NCERT Class 12 Maths

Solutions

Chapter - 6

Application of Derivatives

Exercise 6.4

Note. 1. Symbol for approximate value is ~.

2. Δx, a small increment (change) in the value of x, (positive or negative) is ~ dx.

3. Similarly, Δy ~ dy.

1. **Using differentials, find the approximate value of each of the following up to 3 places of decimal.**

(*i*) $\sqrt{25.3}$ (*ii*) $\sqrt{49.5}$ (*iii*) $\sqrt{0.6}$

(*iv*) $(0.009)^{1/3}$ (*v*) $(0.999)^{1/10}$ (*vi*) $(15)^{1/4}$

(*vii*) $(26)^{1/3}$ (*viii*) $(255)^{1/4}$ (*ix*) $(82)^{1/4}$

(x) **$(401)^{1/2}$** **(xi)** **$(0.0037)^{1/2}$** **(xii)** **$(26.57)^{1/3}$**
($xiii$) **$(81.5)^{1/4}$** **(xiv)** **$(3.968)^{3/2}$** **(xv)** **$(32.15)^{1/5}$.**

Sol. (i) To find approximate value of $\sqrt{25.3}$.

Let $y = \sqrt{x}$ ...(i) by looking at square root of 25.3

$\therefore \frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}} \Rightarrow dy = \frac{dx}{2\sqrt{x}}$ ...(ii)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$y + \Delta y = \sqrt{x + \Delta x} = \sqrt{25.3} = \sqrt{25 + 0.3}$ ...(iii)

(25.3 has been written as 25 + 0.3 because we know the square root of 25 as = 5)

Comparing $\sqrt{x + \Delta x}$ with $\sqrt{25 + 0.3}$, we have

$x = 25$ and $\Delta x = 0.3$ ...(iv)

From eqn. (iii), $\sqrt{25.3} = y + \Delta y \sim y + dy \sim \sqrt{x} + \frac{dx}{2\sqrt{x}}$

(From (i) and (ii))

$\sim \sqrt{x} + \frac{\Delta x}{2\sqrt{x}} \qquad \sim \sqrt{25} + \frac{0.3}{2\sqrt{25}}$ (From (iv))

$\sim 5 + \frac{0.3}{2(5)} = 5 + \frac{0.3}{10} = 5 + 0.03$

$\therefore \qquad \sqrt{25.3} \sim 5.03.$

(ii) To find approximate value of $\sqrt{49.5}$

Let $y = \sqrt{x}$ ...(i)

$\therefore \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dy = \frac{dx}{2\sqrt{x}}$ ...(ii)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$y + \Delta y = \sqrt{x + \Delta x} = \sqrt{49.5} = \sqrt{49 + 0.5}$ ...(iii)

Comparing $\sqrt{x + \Delta x}$ with $\sqrt{49 + 0.5}$,

$x = 49$ and $\Delta x = 0.5$ ...(iv)

From eqn. (iii), $\sqrt{49.5} = y + \Delta y \sim y + dy$

$\sim \sqrt{x} + \frac{dx}{2\sqrt{x}}$ (From (i) and (ii))

$\sim \sqrt{x} + \frac{dx}{2\sqrt{x}} \sim \sqrt{x} + \frac{\Delta x}{2\sqrt{x}}$

$\therefore \qquad \sqrt{49.5} \sim \sqrt{49} + \frac{0.5}{2\sqrt{49}}$ [By (iv)]

$= 7 + \frac{0.5}{2(7)} = 7 + \frac{0.5}{14} = 7 + 0.0357 = 7.0357$

(iii) To find approximate value of $\sqrt{0.6}$

Let $y = \sqrt{x}$ ...(i)

$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \quad \Rightarrow \quad dy = \frac{dx}{2\sqrt{x}}$ ...(*ii*)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = \sqrt{x + \Delta x} = \sqrt{0.6} = \sqrt{0.60}$

$= \sqrt{0.64 - 0.04}$ ...(*iii*) ($\because$ $0.64 - 0.60 = 0.04$)

Comparing $\sqrt{x + \Delta x}$ with $\sqrt{0.64 - 0.04}$,

we have $x = 0.64$ and $\Delta x = -0.04$ ...(*iv*)

From eqn. (*iii*), $\sqrt{0.6} = y + \Delta y \sim y + dy$

$\sim \sqrt{x} + \frac{dx}{2\sqrt{x}}$ (From (*i*) and (*ii*))

$\sim \sqrt{x} + \frac{\Delta x}{2\sqrt{x}} \sim \sqrt{0.64} - \frac{0.04}{2\sqrt{0.64}}$

$\therefore \quad \sqrt{0.6} \sim 0.8 - \frac{0.04}{2(0.8)} = 0.8 - \frac{0.04}{1.6} = 0.8 - \frac{4}{100} \times \frac{10}{16}$

$= 0.8 - \frac{1}{40} = 0.8 - 0.025 = 0.775.$

(*iv*) To find approximate value of $(0.009)^{1/3}$

Let $y = x^{1/3}$ ...(*i*) by looking at power (index) $\frac{1}{3}$ of 0.009.

$\therefore \quad \frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \frac{1}{3x^{2/3}} \Rightarrow dy = \frac{dx}{3x^{2/3}} \sim \frac{\Delta x}{3(x^{1/3})^2}$ ...(*ii*)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = (x + \Delta x)^{1/3} = (0.009)^{1/3} = (0.008 + 0.001)^{1/3}$ ...(*iii*)

0.009 has been written as 0.008 + 0.001 because we know that the cube root of 0.008 *i.e.,* $(0.008)^{1/3} = 0.2$

Comparing $(x + \Delta x)^{1/3}$ with $(0.008 + 0.001)^{1/3}$, we have

$x = 0.008$ and $\Delta x = 0.001$ ...(*iv*)

From eqn. (*iii*), $(0.009)^{1/3} = y + \Delta y$

$\sim y + dy = x^{1/3} + \frac{dx}{3x^{2/3}} \sim x^{1/3} + \frac{\Delta x}{3(x^{1/3})^2}$

(From (*i*) and (*ii*)) $\sim (0.008)^{1/3} + \frac{(0.001)}{3((0.008)^{1/3})^2}$

$\therefore \quad (0.009)^{1/3} \sim 0.2 + \frac{0.001}{3(0.2)^2} = 0.2 + \frac{0.001}{3(0.04)}$

$= 0.2 + \frac{0.001}{0.12}$ $[(0.008)^{1/3} = ((0.2)^3)^{1/3} = 0.2]$

$\sim 0.2 + 0.0083 = 0.2083.$

(*v*) To find approximate value of $(0.999)^{1/10}$

Let $y = x^{1/10}$ ...(*i*)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{10} x^{-9/10} = \frac{1}{10x^{9/10}}$$

$$\Rightarrow \quad dy = \frac{dx}{10(x^{1/10})^9} \sim \frac{\Delta x}{10(x^{1/10})^9} \quad \text{...(ii)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = (x + \Delta x)^{1/10} = (0.999)^{1/10} = (1 - 0.001)^{1/10}$ ...(*iii*)

Comparing $x = 1$ and $\Delta x = -0.001$ ...(*iv*)

From eqn. (*iii*), $(0.999)^{1/10} = y + \Delta y \sim y + dy$

$$\sim x^{1/10} + \frac{\Delta x}{10(x^{1/10})^9} \quad \text{[From (i) and (ii)]}$$

$$\sim (1)^{1/10} - \frac{0.001}{10(1^{1/10})^9} = 1 - \frac{0.001}{10}$$

$$= 1 - 0.0001 = 0.9999.$$

(*vi*) To find approximate value of $(15)^{1/4}$

Let $y = x^{1/4}$ ...(*i*)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{4} x^{1/4 - 1} = \frac{1}{4} x^{-3/4} = \frac{1}{4x^{3/4}}$$

$$\therefore \quad dy = \frac{dx}{4(x^{1/4})^3} \sim \frac{\Delta x}{4(x^{1/4})^3} \quad \text{...(ii)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = (x + \Delta x)^{1/4} = (15)^{1/4} = (16 - 1)^{1/4}$ ...(*iii*)

Comparing, $x = 16$ and $\Delta x = -1$ ...(*iv*)

From eqn. (*iii*), $(15)^{1/4} = y + \Delta y \sim y + dy$

$$= x^{1/4} + \frac{\Delta x}{4(x^{1/4})^3} \quad \text{(From (i) and (ii))}$$

$$\sim (16)^{1/4} - \frac{1}{4((16)^{1/4})^3} \quad \text{(From (iv))}$$

$$= 2 - \frac{1}{4 \times 2^3} \quad (\because \ (16)^{1/4} = (2^4)^{1/4} = 2)$$

$$\therefore \quad (15)^{1/4} \sim 2 - \frac{1}{32} = \frac{64 - 1}{32} = \frac{63}{32} = 1.96875.$$

(*vii*) To find approximate value of $(26)^{1/3}$

Let $y = x^{1/3}$ ...(*i*)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}}$$

$$\therefore \quad dy = \frac{dx}{3x^{2/3}} \sim \frac{\Delta x}{3(x^{1/3})^2} \quad \text{...(ii)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = (x + \Delta x)^{1/3} = (26)^{1/3} = (27 - 1)^{1/3}$ ...(*iii*)

Comparing, $x = 27$ and $\Delta x = -1$ ...(*iv*)

From (iii), $(26)^{1/3} = y + \Delta y \sim y + dy$

$$\sim x^{1/3} + \frac{\Delta x}{3(x^{1/3})^2} \qquad \text{[From } (iii) \text{ and } (ii)\text{]}$$

$$\sim (27)^{1/3} - \frac{1}{3((27)^{1/3})^2} \qquad \text{[From } (iii)\text{]}$$

$$\therefore \quad (26)^{1/3} \sim 3 - \frac{1}{3(3)^2} \qquad [\because \ (27)^{1/3} = (3^3)^{1/3} = 3]$$

$$= 3 - \frac{1}{27} = \frac{81-1}{27} = \frac{80}{27} = 2.9629.$$

$(viii)$ To find approximate value of $(255)^{1/4}$

Let $\quad y = x^{1/4}$ ...(i)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$$

$$\therefore \quad dy = \frac{dx}{4x^{3/4}} \sim \frac{\Delta x}{4(x^{1/4})^3} \qquad ...(ii)$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$$y + \Delta y = (x + \Delta x)^{1/4} = (255)^{1/4} = (256 - 1)^{1/4} \qquad ...(iii)$$

Comparing, $x = 256$ and $\Delta x = -1$ ...(iv)

From (iii), $(255)^{1/4} = y + \Delta y$

$$\sim x^{1/4} + \frac{\Delta x}{4(x^{1/4})^3} \qquad \text{(From } (i) \text{ and } (ii)\text{)}$$

$$\sim (256)^{1/4} - \frac{1}{4((256)^{1/4})^3} \sim 4 - \frac{1}{4(4)^3}$$

$$[\because \ (256)^{1/4} = (4^4)^{1/4} = 4]$$

$$\sim 4 - \frac{1}{256} = \frac{1024-1}{256} = \frac{1023}{256} \sim 3.9961.$$

(ix) To find approximate value of $(82)^{1/4}$

Let $\quad y = x^{1/4}$ ...(i)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{4}x^{-3/4} = \frac{1}{4x^{3/4}}$$

$$\therefore \quad dy = \frac{dx}{4(x^{1/4})^3} \sim \frac{\Delta x}{4(x^{1/4})^3} \qquad ...(ii)$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$$y + \Delta y = (x + \Delta x)^{1/4} = (82)^{1/4} = (81 + 1)^{1/4} \qquad ...(iii)$$

Comparing, $x = 81$ and $\Delta x = 1$ ...(iv)

From (iii), $(81)^{1/4} = y + \Delta y \sim y + dy$

$$\sim x^{1/4} + \frac{\Delta x}{4(x^{1/4})^3} \qquad \text{[From } (i) \text{ and } (ii)\text{]}$$

$$\sim (81)^{1/4} + \frac{1}{4((81)^{1/4})^3} = 3 + \frac{1}{4(3)^3}$$

$$\sim 3 + \frac{1}{108} = \frac{324+1}{108} = \frac{325}{108} = 3.0092.$$

(x) To find approximate value of $(401)^{1/2} = \sqrt{401}$

Let $y = x^{1/2} = \sqrt{x}$ ...(i)

$\therefore \quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$\therefore \quad dy = \frac{dx}{2\sqrt{x}} \sim \frac{\Delta x}{2\sqrt{x}}$ ...(ii)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$y + \Delta y = \sqrt{x + \Delta x} = \sqrt{401} = \sqrt{400 + 1}$ ...(iii)

Comparing, $x = 400$ and $\Delta x = 1$ ...(iv)

From (iii), $\sqrt{401} = y + \Delta y \sim y + dy$

$\sim \sqrt{x} + \frac{\Delta x}{2\sqrt{x}}$ [From (i) and (ii)]

$\sim \sqrt{400} + \frac{1}{2\sqrt{400}} = 20 + \frac{1}{40} = \frac{800 + 1}{40} = \frac{801}{40} \sim 20.025.$

(xi) To find approximate value of $(0.0037)^{1/2} = \sqrt{0.0037}$

Let $y = \sqrt{x}$ ...(i)

$\therefore \quad \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dy = \frac{dx}{2\sqrt{x}} \sim \frac{\Delta x}{2\sqrt{x}}$ ...(ii)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$y + \Delta y = \sqrt{x + \Delta x} = \sqrt{0.0037} = \sqrt{0.0036 + 0.0001}$ ...(iii)

$(\because \quad 0.0037 - 0.0036 = 0.0001)$

Comparing with $x + \Delta x$, $x = 0.0036$ and $\Delta x = 0.0001$ ...(iv)

From (iii), $\sqrt{0.0037} = y + \Delta y \sim y + dy$

$= \sqrt{x} + \frac{\Delta x}{2\sqrt{x}}$ (From (i) and (ii))

$\sim \sqrt{0.0036} + \frac{0.0001}{2\sqrt{0.0036}}$

$= 0.06 + \frac{0.0001}{2(0.06)}$ $[(0.06)^2 = 0.0036]$

$\sim 0.06 + \frac{0.0001}{0.12} \sim 0.06 + 0.000833 \sim 0.060833.$

(xii) To find approximate value of $(26.57)^{1/3}$

Let $y = x^{1/3}$ ...(i)

$\therefore \quad \frac{dy}{dx} = \frac{1}{3}x^{-2/3} = \frac{1}{3.x^{2/3}}$

$\Rightarrow \quad dy = \frac{dx}{3.x^{(2/3)}} \sim \frac{\Delta x}{3(x^{1/3})^2}$ ...(ii)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (i),

$y + \Delta y = (x + \Delta x)^{1/3} = (26.57)^{1/3}$

$= (27 - 0.43)^{1/3}$ ...(iii) $[\because \quad 27 - 26.57 = 0.43]$

Comparing with $x + \Delta x$, $x = 27$ and $\Delta x = -0.43$ ...(iv)

From (*iii*), $(26.57)^{1/3} = y + \Delta y \sim x^{1/3} + \dfrac{\Delta x}{3(x^{1/3})^2}$

(From (*i*) and (*ii*))

$$\sim (27)^{1/3} - \frac{0.43}{3((27)^{1/3})^2} \sim 3 - \frac{0.43}{3(3)^2}$$

$$\sim 3 - \frac{0.43}{27} \sim 3 - 0.0159 \sim 2.9841.$$

(*xiii*) To find approximate value of $(81.5)^{1/4}$

Let $\quad y = x^{1/4}$ ...(*i*)

$$\therefore \quad \frac{dy}{dx} = \frac{1}{4} \,.\, x^{-3/4} = \frac{1}{4x^{(3/4)}}$$

$$\therefore \quad dy = \frac{dx}{4(x^{3/4})} \sim \frac{\Delta x}{4(x^{1/4})^3} \quad \text{...(ii)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$$y + \Delta y = (x + \Delta x)^{1/4} = (81.5)^{1/4} = (81 + 0.5)^{1/4} \quad \text{...(iii)}$$

Comparing with $x + \Delta x$ we have $x = 81$

and $\Delta x = 0.5$ ...(*iv*)

From (*iii*), $(81.5)^{1/4} = y + \Delta y \sim y + dy$

$$\sim x^{1/4} + \frac{\Delta x}{4(x^{1/4})^3} \quad \text{(From (i) and (ii))}$$

$$\sim (81)^{1/4} + \frac{0.5}{4((81)^{1/4})^3} \sim 3 + \frac{0.5}{4(3)^3}$$

$$\sim 3 + \frac{0.5}{108} \sim 3 + 0.00462 \sim 3.00462.$$

(*xiv*) To find approximate value of $(3.968)^{3/2}$

Let $y = x^{3/2} = x^{2/2 + 1/2} = x^{1 + 1/2}$

$$= x^1 \, x^{1/2} = x\sqrt{x} \quad \text{...(i)}$$

On looking at power (index) $\dfrac{3}{2}$ of 3.968

$$\therefore \quad \frac{dy}{dx} = \frac{3}{2}x^{1/2} \quad \therefore \quad dy = \frac{3}{2}x^{1/2}\,dx \sim \frac{3}{2}\sqrt{x}\ \Delta x \quad \text{...(ii)}$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$$y + \Delta y = (x + \Delta x)^{3/2} = (3.968)^{3/2} = (4 - 0.032)^{3/2} \quad \text{...(iii)}$$

Comparing with $x + \Delta x$, we have $x = 4$ and

$$\Delta x = -\,0.032 \quad \text{...(iv)}$$

From (*iii*), $(3.968)^{3/2} = y + \Delta y \sim y + dy$

$$\sim x\sqrt{x} + \frac{3}{2}\sqrt{x}\ \Delta x \quad \text{(From (i) and (ii))}$$

$$\sim 4\sqrt{4} + \frac{3}{2}\sqrt{4}\,(-\,0.032) \quad \text{[By (iv)]}$$

$$\sim 4(2) - \frac{3}{2}(2)(0.032) \sim 8 - 3(0.032)$$
$$\sim 8 - 0.096 \sim 7.904.$$

(*xv*) To find approximate value of $(32.15)^{1/5}$

Let $y = x^{1/5}$ ...(*i*)

$\therefore \frac{dy}{dx} = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}} \therefore \quad dy = \frac{dx}{5x^{4/5}} \sim \frac{\Delta x}{5(x^{1/5})^4}$ ...(*ii*)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = (x + \Delta x)^{1/5} = (32.15)^{1/5} = (32 + 0.15)^{1/5}$ ...(*iii*)

Comparing with $x + \Delta x$, we have $x = 32$ and $\Delta x = 0.15$ ...(*iv*)

From (*iii*), $(32.15)^{1/5} = y + \Delta y \sim y + dy$

$\sim x^{1/5} + \frac{\Delta x}{5(x^{1/5})^4}$ (From (*i*) and (*ii*))

$\sim (32)^{1/5} + \frac{0.15}{5((32)^{1/5})^4} \sim 2 + \frac{0.15}{5(2)^4} (\because \ (32)^{1/5} = (2^5)^{1/5} = 2)$

$\sim 2 + \frac{0.15}{80} \sim 2 + 0.001875 \sim 2.001875.$

2. Find the approximate value of $f(2.01)$ where $f(x) = 4x^2 + 5x + 2$.

Sol. Let $y = f(x) = 4x^2 + 5x + 2$ ...(*i*)

$\therefore \frac{dy}{dx} = f'(x) = 8x + 5$

$\therefore dy = (8x + 5)dx \sim (8x + 5)\Delta x$ ...(*ii*)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = f(x + \Delta x) = f(2.01) = f(2 + 0.01)$ ...(*iii*)

Comparing $f(x + \Delta x)$ with $f(2 + 0.01)$, we have

$x = 2$ and $\Delta x = 0.01$ ...(*iv*)

From (*iii*), $f(2.01) = y + \Delta y \sim y + dy$

$\sim (4x^2 + 5x + 2) + (8x + 5)\ \Delta x$ (From (*i*) and (*ii*))

Putting $x = 2$ and $\Delta x = 0.01$ from (*iv*),

$\sim (4(4) + 5(2) + 2) + (8(2) + 5)\ (0.01)$

$\sim 28 + 21(0.01) \sim 28 + 0.21 \sim 28.21.$

3. Find the approximate value of $f(5.001)$ where $f(x) = x^3 - 7x^2 + 15$.

Sol. Let $y = f(x) = x^3 - 7x^2 + 15$ ...(*i*)

$\therefore \frac{dy}{dx} = f'(x) = 3x^2 - 14x$

$\therefore dy = (3x^2 - 14x)dx \sim (3x^2 - 14x)\ \Delta x$ ...(*ii*)

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$y + \Delta y = f(x + \Delta x) = f(5.001) = f(5 + 0.001)$ ...(*iii*)

Comparing $f(x + \Delta x)$ with $f(5 + 0.001)$, we have

$x = 5$ and $\Delta x = 0.001$ ...(*iv*)

From (*iii*), $f(5.001) = y + \Delta y \sim y + dy$

$$\sim (x^3 - 7x^2 + 15) + (3x^2 - 14x)\Delta x \text{ (From (i) and (ii))}$$

Putting $x = 5$ and $\Delta x = 0.001$ from (*iv*), we have

$$\sim (125 - 175 + 15) + (75 - 70)(0.001)$$

$$\sim -35 + 5(0.001) \sim -35 + 0.005$$

$$\sim -34.995.$$

4. Find the approximate change in the volume of a cube of side x metres caused by increasing the side by 1%.

Sol. We know that volume of a cube of side x metres is given by

$$V = x^3 \quad ...(i)$$

$$\therefore \quad \frac{dV}{dx} = 3x^2 \quad ...(ii)$$

Given: Increase in side = 1% of $x = \frac{1}{100}x$

(Positive sign is being taken because it is given that side of cube is increasing)

i.e., $\quad \Delta x = \frac{x}{100} \quad ...(iii)$

We know that approximate change in volume V of cube

$$= \Delta V \sim dV = \frac{dV}{dx} dx$$

$$\sim \frac{dV}{dx} \Delta x \sim 3x^2 \left(\frac{x}{100}\right) \quad | \text{ From (ii) and (iii)}$$

$$\sim \frac{3}{100} x^3$$

$$\sim 0.03x^3 \text{ m}^3.$$

5. Find the approximate change in the surface area of a cube of side x metres caused by decreasing the side by 1%.

Sol. We know that surface area of a cube of side x is given by $S = 6x^2$

$$\therefore \quad \frac{dS}{dx} = 12x$$

Decrease in side = – 1% of x = – $0.01x$ [Negative sign is being taken because it is given that side of the cube is decreasing]

$\Rightarrow \quad \Delta x = -0.01x$

Approximate change in S = Approximate value of ΔS

$$= dS = \left(\frac{dS}{dx}\right) dx$$

$$= (12x)(-0.01x) \quad [\because \ dx = \Delta x]$$

$$= -0.12x^2 \text{ m}^2$$

Hence, the approximate change in surface is $-0.12x^2$ m^2, *i.e.,* the surface **decreases** by approximately $0.12x^2$ m^2.

6. If the radius of a sphere is measured as 7 m with an error of 0.02 m, then find the approximate error in calculating its volume.

Sol. Let r be the radius of the sphere and Δr be the error in measuring the radius.

Then $r = 7$ m and $\Delta r = 0.02$ m.

Volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$

$$\therefore \quad \frac{dV}{dr} = \frac{4}{3}\pi.3r^2$$

Approximate error in calculating the volume

$$= \text{Approximate value of } \Delta V$$
$$= dV = \left(\frac{dV}{dr}\right) dr = \left(\frac{4}{3}\pi 3r^2\right) dr$$
$$= 4\pi(7)^2 (0.02) \qquad [\because \; dr \sim \Delta r]$$
$$= 3.92\pi \text{ m}^3 = 3.92 \times \frac{22}{7} \text{ m}^3$$
$$= 12.32 \text{ m}^3$$

Hence, the approximate error in calculating volume is 12.32 m^3.

7. If the radius of a sphere is measured as 9 m with an error of 0.03 m, then find the approximate error in calculating its surface area.

Sol. Let x m be the radius of the sphere.

$\therefore$ S, surface area of sphere = $\mathbf{4\pi x^2}$

$$\therefore \quad \frac{dS}{dx} = 4\pi(2x) = 8\pi x$$

$$\therefore \quad dS = 8\pi x \, dx \sim 8\pi x \, \Delta x \qquad ...(i)$$

Given: $x = 9$ m and error $\Delta x = 0.03$ m ...(*ii*)

Putting $x = 9$ and $\Delta x = 0.03$ from (*ii*) in (*i*),

Error ΔS in surface area of sphere

$$\sim dS = 8\pi(9)(0.3) = 72(0.03)\pi = 2.16\pi \text{ m}^2.$$

(**Note.** Error can be positive or negative)

8. If $f(x) = 3x^2 + 15x + 5$, then the approximate value of $f(3.02)$ is

(A) 47.66 (B) 57.66 (C) 67.66 (D) 77.66.

Sol. Let $\quad y = f(x) = 3x^2 + 15x + 5$ ...(*i*)

$$\therefore \quad \frac{dy}{dx} = f'(x) = 6x + 15$$

$$\therefore \quad dy = (6x + 15)dx \sim (6x + 15)\Delta x \qquad ...(ii)$$

Changing x to $x + \Delta x$ and y to $y + \Delta y$ in (*i*),

$$y + \Delta y = f(x + \Delta x) = f(3.02) = f(3 + 0.02) \qquad ...(iii)$$

Comparing $f(x + \Delta x)$ with $f(3 + 0.02)$, we have

$$x = 3 \text{ and } \Delta x = 0.02 \qquad ...(iv)$$

From (*iii*), $f(3.02) = y + \Delta y \sim y + dy$

$$\sim (3x^2 + 15x + 5) + (6x + 15)\Delta x \qquad (\text{From } (i) \text{ and } (ii))$$
$$\sim (3(9) + 15(3) + 5) + (6(3) + 15)(0.02)$$
$$= (72 + 5) + (33)(0.02)$$

$$\simeq 77 + 0.66 \simeq 77.66$$

$\therefore$ Option (D) is the correct answer.

9. The approximate change in the volume of a cube of side x metres caused by increasing the side by 3% is
(A) 0.06 x^3 m^3 (B) 0.6 x^3 m^3 (C) 0.09 x^3 m^3 (D) 0.9 x^3 m^3.

Sol. We know that volume of a cube of side x metres is given by

$$V = x^3 \quad ...(i)$$

$$\therefore \quad \frac{dV}{dx} = 3x^2 \quad ...(ii)$$

Given: Increase in side of cube = 3% = $\frac{3}{100}x$

(Positive sign is being taken because it is given that side of cube is increasing)

i.e., $$\Delta x = \frac{3x}{100} \quad ...(iii)$$

We know that approximate change in volume of cube

$$= \Delta V \simeq dV = \frac{dV}{dx}dx$$

$$\simeq \frac{dV}{dx}\Delta x \simeq 3x^2\left(\frac{3x}{100}\right) \quad | \text{ From } (i) \text{ and } (iii)$$

$$\simeq \frac{9}{100}x^3 \simeq 0.09x^3 \text{ m}^3.$$

$\therefore$ Option (C) is the correct answer.