Exercise 12.1

Solve the following Linear Programming Problems graphically:

1. **Maximise Z = $3x + 4y$**
subject to the constraints: $x + y \leq 4, x \geq 0, y \geq 0$.

Sol. Maximise Z = $3x + 4y$ ...(*i*)

subject to the constraints:

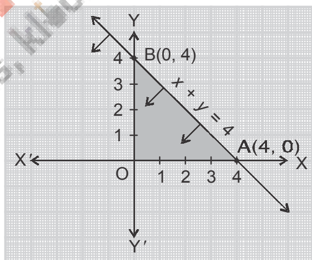
$$x + y \leq 4 \quad ...(ii)$$

$$x \geq 0, y \geq 0 \quad ...(iii)$$

Step I. Constraint (*iii*) namely $x \geq 0, y \geq 0 \Rightarrow$ Feasible region is in first quadrant.

Table of values for line $x + y = 4$ corresponding to constraint (*ii*)

x	0	4
y	4	0



So let us draw the line joining the points (0, 4) and (4, 0).

Now let us test for origin ($x = 0, y = 0$) in constraint (*ii*) $x + y \leq 4$. This gives us $0 \leq 4$ which is true. Therefore region for constraint (*ii*) is on the origin side of the line.

The shaded region in the figure is the feasible region determined by the system of constraints (*ii*) and (*iii*). The feasible region OAB is bounded.

Step II. The coordinates of the corner points O, A and B are (0, 0), (4, 0) and (0, 4) respectively.

Step III. Now we evaluate Z at each corner point.

Corner Point	Z = $3x + 4y$	
O(0, 0)	0	
A(4, 0)	12	
B(0, 4)	16 = M	← Maximum

Hence, by Corner Point Method, the maximum value of Z is 16 attained at the corner point B(0, 4).$\Rightarrow$Maximum Z = 16 at (0, 4).

2. Minimise Z = – 3x + 4y
subject to $x + 2y \le 8$, $3x + 2y \le 12$, $x \ge 0$, $y \ge 0$.

Sol. Minimise Z = $-3x + 4y$ ...(*i*)

subject to: $x + 2y \le 8$ ...(*ii*), $3x + 2y \le 12$ ...(*iii*), $x \ge 0$, $y \ge 0$..(*iv*)

Step I. Constraint (*iv*) namely $x \ge 0$, $y \ge 0$ $\Rightarrow$ Feasible region is in first quadrant.

Table of values for line $x + 2y = 8$ of constraint (*ii*)

x	0	8
y	4	0

Let us draw the line joining the points (0, 4) and (8, 0).

Now let us test for origin (0, 0) in constraint (*ii*) which gives $0 \le 8$ which is true.

$\therefore$ Region for constraint (*ii*) is on the origin side of the line.

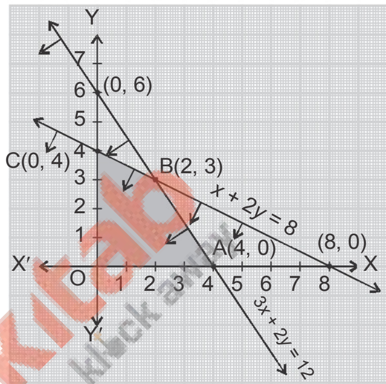


Table of values for line $3x + 2y = 12$ of constraint (*iii*)

x	0	4
y	6	0

Let us draw the line joining the points (0, 6) and (4, 0).

Now let us test for origin (0, 0) in constraint (*iii*) which gives $0 \le 12$ and which is true.

$\therefore$ Region for constraint (*iii*) is also on the origin side of the line. The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region OABC is bounded.

Step II. The coordinates of the corner points O, A and C are (0, 0), (4, 0) and (0, 4) respectively.

Now let us find corner point B, intersection of lines

$$x + 2y = 8 \quad \text{and} \quad 3x + 2y = 12$$

Subtracting $2x = 4 \Rightarrow x = \frac{4}{2} = 2$.

Putting $x = 2$ in first equation $2 + 2y = 8$

$\Rightarrow 2y = 6 \Rightarrow y = 3$

$\therefore$ Corner point B is (2, 3)

Step III. Now let us evaluate Z at each corner point.

Corner Point	Z = $-3x + 4y$	
O(0, 0)	0	
A(4, 0)	$-12 = m$	← Minimum
B(2, 3)	6	
C(0, 4)	16	

Hence, by Corner Point Method, the minimum value of Z is – 12 attained at the point A(4, 0).

$\Rightarrow$ Minimum Z = – 12 at (4, 0).

3. Maximise Z = 5*x* + 3*y*
subject to $3x + 5y \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.

Sol. Maximise $Z = 5x + 3y$ ...(*i*)

subject to:

$3x + 5y \leq 15$ ...(*ii*)

$5x + 2y \leq 10$ ...(*iii*)

$x \geq 0, y \geq 0$ ...(*iv*)

Step I. Constraint (*iv*) namely $x \geq 0$ and $y \geq 0$

$\Rightarrow$ Feasible region is in first quadrant.

Table of values for line $3x + 5y = 15$ of constraint (*ii*)

x	0	5
y	3	0

Let us draw the line joining the points (0, 3) and (5, 0).

Let us test for origin (0, 0) in constraint (*ii*) which gives $0 \leq 15$ and which is true.

$\therefore$ Region for constraint (*ii*) contains the origin.

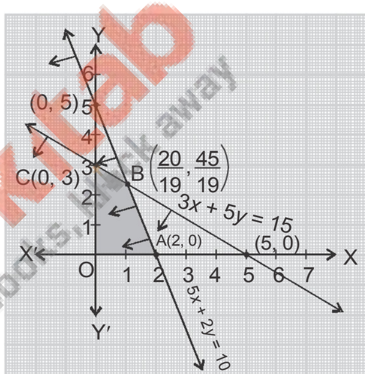


Table of values for line $5x + 2y = 10$ of constraint (*iii*).

x	0	2
y	5	0

Let us draw the line joining the points (0, 5) and (2, 0).

Let us test for origin (0, 0) in constraint (*iii*) which gives $0 \leq 10$ and which is true.

$\therefore$ Region for constraint (*iii*) also contains the origin.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) and (*iv*). The feasible region OABC is bounded.

Step II. The coordinates of the corner points O, A and C are (0, 0), (2, 0) and (0, 3) respectively.

Now **let us find corner point B;** intersection of lines

$3x + 5y = 15$ and $5x + 2y = 10$

Ist eqn. × 2 – IInd eqn. × 5 gives $-19x = -20 \Rightarrow x = \frac{20}{19}$

Putting $x = \frac{20}{19}$ in first eqn. $\Rightarrow \frac{60}{19} + 5y = 15$

$\Rightarrow \quad 5y = 15 - \frac{60}{19} = \frac{285 - 60}{19} = \frac{225}{19}$

$\Rightarrow \quad y = \frac{45}{19}$. Therefore corner point $B\left(\frac{20}{19}, \frac{45}{19}\right)$.

Step III. Now we evaluate Z at each corner point.

Corner Point	Z = 5x + 3y	
O(0, 0)	0	
A(2, 0)	10	
$B\left(\frac{20}{19}, \frac{45}{19}\right)$	$\frac{100+135}{19} = \frac{235}{19} = M$	← Maximum
C(0, 3)	9	

Hence, by Corner Point Method, the maximum value of Z is $\frac{235}{19}$ attained at the corner point $B\left(\frac{20}{19}, \frac{45}{19}\right)$.

$\Rightarrow \quad$ Maximum Z = $\frac{235}{19}$ at $\left(\frac{20}{19}, \frac{45}{19}\right)$.

4. Minimise Z = 3*x* + 5*y*

such that *x* + 3*y* ≥ 3, *x* + *y* ≥ 2, *x*, *y* ≥ 0.

Sol. Minimise Z = $3x + 5y$ ...(*i*)

such that: $x + 3y \geq 3$ ...(*ii*), $x + y \geq 2$ ...(*iii*), $x, y \geq 0$ ...(*iv*)

Step I. The constraint (*iv*) $x, y \geq 0 \Rightarrow$ Feasible region is in first quadrant.

Table of values for line *x* + 3*y* = 3 of constraint (*ii*)

x	0	3
y	1	0

Let us draw the line joining the points (0, 1) and (3, 0).

Now let us test for origin ($x = 0, y = 0$) in constraint (*ii*) $x + 3y \geq 3$, which gives us $0 \geq 3$ and which is not true.

∴ Region for constraint (*ii*) does not contain the origin *i.e.,* the region for constraint (*ii*) is **not** the origin side of the line.

Table of values for line *x* + *y* = 2 of constraint (*iii*)

x	0	2
y	2	0

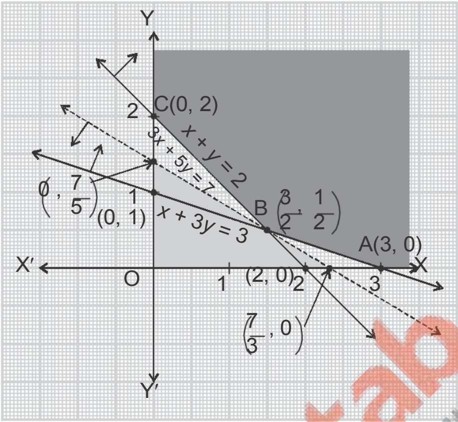
Let us draw the line joining the points (0, 2) and (2, 0).

Now let us test for origin ($x = 0, y = 0$) in constraint (*iii*), $x + y \geq 2$, which gives us $0 \geq 2$ and which is not true.

∴ Region for constraint (*iii*) does not contain the origin *i.e.,* is **not** the origin side of the line.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is unbounded.

Step II. The coordinates of the corner points A and C are (3, 0) and (0, 2) respectively.



Now let us find corner point B, the point of intersection of lines

$$x + 3y = 3 \quad \text{and} \quad x + y = 2$$

Subtracting, $2y = 1 \Rightarrow y = \frac{1}{2}$.

Putting $y = \frac{1}{2}$ in $x + y = 2$, we have $x = 2 - y = 2 - \frac{1}{2} = \frac{3}{2}$

$\therefore$ Corner point B is $\left(\frac{3}{2}, \frac{1}{2}\right)$.

Step III. Now, we evaluate Z at each corner point.

Corner Point	Z = 3x + 5y	
A(3, 0)	9	
B$\left(\frac{3}{2}, \frac{1}{2}\right)$	$\frac{9}{2} + \frac{5}{2} = 7 = m$	← Smallest
C(0, 2)	10	

From this table, we find that 7 is the smallest value of Z at the corner B$\left(\frac{3}{2}, \frac{1}{2}\right)$. Since the feasible region is unbounded, 7 may or may not be the minimum value of Z.

Step IV. To decide this, we graph the inequality $Z < m$

i.e., $3x + 5y < 7$.

Table of values for line $3x + 5y = 7$ corresponding to constraint $3x + 5y < 7$

x	0	$\frac{7}{3}$
y	$\frac{7}{5}$	0

Let us draw the dotted line joining the

points $\left(0, \frac{7}{5}\right)$ and $\left(\frac{7}{3}, 0\right)$. This line is to be shown dotted as constraint involves < and not ≤, so boundary of line is to be excluded.

Let us test for origin ($x = 0$, $y = 0$) in constraint $3x + 5y < 7$, we have $0 < 7$ which is true. Therefore region for this constraint is on the origin side of the line $3x + 5y = 7$.

We observe that the half-plane determined by $Z < m$ has no point in common with the feasible region. Hence $m = 7$ is the minimum value of Z attained at the point $B\left(\frac{3}{2}, \frac{1}{2}\right)$.

$\Rightarrow$ Minimum Z = 7 at $\left(\frac{3}{2}, \frac{1}{2}\right)$.

5. Maximise Z = 3*x* + 2*y*

subject to $x + 2y \le 10$, $3x + y \le 15$, $x, y \ge 0$.

Sol. Maximise $Z = 3x + 2y$ ...(*i*)

subject to:

$x + 2y \le 10$ ...(*ii*), $3x + y \le 15$ ...(*iii*), $x, y \ge 0$ ...(*iv*)

Step I. Constraint (*iv*) $x, y \ge 0$ $\Rightarrow$ Feasible region is in first quadrant.

Table of values for the line $x + 2y = 10$ corresponding to constraint (*ii*)

x	0	10
y	5	0

Let us draw the line joining the points (0, 5) and (10, 0).

Let us test for origin ($x = 0$, $y = 0$) in constraint (*ii*), we have $0 \le 10$ which is true.

∴ Region for constraint (*ii*) is on the origin side of this line.

Table of values for line $3x + y = 15$ corresponding to constraint (*iii*)

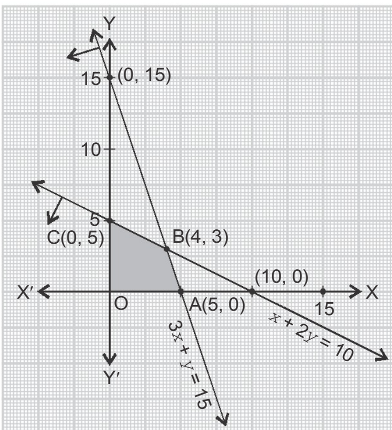
x	0	5
y	15	0

Let us draw the line joining the points (0, 15) and (5, 0).

Let us test for origin ($x = 0$, $y = 0$) in constraint (*iii*), we have $0 \le 15$ which is true.

∴ Region for constraint (*iii*) is on the origin side of this line.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region OABC is bounded.



Step II. The coordinates of the corner points O, A and C are (0, 0), (5, 0) and (0, 5) respectively.

Now let us find corner point B, intersection of the lines

$$x + 2y = 10$$

and $$3x + y = 15$$

First equation – 2 × second equation gives

$-5x = 10 - 30 \Rightarrow -5x = -20 \Rightarrow x = 4$

Putting $x = 4$ in $x + 2y = 10$, we have

$4 + 2y = 10 \Rightarrow 2y = 6 \Rightarrow y = 3$

∴ Corner point B is B(4, 3).

Step III. Now we evaluate Z at each corner point.

Corner Point	$Z = 3x + 2y$	
O(0, 0)	0	
A(5, 0)	15	
B(4, 3)	18 = M	← Maximum
C(0, 5)	10	

Hence, by Corner Point Method, the maximum value of Z is 18 attained at the point B(4, 3).

⇒ Maximum Z = 18 at (4, 3).

6. Minimise $Z = x + 2y$
subject to $2x + y \geq 3$, $x + 2y \geq 6$, $x, y \geq 0$.
Show that the minimum of Z occurs at more than two points.

Sol. Minimise $Z = x + 2y$ ...(*i*)

subject to:

$2x + y \geq 3$ ...(*ii*), $x + 2y \geq 6$ ...(*iii*), $x, y \geq 0$ ...(*iv*)

Step I. Constraint (*iv*) $x, y \geq 0$ ⇒ Feasible region is in first quadrant.

Table of values for the line $2x + y = 3$ corresponding to constraint (*ii*).

x	0	$\frac{3}{2}$
y	3	0

Let us draw the line joining the points (0, 3) and $\left(\frac{3}{2}, 0\right)$.

Now let us test for origin ($x = 0, y = 0$) in constraint (*ii*) $2x + y \geq 3$, we have $0 \geq 3$ which is not true.

∴ The region of constraint (*ii*) is on that side of the line which does not contain the origin *i.e.,* the region other than the origin side of the line.

Table of values for the line $x + 2y = 6$ corresponding to constraint (*ii*).

x	0	6
y	3	0

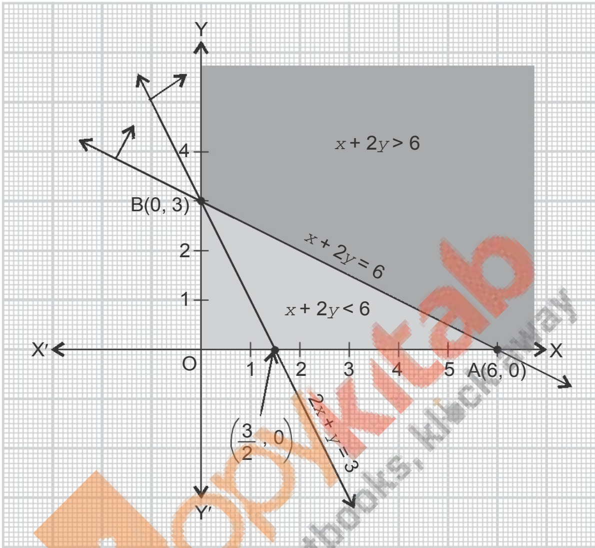
Let us draw the line joining the points (0, 3) and (6, 0).

Now let us test for origin ($x = 0, y = 0$) in constraint (*iii*) $x + 2y \geq 6$, we have $0 \geq 6$ which is not true.

$\therefore$ Region for constraint (*iii*) is the region other than the origin side of the line *i.e.,* region not containing the origin.

The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*iv*). The feasible region is unbounded.

Step II. The coordinates of the corner points A and B are (6, 0) and (0, 3) respectively.



Step III. Now, we evaluate Z at each corner point.

Corner Point	$Z = x + 2y$	
A(6, 0)	6	$= m$ ← Smallest
B(0, 3)	6	

From this table, we find that 6 is the smallest value of Z at each of the two corner points. Since the feasible region is unbounded, 6 may or may not be the minimum value of Z.

Step IV. To decide this, we graph the inequality $Z < m$ *i.e.,* $x + 2y < 6$.

The line $x + 2y = 6$ for this constraint $Z < m$ ($\Rightarrow x + 2y < 6$) is the same as the line AB for constraint (*iii*).

Let us test for origin ($x = 0, y = 0$) for this constraint, we have $0 < 6$ which is true.

Therefore region for this constraint is the (half-plane on) origin side of this line.

Points on the line segment AB are included in the feasible region and not in the half-plane determined by $x + 2y < 6$.

We observe that the half-plane determined by $Z < m$ has no point in common with the feasible region. Hence $m = 6$ is the minimum value of Z attained at each of the points A(6, 0) and B(0, 3).

$\Rightarrow$ Minimum Z = 6 at (6, 0) and (0, 3).

Remark. In fact, Z = 6 at all points on the line segment AB for example $\left(1, \frac{5}{2}\right)$, (2, 2), $\left(3, \frac{3}{2}\right)$ etc.

7. Minimise and Maximise Z = 5*x* + 10*y* subject to $x + 2y \le 120$, $x + y \ge 60$, $x - 2y \ge 0$, $x, y \ge 0$.

Sol. Minimise and Maximise Z = $5x + 10y$ ...(*i*)

subject to: $x + 2y \le 120$ ...(*ii*)

$x + y \ge 60$ ...(*iii*), $x - 2y \ge 0$ ...(*iv*), $x, y \ge 0$ ...(*v*)

Step I. Constraint (*v*) $x, y \ge 0$ $\Rightarrow$ Feasible region is in first quadrant.

Table of values for line $x + 2y = 120$ of constraint (*ii*)

x	0	120
y	60	0

Let us draw the line joining the points (0, 60) and (120, 0).

Let us test for origin ($x = 0$, $y = 0$) in constraint (*iii*) $x + 2y \le 120$ we have $0 \le 120$ which is true.

$\therefore$ Region for constraint (*ii*) is on the origin side of the line

$$x + 2y = 120.$$

Table of values for line $x + y = 60$ of constraint (*iii*)

x	0	60
y	60	0

Let us draw the line joining the points (0, 60) and (60, 0).

Let us test for origin ($x = 0$, $y = 0$) in constraint (*iii*) $x + y \ge 60$, we have $0 \ge 60$ which is not true.

$\therefore$ Region for constraint (*iii*) is the half-plane on the non-origin side of the line $x + y = 60$ (*i.e.*, on the side of the line opposite to the origin side).

Table of values for line $x - 2y = 0$ of constraint (*iv*)

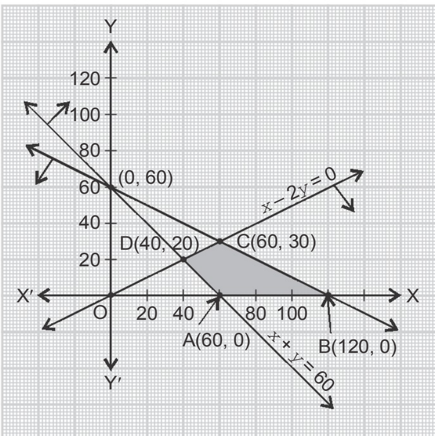
x	0	0	60
y	0	0	30

($\because$ The line $x - 2y = 0$ is passing through the origin, so we have taken still another point (60, 30) on the line).

Let us draw the line joining the points (0, 0) and (60, 30).

Let us test for (60, 0) (a point other than origin) in constraint (*iv*), we have $60 \ge 0$ which is true.

$\therefore$ Region for constraint (*iv*) is the half-plane on that side of the line which containing the point (60, 0).



The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*v*). The feasible region ABCD is bounded.

Step II. The coordinates of the corner points A and B are (60, 0) and (120, 0) respectively.

Corner point C is the intersection of the line $x - 2y = 0$ *i.e.*, $x = 2y$ and $x + 2y = 120$. Putting $x = 2y$ in $x + 2y = 120$, we have $2y + 2y = 120 \Rightarrow 4y = 120$

$\Rightarrow y = 30$ and therefore $x = 2y = 60$.

$\therefore$ Corner point C (60, 30).

Similarly for corner point D, putting $x = 2y$ in $x + y = 60$, we have $2y + y = 60 \Rightarrow 3y = 60 \Rightarrow y = 20$ and therefore $x = 2y = 40$. Therefore corner point D is (40, 20).

Step III. Now, we evaluate Z at each corner point.

Corner Point	$Z = 5x + 10y$	
A(60, 0)	300 = m	← Minimum
B(120, 0)	600	
C(60, 30)	300 + 300 = 600 = M	← Maximum
D(40, 20)	400	

Hence, by Corner Point Method,

Minimum Z = 300 at (60, 0)

Maximum Z = 600 at B(120, 0) and C(60, 30) and hence maximum at all the points on the line segment BC joining the points (120, 0) and (60, 30).

8. Minimise and Maximise $Z = x + 2y$

subject to $x + 2y \geq 100$, $2x - y \leq 0$, $2x + y \leq 200$; $x, y \geq 0$.

Sol. Minimise and Maximise $Z = x + 2y$ ...(*i*)

subject to:

$x + 2y \geq 100$ ...(*ii*)

$2x - y \leq 0$ ...(*iii*)

$2x + y \leq 200$ ...(*iv*)

$x, y \geq 0$ ...(*v*)

Step I. The constraint (*v*) $x, y \geq 0 \Rightarrow$ Feasible region is in first quadrant.

Table of values for the line $x + 2y = 100$ for constraint (*ii*).

x	0	100
y	50	0

Let us draw the line joining the points (0, 50) and (100, 0).

Let us test for origin ($x = 0$, $y = 0$) in constraint (*ii*) $x + 2y \geq 100$, we have $0 \geq 100$ which is not true.

$\therefore$ Region for constraint (*i*) is that half-plane which does not contain the origin.

Table of values for the line $2x - y = 0$ *i.e.*, $2x = y$ of constraint (*iii*).

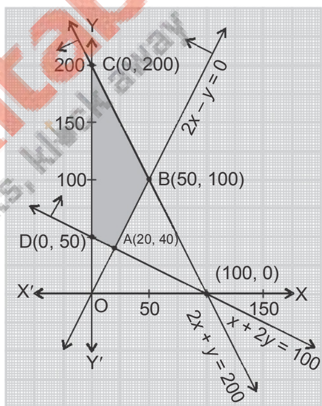
x	0	20
y	0	40

Let us draw the line joining the points (0, 0) and (20, 40).
Because this line passes through the origin, so we shall have the test for some point say (100, 0) other than the origin.
Putting $x = 100$ and $y = 0$ in constraint (*iii*) $2x - y \leq 0$, we have $200 \leq 0$ which is not true.
$\therefore$ Region for constraint (*iii*) is the half plane on the side of the line which does not contain the point (100, 0).

Table of values for the line $2x + y = 200$ of constraint (*iv*).

x	0	100
y	200	0

Let us draw the line joining the points (0, 200) and (100, 0).
Let us test for origin ($x = 0$, $y = 0$) in constraint (*iv*) $2x + y \leq 200$, we have $0 \leq 200$ which is true. Therefore region for constraint (*iv*) is the half-plane on origin side of the line.
The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*v*). The feasible region ABCD is bounded.



Step II. The coordinates of the two corner points are C(0, 200) and D(0, 50).
Corner point A is the intersection of boundary lines $x + 2y = 100$ and $2x - y = 0$ *i.e.,* $y = 2x$.
Solving them, putting $y = 2x$, $x + 4x = 100$
$\Rightarrow \quad 5x = 100 \quad \Rightarrow \quad x = 20.$
$\therefore \quad y = 2x = 2 \times 20 = 40.$
Therefore corner point A(20, 40).
Corner point B is the intersection of the boundary lines $2x + y = 200$ and $2x - y = 0$ *i.e.,* $y = 2x$.
Solving them, putting $y = 2x$, $2x + 2x = 200 \Rightarrow 4x = 200$
$\Rightarrow \quad x = 50$ and therefore $y = 2x = 100$. Therefore corner point B is (50, 100).

Step III. Now, we evaluate Z at each corner point.

Corner Point	Z = $x + 2y$	
A(20, 40)	100 = m	← Minimum
B(50, 100)	250	
C(0, 200)	400 = M	← Maximum
D(0, 50)	100 = m	← Minimum

By Corner Point Method,
Minimum Z = 100 at all the points on the line segment joining the points (20, 40) and (0, 50).
(See Step III, Example 7, Page 770.
Maximum Z = 400 at (0, 200).

9. Maximise Z = – x + 2y, subject to the constraints:

$x \geq 3$, $x + y \geq 5$, $x + 2y \geq 6$, $y \geq 0$.

Sol. Maximise $Z = -x + 2y$ ...(*i*)

subject to the constraints:

$x \geq 3$ ...(*ii*), $x + y \geq 5$ ...(*iii*), $x + 2y \geq 6$ ...(*iv*), $y \geq 0$ ...(*v*)

Step I. Constraint (*v*), $y \geq 0$ $\Rightarrow$ Positive side of y-axis

$\Rightarrow$ Feasible region is in first and second quadrants.

Region for constraint (*ii*) $x \geq 3$.

We know that graph of the line $x = 3$ is a vertical line parallel to y-axis at a distance 3 from origin along OX.

$\therefore$ Region for $x \geq 3$ is the half-plane on right side of the line $x = 3$.

Table of values for line $x + y = 5$ of constraint (*iii*)

x	0	5
y	5	0

Let us draw the line joining the points (0, 5) and (5, 0).

Let us test for origin (0, 0) in constraint (*ii*).

Putting $x = 0$ and $y = 0$ in $x + y \geq 5$, we have $0 \geq 5$ which is not true.

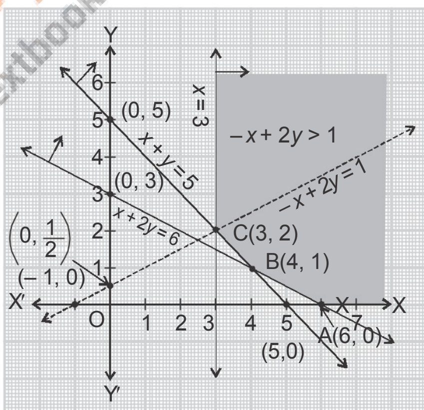
$\therefore$ Region for constraint (*iii*) is the half plane on the non-origin side of the line $x + y = 5$.

Table of values for the line $x + 2y = 6$ of constraint (*iii*)

x	0	6
y	3	0

Let us test for origin (0, 0) in constraint (*iv*) $x + 2y \geq 6$, we have $0 \geq 6$ which is not true.

$\therefore$ Region for constraint (*iv*) is again the half plane on the non-origin side of the line $x + 2y = 6$.



The shaded region in the figure is the feasible region determined by the system of constraints from (*ii*) to (*v*). The feasible region is unbounded.

Step II. The coordinates of the corner point A are (6, 0).

Corner point B is the intersection of the boundary lines

$$x + y = 5 \quad \text{and} \quad x + 2y = 6$$

Let us solve them for x and y.

Subtracting the two equations $2y - y = 6 - 5$ or $y = 1$.

Putting $y = 1$ in $x + y = 5$, we have $x + 1 = 5$ or $x = 4$. Therefore, vertex B is (4, 1).

Corner point C is the intersection of the boundary lines $x + y = 5$ and $x = 3$.

Solving for x and y; putting $x = 3$ in $x + y = 5$; $3 + y = 5$ or $y = 2$. Therefore corner point C is (3, 2).

Step III. Now, we evaluate Z at each corner point.

Corner Point	$Z = -x + 2y$	
A(6, 0)	– 6	
B(4, 1)	– 2	
C(3, 2)	1 = M	← Maximum

From this table, we find that 1 is the maximum value of Z at (3, 2).

Step IV. Since the feasible region is unbounded, 1 may or may not be the maximum value of Z. To decide this, we graph the inequality Z > M *i.e.*, $-x + 2y > 1$.

Table of values for the line $-x + 2y = 1$ corresponding to constraint Z > M *i.e.*, $-x + 2y > 1$.

x	0	– 1
y	$\frac{1}{2}$	0

Let us draw the **dotted** line joining the points $\left(0, \frac{1}{2}\right)$ and (– 1, 0). The line is to be shown dotted because boundary of the line is to be excluded as equality sign is missing in the constraint Z > M. We observe that the half-plane determined by Z > M has points in common with the feasible region. Therefore, $Z = -x + 2y$ has no maximum value subject to the given constraints.

10. Maximise $Z = x + y$,
subject to $x - y \le -1$, $-x + y \le 0$, $x, y \ge 0$.

Sol. Maximise $Z = x + y$ ...(*i*)

subject to:

$x - y \le -1$ ...(*ii*), $-x + y \le 0$ ...(*iii*), $x, y \ge 0$ ...(*iv*)

Step I. Constraint (*iv*) $x, y \ge 0$.

⇒ Feasible region is in first quadrant.

Table of values for the line $x - y = -1$ of constraint (*ii*)

x	0	– 1
y	1	0

Let us draw the straight line joining the points (0, 1) and (– 1, 0).

Let us test for origin (0, 0) in constraint (*ii*) $x - y \le -1$, we have $0 \le -1$ which is not true.

Therefore region for constraint (*ii*) is the region on that side of the line which is away from the origin (as shown shaded in the figure)

Table of values for the line $-x + y = 0$ *i.e.*, $y = x$ of constraint (*iii*)

x	0	2
y	0	2

Let us draw the line joining the points (0, 0) and (2, 2).

Let us test for the point (2, 0) (say) [and not origin as line passes through (0, 0)] in constraint (*iii*) $-x + y \le 0$, we have $-2 \le 0$ which is true.

∴ Region for constraint (*iii*) is towards the point (2, 0) side of the line (shown shaded in the figure).

From the figure, we observe that there is no point common in the two shaded regions. Thus, the problem has no feasible region and hence no feasible solution *i.e.,* no maximum value of Z.