14 | 0 |

Let us draw the straight line joining the points (0, 14) and (28, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*ii*)

*i.e.*, $x + 2y \le 28$; we have $0 \le 28$ which is true.

$\therefore$ Region for constraint (*ii*) is the region towards the origin side of the line $x + 2y = 28$.

**Table of values of equation $3x + y = 24$ of constraint (*iii*)**

| | | |
|---|---|---|
| x | 0 | 8 |
| y | 24 | 0 |

Let us draw the line joining the points (0, 24) and (8, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $3x + y \le 24$, we have $0 \le 24$ which is true.

$\therefore$ Region for constraint (*iii*) is the region towards the origin side of the line.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (8, 0) and (0, 14) respectively.

**Corner point B:** It is the point of intersection of the boundary lines

$$x + 2y = 28 \quad \text{and} \quad 3x + y = 24.$$

First eqn. – 2 × second eqn. gives

$$x + 2y - 2(3x + y) = 28 - 2 \times 24$$

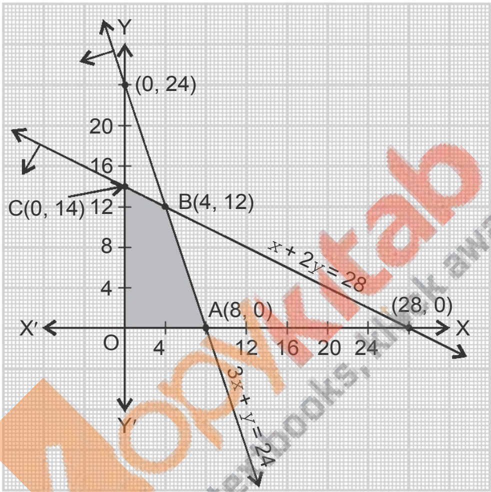
$\Rightarrow \quad x + 2y - 6x - 2y = 28 - 48 \quad \Rightarrow \quad -5x = -20$

$\Rightarrow \quad x = 4.$

Putting $x = 4$ in $x + 2y = 28$, $4 + 2y = 28$

$\Rightarrow \quad 2y = 24 \quad \Rightarrow \quad y = 12$

$\therefore$ Corner point B is (4, 12).





**Step IV.** (*i*) Now, we evaluate Z at each corner point.

| Corner Point | $Z = x + y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(8, 0) | 8 | |
| B(4, 12) | 16 = M | ← Maximum |
| C(0, 14) | 14 | |

By Corner Point Method, maximum Z = 16 at (4, 12).

(*ii*) Now, we evaluate P at each corner point.

| Corner Point | $P = 20x + 10y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(8, 0) | 160 | |
| B(4, 12) | 200 = M | ← Maximum |
| C(0, 14) | 140 | |

By Corner Point Method, maximum P = 200 at (4, 12).

Hence, the factory should make 4 tennis rackets and 12 cricket bats to make use of full capacity and then the profit is also maximum equal to ₹ 200.

**4. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of ₹ 17.50 per package on nuts and ₹ 7.00 per package on bolts. How many packages of each should be produced each day so as to maximise his profit, if he operates his machines for at the most 12 hours a day?**

**Sol. Sol. Step I. Mathematical Formulation of L.P.P.**

Suppose the manufacturer produces $x$ packages of nuts and $y$ packages of bolts each day. The given data is condensed in the following table:

| Item | Number of packages | Number of hours per package | | Profit |
|---|---|---|---|---|
| | | on Machine A | on Machine B | (₹/package) |
| Nuts | $x$ | 1 | 3 | 17.50 |
| Bolts | $y$ | 3 | 1 | 7.00 |

Total profit (in ₹) $= 17.5x + 7y$

Let $Z = 17.5x + 7y$

We have the following mathematical model for the given problem.

Maximise $Z = 17.5x + 7y$ ...(*i*)

subject to the constraints:

$x + 3y \le 12$ (Machine A constraint) ...(*ii*)

(**Given:** He operates his machine A for **at most** 12 hours *i.e.*, $\le$ 12 hours)

$3x + y \le 12$ (Machine B constraint) ...(*iii*)

(**Given:** He operates his machine B also for **at the most** 12 hours *i.e.*, $\le$ 12 hours)

$x, y \ge 0$ ...(*iv*)

($\because$ Number of packages of nuts and bolts can't be negative)

Constraint (*iv*) $x, y \ge 0$

$\Rightarrow$ Feasible region is in first quadrant.

**Step-II. Table of values for the line $x + 3y = 12$ of constraint (*ii*)**

| $x$ | 0 | 12 |
|---|---|---|
| $y$ | 4 | 0 |

Let us draw the straight line joining the points (0, 4) and (12, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*ii*).

$x + 3y \le 12$, we have $0 \le 12$ which is true.

$\therefore$ Region for constraint (*ii*) is the region on the origin side of the line $x + 3y = 12$.

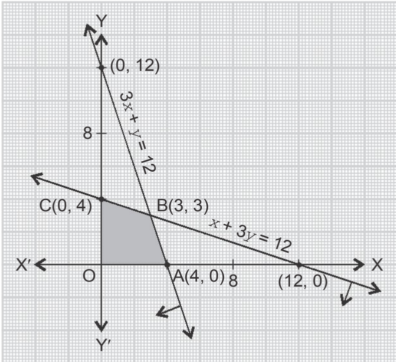
**Table of values for the line $3x + y = 12$ of constraint (*iii*)**

| $x$ | 0 | 4 |
|---|---|---|
| $y$ | 12 | 0 |

Let us draw the straight line joining the points (0, 12) and (4, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $3x + y \le 12$, we have $0 \le 12$ which is true.

$\therefore$ Region for constraint (*iii*) is also on the origin side of the line $3x + y = 12$.




The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (4, 0) and (0, 4) respectively.

**Corner point B:** It is the point of intersection of the boundary lines

$$x + 3y = 12 \quad \text{and} \quad 3x + y = 12$$

Solving them for $x$, $y$:

Ist Eqn. – 3 × second Eqn. gives

$$x + 3y - 3(3x + y) = 12 - 36$$

$$\Rightarrow \quad x + 3y - 9x - 3y = -24 \quad \Rightarrow \quad -8x = -24$$

$$\Rightarrow \quad x = \frac{-24}{-8} = 3$$

Putting $x = 3$ in $x + 3y = 12$, $3 + 3y = 12$

$$\Rightarrow \quad 3y = 9 \quad \Rightarrow \quad y = \frac{9}{3} = 3$$

∴ Corner point B is (3, 3).

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 17.5x + 7y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(4, 0) | 70 | |
| B(3, 3) | 73.5 = M | ← Maximum |
| C(0, 4) | 28 | |

By Corner Point Method, maximum Z = 73.5 at (3, 3).

Hence, the profit is maximum equal to ₹ 73.50 when 3 packages of nuts and 3 packages of bolts are manufactured.

**5. A factory manufactures two types of screws, A and B. Each type of screw requires the use of two machines, an automatic and a hand operated. It takes 4 minutes on the automatic and 6 minutes on hand operated machines to**

**manufacture a package of screws A, while it takes 6 minutes on automatic and 3 minutes on the hand operated machines to manufacture a package of screws B. Each machine is available for at the most 4 hours on any day. The manufacturer can sell a package of screws A at a profit of ₹ 7 and screws B at a profit of ₹ 10. Assuming that he can sell all the screws he manufactures, how many packages of each type should the factory owner produce in a day in order to maximise his profit? Determine the maximum profit.**

**Sol. Step I. Mathematical Formulation of L.P.P.**

Suppose the factory owner produces $x$ packages of screw A and $y$ packages of screw B in a day. The given data is condensed in the following table:

| Type of screw | Number of packages | Time in minutes per item | | Profit (₹/item) |
|---|---|---|---|---|
| | | on automatic machine | on hand operated machine | |
| A | $x$ | 4 | 6 | 7 |
| B | $y$ | 6 | 3 | 10 |

Total profit $= 7x + 10y$

Let $Z = 7x + 10y$

We have the following mathematical model for the given problem.

Maximise $Z = 7x + 10y$ ...(*i*)

subject to the constraints:

$4x + 6y \le 240$

[$\because$ Each machine *i.e.,* automatic machine is also available for atmost *i.e.,* $\le$ 4 hours *i.e.,* $4 \times 60 = 240$ minutes]

or $2x + 3y \le 120$ (Automatic machine constraint) ...(*ii*)

$6x + 3y \le 240$

(Same argument as given above for constraint (*ii*))

or $2x + y \le 80$ ...(*iii*)

(Hand operated machine constraint)

$x, y \ge 0$ ...(*iv*)

($\because$ Number of screws A and B can't be negative)

**Step II. Table of values for the line $2x + 3y = 120$ of constraint (*ii*)**

| $x$ | 0 | 60 |
|---|---|---|
| $y$ | 40 | 0 |

Let us draw the straight line joining the points (0, 40) and (60, 0).

Let us test for origin (put $x = 0$, $y = 0$) in constraint (*ii*) $2x + 3y \le 120$, we have $0 \le 120$ which is true.

$\therefore$ Region for constraint (*ii*) is on the origin side of the line

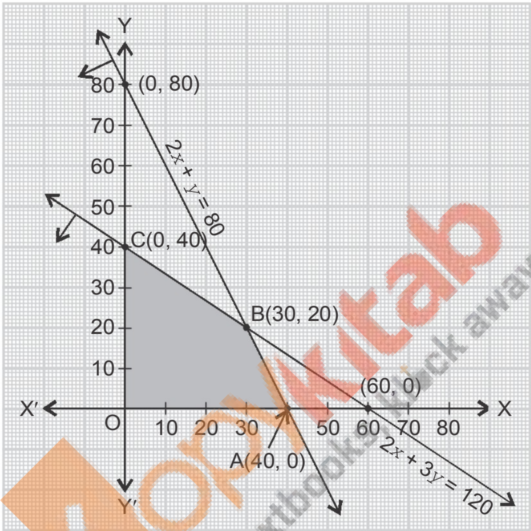
$2x + 3y = 120.$

**Table of values for the line $2x + y = 80$ of constraint (*iii*)**

| | | |
|---|---|---|
| $x$ | 0 | 40 |
| $y$ | 80 | 0 |

Let us draw the straight line joining the points (0, 80) and (40, 0). Let us test for origin (put $x = 0, y = 0$) in constraint (*iii*) $2x + y \leq 80$, we have $0 \leq 80$ which is true.

$\therefore$ Region for constraint (*iii*) is also towards the origin side of the line $2x + y = 80$.




The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (40, 0) and (0, 40) respectively.

**Corner Point B:** It is the point of intersection of boundary lines

$$2x + 3y = 120 \quad \text{and} \quad 2x + y = 80$$

Let us solve them for $x$ and $y$. Subtracting $2y = 40$

$\Rightarrow \quad y = 20$

Putting $y = 20$ in $2x + 3y = 120$; $2x + 60 = 120$

$\Rightarrow \quad 2x = 60 \quad \Rightarrow \quad x = 30.$

Therefore corner point B is (30, 20).

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 7x + 10y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(40, 0) | 280 | |
| B(30, 20) | 410 = M | ← Maximum |
| C(0, 40) | 400 | |

By Corner Point Method, maximum Z = 410 at (30, 20).

Hence, the profit is maximum equal to ₹ 410 when 30 packages of screws A and 20 packages of screws B are produced in a day.

**6. A cottage industry manufactures pedestal lamps and wooden shades, each requiring the use of a grinding/cutting machine and a sprayer. It takes 2 hours on grinding/cutting machine and 3 hours on the sprayer to manufacture a pedestal lamp. It takes 1 hour on the grinding/cutting machine and 2 hours on the sprayer to manufacture a shade. On any day, the sprayer is available for at the most 20 hours and the grinding/cutting machine for at the most 12 hours. The profit from the sale of a lamp is ₹ 5 and that from a shade is ₹ 3. Assuming that the manufacturer can sell all the lamps and shades that he produces, how should he schedule his daily production in order to maximise his profit?**

**Sol. Step I. Mathematical formulation of L.P.P.**

Suppose the manufacturer produces $x$ pedestal lamps and $y$ wooden shades. The given data is condensed in the following table:

| Item | Number | Time on grinding/ cutting machine (hrs/item) | Time on sprayer (hrs/item) | Profit (₹/item) |
|---|---|---|---|---|
| Pedestal lamps | $x$ | 2 | 3 | 5 |
| Wooden shades | $y$ | 1 | 2 | 3 |

Total profit = $5x + 3y$

Let $Z = 5x + 3y$

We have the following mathematical model for the given problem:

Maximise $Z = 5x + 3y$ ...(*i*)

subject to the constraints:

$2x + y \le 12$ (Grinding/cutting machine constraint) ...(*ii*)

[**Given:** Cutting/grinding machine is available for at the most (*i.e.,* $\le$) 12 hours]

$3x + 2y \le 20$ (Sprayer constraint) ...(*iii*)

[**Given:** The sprayer is available for at the most 20 hours *i.e.,* $\le$ 20]

$x, y \ge 0$ ...(*iv*) ($\because$ Number of pedestal lamps and wooden shades can't be negative)

**Step II.** The constraint (*iv*) $x, y \ge 0$ $\Rightarrow$ The feasible region is in first quadrant.

**Table of values for the line $2x + y = 12$ of constraint (*ii*)**

| $x$ | 0 | 6 |
|---|---|---|
| $y$ | 12 | 0 |

Let us draw the line joining the points (0, 12) and (6, 0).

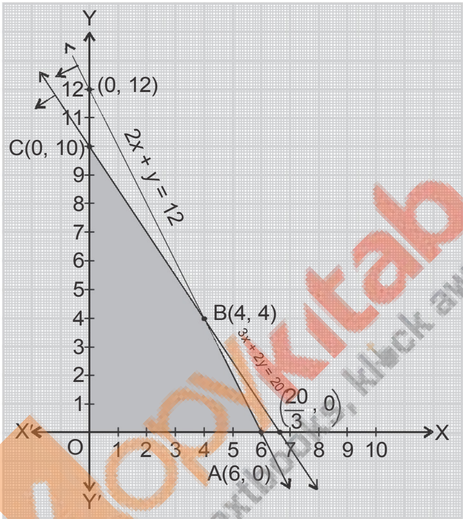
Let us test for origin ($x = 0$, $y = 0$) in constraint (*ii*) $2x + y \le 12$, we have $0 \leq 12$ which is true.

$\therefore$ Region for constraint (*ii*) is on the origin side of the line $2x + y = 12$.

**Table of values for the line $3x + 2y = 20$ of constraint (*iii*)**

| | | |
|---|---|---|
| $x$ | 0 | $\frac{20}{3}$ |
| $y$ | 10 | 0 |

Let us draw the line joining the points (0, 10) and $\left(\frac{20}{3}, 0\right)$.




Let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $3x + 2y \leq 20$, we have $0 \leq 20$ which is true.

$\therefore$ Region for constraint (*iii*) is on the origin side of the line

$$3x + 2y = 20.$$

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (6, 0) and (0, 10) respectively.

**Corner point B:** It is the point of intersection of boundary lines

$$2x + y = 12$$

and $$3x + 2y = 20$$

2 × First eqn. – Second eqn. gives

$$4x + 2y - 3x - 2y = 24 - 20 \quad \Rightarrow \quad x = 4.$$

Putting $x = 4$ in $2x + y = 12$, we have $8 + y = 12$

$\Rightarrow \quad y = 4.$

$\therefore$ Corner point B is (4, 4).

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 5x + 3y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(6, 0) | 30 | |
| B(4, 4) | 32 = M | ← Maximum |
| C(0, 10) | 30 | |

By Corner Point Method, maximum Z = 32 at (4, 4).

Hence, the profit is maximum when 4 pedestal lamps and 4 wooden shades are manufactured. Maximum profit is ₹ 32.

**7. A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours for assembling. The profit is ₹ 5 each for type A and ₹ 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximise the profit?**

**(*Important*)**

**Sol. Step I. Mathematical formulation of L.P.P.**

Suppose the company manufactures $x$ souvenirs of type A and $y$ souvenirs of type B. The given data is condensed in the following table:

| Type | Number | Time for cutting (min/item) | Time for assembling (min/item) | Profit (₹/item) |
|---|---|---|---|---|
| A | $x$ | 5 | 10 | 5 |
| B | $y$ | 8 | 8 | 6 |

Total profit = $5x + 6y$

Let $Z = 5x + 6y$

We have the following mathematical model for the given problem:

Maximise $Z = 5x + 6y$ ...(*i*)

subject to the constraints:

$5x + 8y \le 200$ (Cutting constraint) ...(*ii*)

[**Given:** (Maximum) time available for cutting is 3 hours, 20 minutes = $3 \times 60 + 20 = 200$ minutes]

$10x + 8y \le 240$ (Assembling constraint) ...(*iii*)

[**Given:** (Maximum) Time available for assembly is 4 hours $= 4 \times 60 = 240$ minutes]

$x, y \ge 0$ ...(*iv*)

($\because$ Number of souvenirs can't be negative)

**Step II.** Constraint (*iv*) $x, y \ge 0 \Rightarrow$ Feasible region is in first quadrant.

**Table of values for the line $5x + 8y = 200$ of constraint (*ii*)**

| $x$ | 0 | 40 |
|---|---|---|
| $y$ | 25 | 0 |

Let us draw the line joining the points (0, 25) and (40, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*ii*) $5x + 8y \leq 200$ we have $0 \leq 200$ which is true.

$\therefore$ Region for constraint (*ii*) is on the origin side of the line $5x + 8y = 200$.

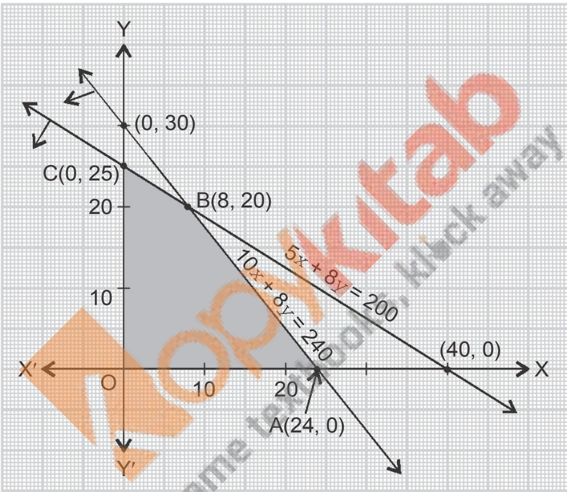
**Table of values for the line $10x + 8y = 240$ of constraint (*iii*)**

| $x$ | 0 | 24 |
|---|---|---|
| $y$ | 30 | 0 |

Let us draw the line joining the points (0, 30) and (24, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $10x + 8y \leq 240$, we have $0 \leq 240$ which is true.

$\therefore$ Region for constraint (*iii*) is on the origin side of the line $10x + 8y = 240$.




The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (24, 0) and (0, 25) respectively.

**Corner point B:** It is the point of intersection of the boundary lines

$$5x + 8y = 200 \quad \text{and} \quad 10x + 8y = 240$$

Subtracting, $\quad -5x = -40 \quad \Rightarrow \quad x = \dfrac{-40}{-5} = 8.$

Putting $x = 8$ in $5x + 8y = 200$, we have

$$40 + 8y = 200 \quad \Rightarrow \quad 8y = 160 \quad \Rightarrow \quad y = \frac{160}{8} = 20$$

$\therefore$ Corner point B(8, 20).

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 5x + 6y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(24, 0) | 120 | |
| B(8, 20) | 160 = M | ← Maximum |
| C(0, 25) | 150 | |

By Corner Point Method, maximum Z = 160 at (8, 20).

Hence, the profit is maximum when 8 souvenirs of type A and 20 souvenirs of type B are manufactured.

Maximum profit = ₹ 160.

**8. A merchant plans to sell two types of personal computers a desktop model and a portable model that will cost ₹ 25,000 and ₹ 40,000 respectively. He estimates that the total monthly demand of computers will not exceed 250 units. Determine the number of units of each type of computers which the merchant should stock to get maximum profit if he does not want to invest more than ₹ 70 lakhs and if his profit on the desktop model is ₹ 4500 and on portable model is ₹ 5000.**

**Sol. Step I. Mathematical Formulation of L.P.P.**

Suppose the merchant stocks $x$ units of desktop model and $y$ units of portable model. The given data is condensed in the following table.

| Type of Model | Number of units | Cost (₹/unit) | Profit (₹/unit) |
|---|---|---|---|
| Desktop | $x$ | 25000 | 4500 |
| Portable | $y$ | 40000 | 5000 |

Total profit $= 4500x + 5000y$

Let $Z = 4500x + 5000y$

We have the following mathematical model for the given problem:

Maximise profit $Z = 4500x + 5000y$ ...(*i*)

subject to the constraints:

$x + y \leq 250$ (Demand constraint) ...(*ii*)

[**Given:** Total monthly demand of computers will not exceed 250 *i.e.,* $\leq 250$]

$25000x + 40000y \leq 70,00,000$

[**Given:** He does not want to invest more than ₹ 70 lakhs = ₹ 70 × 100,000]

Dividing every term by 5000,

or $5x + 8y \leq 1400$ (Investment constraint) ...(*iii*)

$x, y \geq 0$ ...(*iv*)

($\because$ Number of computers can't be negative)

**Step II.** Constraint (*iv*) $x, y \geq 0$ $\Rightarrow$ Feasible region is in first quadrant.

**Table of values for the line $x + y = 250$ of constraint (*ii*)**

| | | |
|---|---|---|
| $x$ | 0 | 250 |
| $y$ | 250 | 0 |

Let us draw the line joining the points (0, 250) and (250, 0).

Let us test for origin ($x = 0$, $y = 0$) in constraint (*ii*) $x + y \le 250$, we have $0 \le 250$ which is true.

$\therefore$ Region for constraint (*ii*) is on the origin side of the line $x + y = 250$.

**Table of values for the line $5x + 8y = 1400$ of constraint (*iii*)**

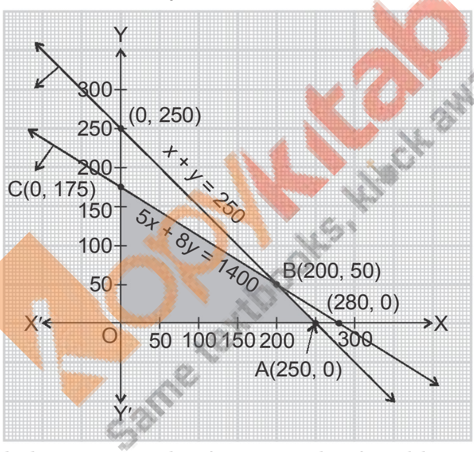
| | | |
|---|---|---|
| $x$ | 0 | 280 |
| $y$ | 175 | 0 |

Let us draw the line joining the points (0, 175) and (280, 0).

Let us test for origin (0, 0) in constraint (*iii*), $5x + 8y \le 1400$, we have $0 \le 1400$ which is true.

$\therefore$ Region for constraint (*iii*) is on the origin side of the line

$$5x + 8y = 1400.$$




The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is bounded.

**Step III.** The coordinates of the corner points O, A and C are (0, 0), (250, 0) and (0, 175) respectively.

**Corner point B:** It is the point of intersection of boundary lines:

$$x + y = 250 \quad \text{and} \quad 5x + 8y = 1400$$

Second Eqn. – 5 × Ist equation gives

$$5x + 8y - 5x - 5y = 1400 - 1250$$

or $\quad 3y = 150 \Rightarrow y = \frac{150}{3} = 50$

Putting $y = 50$ in $x + y = 250$,

we have $x + 50 = 250 \Rightarrow x = 200$

$\therefore$ Corner point B is (200, 50).

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 4500x + 5000y$ | |
|---|---|---|
| O(0, 0) | 0 | |
| A(250, 0) | 11,25,000 | |
| B(200, 50) | 11,50,000 = M | ← Maximum |
| C(0, 175) | 8,75,000 | |

By Corner Point Method, maximum Z = 11,50,000 at (200, 50).

Hence, the merchant should stock 200 units of desktop model and 50 units of portable model for a maximum profit of ₹ 11,50,000.

**9. A diet is to contain at least 80 units of vitamin A and 100 units of minerals. Two foods $F_1$ and $F_2$ are avialable. Food $F_1$ costs ₹ 4 per unit and food $F_2$ costs ₹ 6 per unit. One unit of food $F_1$ contains 3 units of vitamin A and 4 units of minerals. One unit of food $F_2$ contains 6 units of vitamin A and 3 units of minerals. Formulate this as a linear programming problem. Find the minimum cost for diet that consists of mixture of these two foods and also meets the minimal nutritional requirements.**

**Sol. Step I. Mathematical formulation of L.P.P.**

Suppose the diet contains $x$ units of food $F_1$ and $y$ units of food $F_2$. The given data is condensed in the following table:

| Type of Food | Number of units | Cost (₹/unit) | Vitamin A (units) | Minerals (units) |
|---|---|---|---|---|
| $F_1$ | $x$ | 4 | 3 | 4 |
| $F_2$ | $y$ | 6 | 6 | 3 |

Total cost = $4x + 6y$

Let $Z = 4x + 6y$

We have the following mathematical model for the given problem.

Minimise $Z = 4x + 6y$ ...(*i*)

subject to the constraints:

$3x + 6y \geq 80$ (Vitamin A constraint) ...(*ii*)

[**Given:** At least *i.e.,* ≥ 80 units of vitamin A]

$4x + 3y \geq 100$ (Mineral constraint) ...(*iii*)

[**Given:** At least *i.e.,* ≥ 100 units of minerals]

$x, y \geq 0$

($\because$ Units of vitamins and minerals can't be negative) ...(*iv*)

**Step II.** The constraint (*iv*) $x, y \geq 0$.

⇒ Feasible region is in first quadrant.

**Table of values for the line $3x + 6y = 80$ of constraint (*ii*)**

| | | |
|---|---|---|
| $x$ | 0 | $\frac{80}{3}$ |
| $y$ | $\frac{40}{3}$ | 0 |

Let us draw the line joining the points $\left(0, \dfrac{40}{3}\right)$ and $\left(\dfrac{80}{3}, 0\right)$.

Let us test for origin ($x = 0, y = 0$) in constraint (*ii*) $3x + 6y \geq 80$, we have $0 \geq 80$ which is not true.

$\therefore$ Region for constraint (*ii*) is the half-plane not containing the origin *i.e.,* region on the non-origin side of the line $3x + 6y = 80$.

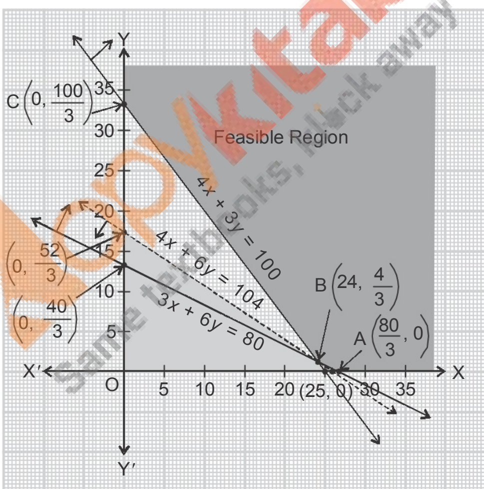
**Table of values for the line $4x + 3y = 100$ of constraint (*iii*)**

| $x$ | 0 | 25 |
|---|---|---|
| $y$ | $\dfrac{100}{3}$ | 0 |

Let us draw the line joining the points $\left(0, \dfrac{100}{3}\right)$ and (25, 0).

Let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $4x + 3y \geq 100$, we have $0 \geq 100$ which is not true.

$\therefore$ Region for constraint (*iii*) is the half-plane again on the non-origin side of the line $4x + 3y = 100$.




The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*).

The feasible region is unbounded.

**Step III.** The coordinates of the corner points A and C are

$\left(\dfrac{80}{3}, 0\right)$ and $\left(0, \dfrac{100}{3}\right)$ respectively.

**To find corner point B:** Corner point B is the point of intersection of the boundary lines

$$3x + 6y = 80 \qquad \text{and} \qquad 4x + 3y = 100$$

First Eqn. – 2 × Second eqn. gives

$$3x + 6y - 8x - 6y = 80 - 200$$

or $$-5x = -120 \Rightarrow x = \frac{-120}{-5} = 24$$

Putting $x = 24$ in $3x + 6y = 80$, we have

$$72 + 6y = 80 \Rightarrow 6y = 8 \Rightarrow y = \frac{8}{6} = \frac{4}{3}$$

∴ Corner point B is $\left(24, \frac{4}{3}\right)$.

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 4x + 6y$ | |
|---|---|---|
| $A\left(\frac{80}{3}, 0\right)$ | $\frac{320}{3}$ | |
| $B\left(24, \frac{4}{3}\right)$ | $104 = m$ | ← Smallest |
| $C\left(0, \frac{100}{3}\right)$ | 200 | |

From this table, we find that 104 is the smallest value of Z at the corner $B\left(24, \frac{4}{3}\right)$.

**Step V.** Since the feasible region is unbounded, 104 may or may not be the minimum value of Z. To decide this, we graph the inequality $Z < m$ *i.e.*, $4x + 6y < 104$.

**Table of values for the line $4x + 6y = 104$ (of constraint $Z < m$ *i.e.*, $4x + 6y < 104$)**

| $x$ | 0 | 26 |
|---|---|---|
| $y$ | $\frac{52}{3}$ | 0 |

Let us draw the dotted line joining the points $\left(0, \frac{52}{3}\right)$ and (26, 0).

[(26, 0) not being marked in the graph because it is very close to the point $\left(\frac{80}{3}, 0\right)$ = (26.7, 0) already marked and (26, 0) is slightly to the left of $\left(\frac{80}{3}, 0\right)$]

The line is shown dotted because equality sign is absent in the constraint $Z < m$.

Let us test for origin ($x = 0, y = 0$) in constraint $Z < m$ *i.e.*, $4x + 6y < 104$, we have $0 < 104$ which is true.

∴ Region for constraint $Z < m$ *i.e.*, $4x + 6y < 104$ is the origin side of the line $4x + 6y = 104$

We observe that the half plane determined by Z < $m$ has no point in common with the feasible region. Hence $m = 104$ is the minimum value of Z attained at the point $B\left(24, \frac{4}{3}\right)$.

$\therefore$ Minimum cost is ₹ 104 when 24 units of food $F_1$ are mixed with $\frac{4}{3}$ units of food $F_2$.

**10. There are two types of fertilisers $F_1$ and $F_2$. $F_1$ consists of 10% nitrogen and 6% phosphoric acid and $F_2$ consists of 5% nitrogen and 10% phosphoric acid. After testing the soil conditions, a farmer finds that she needs atleast 14 kg of nitrogen and 14 kg of phosphoric acid for her crop. If $F_1$ costs ₹ 6/kg and $F_2$ costs ₹ 5/kg, determine how much of each type of fertiliser should be used so that nutrient requirements are met at a minimum cost. What is the minimum cost?**

**Sol. Step I. Mathematical formulation of L.P.P.**

Suppose the farmer uses $x$ kg of fertiliser $F_1$ and $y$ kg of fertiliser $F_2$. The given data is condensed in the following table.

| Fertiliser | Quantity (kg) | Nitrogen content | Phosphoric acid content | Cost (₹/kg) |
|---|---|---|---|---|
| $F_1$ | $x$ | 10% | 6% | 6 |
| $F_2$ | $y$ | 5% | 10% | 5 |

Total cost $= 6x + 5y$

Let $Z = 6x + 5y$

We have the following mathematical model for the given problem:

Minimise $Z = 6x + 5y$ ...($i$)

subject to the constraints:

$$\frac{10}{100}x + \frac{5}{100}y \ge 14$$

[**Given:** She needs at least *i.e.,* $\ge$ 14 kg of nitrogen for her crops]

Multiplying by 100 and dividing by 5,

$$2x + y \ge 280 \quad \text{(Nitrogen constraint)} \quad ...(ii)$$

$$\frac{6}{100}x + \frac{10}{100}y \ge 14$$

[**Given:** She needs at least 14 kg of phosphoric acid for her crops]

Multiplying by 100 and dividing by 2,

$$3x + 5y \ge 700 \quad \text{(Phosphoric acid constraint)} \quad ...(iii)$$

$$x, y \ge 0 \quad ...(iv)$$

($\because$ Quantity of Nitrogen and Phosphoric acid can't be negative)

**Step II.** Constraint ($iv$) $x, y \ge 0$.

$\Rightarrow$ Feasible region is in first quadrant.

**Table of values for the line $2x + y = 280$ of constraint (*ii*)**

| | | |
|---|---|---|
| $x$ | 0 | 140 |
| $y$ | 280 | 0 |

Let us draw the line joining the points (0, 280) and (140, 0).
Let us test for origin ($x = 0$, $y = 0$) in constraint (*ii*) $2x + y \geq 280$, we have $0 \geq 280$ which is not true.

$\therefore$ Region for constraint (*ii*) is the half-plane not containing the origin *i.e.,* region on the non-origin side of the line $2x + y = 280$.

**Table of values for the line $3x + 5y = 700$ corresponding to constraint (*iii*)**

| | | |
|---|---|---|
| $x$ | 0 | $\frac{700}{3}$ |
| $y$ | 140 | 0 |

Let us draw the line joining the points (0, 140) and $\left(\frac{700}{3}, 0\right)$.

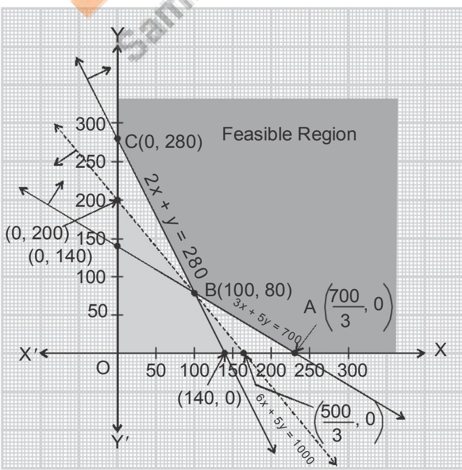
Let us test for origin ($x = 0$, $y = 0$) in constraint (*iii*) $3x + 5y \geq 700$, we have $0 \geq 700$ which is not true.

$\therefore$ Region for constraint (*iii*) is again on the non-origin side of the line $3x + 5y = 700$.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is unbounded.

**Step III.** The coordinates of the corner points. A and C are $\left(\frac{700}{3}, 0\right)$ and (0, 280) respectively.

**To find corner point B:** Let us solve the equations of bounding lines $2x + y = 280$ and $3x + 5y = 700$ for $x$ and $y$.




Second eqn. $-$ 5 $\times$ first eqn. gives

$$3x + 5y - 10x - 5y = 700 - 1400$$

$$\Rightarrow \quad -7x = -700 \Rightarrow x = \frac{-700}{-7} = 100$$

Putting $x = 100$ in $2x + y = 280$, we have

$200 + y = 280 \Rightarrow y = 80 \qquad \therefore$ Corner point B is (100, 80).

**Step IV.** Now, we evaluate Z at each corner point.

| Corner Point | $Z = 6x + 5y$ | |
|---|---|---|
| $A\left(\frac{700}{3}, 0\right)$ | 1400 | |
| B(100, 80) | $1000 = m$ | ← Smallest |
| C(0, 280) | 1400 | |

From this table, we find that 1000 is the smallest value of Z at the corner B(100, 80). Since the feasible region is unbounded, 1000 may or may not be the minimum value of Z.

**Step V.** To decide this, we graph the inequality $Z < m$

*i.e.,* $6x + 5y < 1000$.

**Table of values for the line $6x + 5y = 1000$ (for constraint $Z < m$ i.e., $6x + 5y < 1000$)**

| | | |
|---|---|---|
| $x$ | 0 | $\frac{500}{3}$ |
| $y$ | 200 | 0 |

Let us draw the dotted line joining the points (0, 200) and $\left(\frac{500}{3}, 0\right)$.

The line is drawn dotted because equality sign is absent in the constraint $Z < m$.

We observe that the half-plane determined by $Z < m$ has no point in common with the feasible region. Hence, $m = 1000$ is the minimum value of Z attained at the point B(100, 80).

$\therefore$ Minimum cost is ₹ 1000 when the farmer uses 100 kg of fertiliser $F_1$ and 80 kg of fertiliser $F_2$.

**11. The corner points of the feasible region determined by the following system of linear inequalities:**

**$2x + y \le 10$, $x + 3y \le 15$, $x, y \ge 0$ are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = px + qy$, where $p, q > 0$. Condition on $p$ and $q$ so that the maximum of Z occurs at both (3, 4) and (0, 5) is**

**(A) $p = q$ (B) $p = 2q$ (C) $p = 3q$ (D) $q = 3p$.**

**Sol.** We evaluate Z at each corner point.

| Corner Point | $Z = px + qy$, $p > 0$, $q > 0$ | |
|---|---|---|
| (0, 0) | 0 | |
| (5, 0) | $5p$ | |
| (3, 4) | $3p + 4q$ } = M | ← Maximum |
| (0, 5) | $5q$ } = M | ← Maximum |

$\because$ Maximum of Z occurs at both (3, 4) and (0, 5) (given)

$\therefore \quad 3p + 4q = 5q$

$\therefore \quad q = 3p$

Hence, the correct option is (D).