Exercise 5.4

Differentiate the following functions 1 to 10 w.r.t. *x*

1. $\dfrac{e^x}{\sin x}$.

Sol. Let $y = \dfrac{e^x}{\sin x}$

$$\therefore \quad \frac{dy}{dx} = \frac{(\text{DEN})\dfrac{d}{dx}(\text{NUM}) - (\text{NUM})\dfrac{d}{dx}(\text{DEN})}{(\text{DEN})^2}$$

$$= \frac{\sin x \dfrac{d}{dx} e^x - e^x \dfrac{d}{dx} \sin x}{\sin^2 x} = \frac{\sin x \,.\, e^x - e^x \cos x}{\sin^2 x}$$

$$= e^x \frac{(\sin x - \cos x)}{\sin^2 x}.$$

2. $e^{\sin^{-1} x}$.

Sol. Let $y = e^{\sin^{-1} x}$

$$\therefore \quad \frac{dy}{dx} = e^{\sin^{-1} x} \frac{d}{dx} \sin^{-1} x \qquad \left[\because \frac{d}{dx} e^{f(x)} = e^{f(x)} \frac{d}{dx} f(x)\right]$$

$$= e^{\sin^{-1} x} \cdot \frac{1}{\sqrt{1 - x^2}}.$$

3. e^{x^3}.

Sol. Let $y = e^{x^3} = e^{(x^3)}$

$$\therefore \quad \frac{dy}{dx} = e^{(x^3)} \frac{d}{dx} x^3 \qquad \left[\because \frac{d}{dx} e^{f(x)} = e^{f(x)} \frac{d}{dx} f(x)\right]$$

$$= e^{(x^3)}\, 3x^2 = 3x^2\, e^{(x^3)}.$$

4. $\sin(\tan^{-1} e^{-x})$.

Sol. Let $y = \sin(\tan^{-1} e^{-x})$

$$\therefore \quad \frac{dy}{dx} = \cos(\tan^{-1} e^{-x}) \frac{d}{dx} (\tan^{-1} e^{-x})$$

$$\left[\because \frac{d}{dx} \sin f(x) = \cos f(x) \frac{d}{dx} f(x)\right]$$

$$= \cos(\tan^{-1} e^{-x}) \frac{1}{1 + (e^{-x})^2} \frac{d}{dx} e^{-x}$$

$$\left[\because \frac{d}{dx} \tan^{-1} f(x) = \frac{1}{1 + (f(x))^2} \frac{d}{dx} f(x)\right]$$

$$= \cos(\tan^{-1} e^{-x}) \frac{1}{1+e^{-2x}} e^{-x} \frac{d}{dx}(-x)$$

$$= -\frac{e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \qquad \left[\because \frac{d}{dx}(-x) = -1\right]$$

5. log (cos e^x).

Sol. Let $y = \log(\cos e^x)$

$$\therefore \frac{dy}{dx} = \frac{1}{\cos e^x} \frac{d}{dx}(\cos e^x) \left[\because \frac{d}{dx}\log f(x) = \frac{1}{f(x)}\frac{d}{dx}f(x)\right]$$

$$= \frac{1}{\cos e^x}(-\sin e^x)\frac{d}{dx}e^x \qquad \left[\because \frac{d}{dx}\cos f(x) = -\sin f(x)\frac{d}{dx}f(x)\right]$$

$$= -(\tan e^x)\, e^x = -e^x(\tan e^x)$$

6. $e^x + e^{x^2} + ... + e^{x^5}$.

Sol. Let $y = e^x + e^{x^2} + ... + e^{x^5}$

$$= e^x + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}e^x + \frac{d}{dx}e^{x^2} + \frac{d}{dx}e^{x^3} + \frac{d}{dx}e^{x^4} + \frac{d}{dx}e^{x^5}$$

$$= e^x + e^{x^2}\frac{d}{dx}x^2 + e^{x^3}\frac{d}{dx}x^3 + e^{x^4}\frac{d}{dx}x^4$$

$$+ e^{x^5}\frac{d}{dx}x^5 \left[\because \frac{d}{dx}e^{f(x)} = e^{f(x)}\frac{d}{dx}f(x)\right]$$

$$= e^x + e^{x^2}.\, 2x + e^{x^3}.\, 3x^2 + e^{x^4}.\, 4x^3 + e^{x^5}\, 5x^4$$

$$= e^x + 2x\, e^{x^2} + 3x^2\, e^{x^3} + 4x^3\, e^{x^4} + 5x^4\, e^{x^5}.$$

7. $\sqrt{e^{\sqrt{x}}}$, $x > 0$.

Sol. Let $y = \sqrt{e^{\sqrt{x}}} = (e^{\sqrt{x}})^{1/2}$

$$\therefore \frac{dy}{dx} = \frac{1}{2}(e^{\sqrt{x}})^{-1/2}\frac{d}{dx}e^{\sqrt{x}} \left[\because \frac{d}{dx}(f(x))^n = n(f(x))^{n-1}\frac{d}{dx}f(x)\right]$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}\, e^{\sqrt{x}}\frac{d}{dx}\sqrt{x} \qquad \left[\because \frac{d}{dx}e^{f(x)} = e^{f(x)}\frac{d}{dx}f(x)\right]$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}}\, e^{\sqrt{x}}\frac{1}{2\sqrt{x}} \left[\because \frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\right]$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{x}\,\sqrt{e^{\sqrt{x}}}}.$$

8. log (log x), $x > 1$.

Sol. Let $y = \log(\log x)$

$$\therefore \quad \frac{dy}{dx} = \frac{1}{\log x} \frac{d}{dx} (\log x) \qquad \left[\because \frac{d}{dx} \log f(x) = \frac{1}{f(x)} \frac{d}{dx} f(x)\right]$$

$$= \frac{1}{\log x} \frac{1}{x} = \frac{1}{x \log x}.$$

9. $\dfrac{\cos x}{\log x}$**, $x > 0$.**

Sol. Let $y = \dfrac{\cos x}{\log x}$

$$\therefore \quad \frac{dy}{dx} = \frac{(\text{DEN}) \frac{d}{dx} (\text{NUM}) - (\text{NUM}) \frac{d}{dx} (\text{DEN})}{(\text{DEN})^2}$$

$$= \frac{\log x \frac{d}{dx} (\cos x) - \cos x \frac{d}{dx} \log x}{(\log x)^2}$$

$$= \frac{\log x (- \sin x) - \cos x \cdot \frac{1}{x}}{(\log x)^2}$$

$$= \frac{-\left(\sin x \log x + \frac{\cos x}{x}\right)}{(\log x)^2} = - \frac{(x \sin x \log x + \cos x)}{x (\log x)^2}.$$

10. $\cos (\log x + e^x)$, $x > 0$.

Sol. Let $y = \cos (\log x + e^x)$

$$\therefore \quad \frac{dy}{dx} = - \sin (\log x + e^x) \frac{d}{dx} (\log x + e^x)$$

$$\left[\because \frac{d}{dx} \cos f(x) = - \sin f(x) \frac{d}{dx} f(x)\right]$$

$$= - \sin (\log x + e^x) \cdot \left(\frac{1}{x} + e^x\right)$$

$$= - \left(\frac{1}{x} + e^x\right) \sin (\log x + e^x).$$