NCERT Class 12 Maths

Solution

Chapter - 4

Exercise 4.5

Find adjoint of each of the matrices in Exercises 1 and 2.

1. $\begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{4} \end{bmatrix}$.

Sol. Here $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$| A | = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$\therefore$ A_{11} = Cofactor of $a_{11} = (-1)^2\ 4 = 4,$
A_{12} = Cofactor of $a_{12} = (-1)^3\ 3 = -3$
A_{21} = Cofactor of $a_{21} = (-1)^3\ 2 = -2,$
A_{22} = Cofactor of $a_{22} = (-1)^4\ 1 = 1$

$$\therefore \quad \text{adj. } A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}' = \begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}' = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}.$$

Remark. For writing the Cofactors of the elements of a determinant of order 2, assign a **positive** sign to the Cofactors of **diagonal** elements and a **negative sign** to the Cofactors of **non-diagonal** elements.

2. $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{2} & \mathbf{3} & \mathbf{5} \\ \mathbf{-2} & \mathbf{0} & \mathbf{1} \end{bmatrix}$.

Sol. Here $A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} | A | = \begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$

$\therefore \quad A_{11} = + \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3, \; A_{12} = - \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix}$

$= -(2 + 10) = -12,$ (See Note 2, below)

$A_{13} = + \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6, \; A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = -(-1) = 1,$

$A_{22} = + \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1 + 4 = 5, \; A_{23} = - \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix}$

$= -(-2) = 2,$

$A_{31} = + \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11,$

$A_{32} = - \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5 - 4) = -1, \; A_{33} = + \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix}$

$= 3 + 2 = 5$

$\therefore$ adj. A $= \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}'$

$= \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}.$

Note. 1. Adjoint of matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **is** $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

i.e., To write adjoint of a 2 × 2 matrix, interchange the diagonal elements and change the signs of non-diagonal elements.

The above result can be used as a formula.

2. For writing the Cofactors of the elements of a determinant of order 3 × 3, using the rule $(-1)^{i+j} M_{ij}$, the signs to be assigned to 9 cofactors are alternately + and – beginning with +.

Verify A(adj. A) = (adj. A)A = | A | I in Exercises 3 and 4:

3. $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$.

Sol. Let A $= \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

$\therefore$ By Note 1, above, adj. A $= \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$

$$\left(\because \text{ adj.}\begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ is } \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right)$$

$$\therefore \quad \text{A. (adj. A)} = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -6+6 \\ 24-24 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad ...(i)$$

$$\text{Again (adj. A). A} = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -12+12 & -18+18 \\ 8-8 & 12-12 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad ...(ii)$$

Now $|A| = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix} = 2(-6) - 3(-4) = -12 + 12 = 0$

Again $|A|\,I = |A|\,I_2$ (I is I_2 because A is of order 2×2)

$$= 0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad ...(iii)$$

From (*i*), (*ii*) and (*iii*), A (adj. A) = (adj. A) A = | A | I.

4. $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{3} & \mathbf{0} & \mathbf{-2} \\ \mathbf{1} & \mathbf{0} & \mathbf{3} \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$ $\therefore |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$

Let A_{ij} denote Cofactor of a_{ij}

(For rule of signs to be assigned see Note 2 at the end of solution of Q. No. 2).

$$\therefore \quad A_{11} = + \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = +(0+0) = 0,$$

$$A_{12} = - \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -(9+2) = -11,$$

$$A_{13} = + \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = +(0-0) = 0,$$

$$A_{21} = - \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = -(-3-0) = 3,$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = +(3-2) = 1,$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -(0+1) = -1,$$

$$A_{31} = +\begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = +(2-0) = 2,$$

$$A_{32} = -\begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = -(-2-6) = 8,$$

$$A_{33} = +\begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = +(0+3) = 3$$

$$\therefore \text{ adj. A} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} 0 & -11 & 0 \\ 3 & 1 & -1 \\ 2 & 8 & 3 \end{bmatrix}' = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$$

$$\therefore \text{ A (adj. A)} = \begin{bmatrix} \boxed{1 \quad -1 \quad 2} \\ \boxed{3 \quad 0 \quad -2} \\ \boxed{1 \quad 0 \quad 3} \end{bmatrix} \begin{bmatrix} \boxed{\begin{matrix} 0 \\ -11 \\ 0 \end{matrix}} & \boxed{\begin{matrix} 3 \\ 1 \\ -1 \end{matrix}} & \boxed{\begin{matrix} 2 \\ 8 \\ 3 \end{matrix}} \end{bmatrix}$$

$$= \begin{bmatrix} 0+11+0 & 3-1-2 & 2-8+6 \\ 0-0-0 & 9+0+2 & 6+0-6 \\ 0+0+0 & 3+0-3 & 2+0+9 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \text{...}(i)$$

$$\text{Now (adj. A) A} = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0+9+2 & 0+0+0 & 0-6+6 \\ -11+3+8 & 11+0+0 & -22-2+24 \\ 0-3+3 & 0-0+0 & 0+2+9 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \text{...}(ii)$$

$$\text{Now } |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

Expanding along first row

$= 1(0 - 0) - (-1)(9 + 2) + 2(0 - 0) = 0 + 11 + 0 = 11$

Again | A | I = | A | I_3 ($\because$ A is 3 × 3, therefore I must be I_3)

$$= 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} \quad \text{...}(iii)$$

From (*i*), (*ii*) and (*iii*)

A. (adj. A) = (adj. A) A = | A | I.

Find the inverse of the matrix (if it exists) given in Exercises 5 to 11.

5. $\begin{bmatrix} \mathbf{2} & \mathbf{-2} \\ \mathbf{4} & \mathbf{3} \end{bmatrix}$.

Sol. Let $\quad A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} = 6 - (-8) = 6 + 8 = 14 \neq 0$

$\therefore$ Matrix A is non-singular and hence A^{-1} exists.

We know that adj. $A = \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix} \left(\because \text{ Adj.} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{is} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \right)$

We know that $A^{-1} = \dfrac{1}{|A|}$ adj. $A = \dfrac{1}{14} \begin{bmatrix} 3 & 2 \\ -4 & 2 \end{bmatrix}$.

6. $\begin{bmatrix} \mathbf{-1} & \mathbf{5} \\ \mathbf{-3} & \mathbf{2} \end{bmatrix}$.

Sol. Let $\quad A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix} = -2 - (-15) = -2 + 15 = 13 \neq 0$

$\therefore$ A^{-1} exists.

We know that adj. $A = \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix} \left(\because \text{ Adj.} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \right)$

$\therefore \quad A^{-1} = \dfrac{1}{|A|}$ (adj. A) $= \dfrac{1}{13} \begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix}$.

7. $\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{3} \\ \mathbf{0} & \mathbf{2} & \mathbf{4} \\ \mathbf{0} & \mathbf{0} & \mathbf{5} \end{bmatrix}$.

Sol. Let $\quad A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix} \qquad \therefore \quad |A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$

Expanding along first row

$= 1(10 - 0) - 2(0 - 0) + 3(0 - 0) = 10 \neq 0$

$\therefore$ A^{-1} exists.

$A_{11} = + \begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = + (10 - 0) = 10,$

$A_{12} = - \begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = - (0 - 0) = 0,$

$A_{13} = + \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = (0 - 0) = 0,$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -(10 - 0) = -10,$$

$$A_{22} = +\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = (5 - 0) = 5,$$

$$A_{23} = -\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = -(0 - 0) = 0,$$

$$A_{31} = +\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = (8 - 6) = 2,$$

$$A_{32} = -\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -(4 - 0) = -4,$$

$$A_{33} = +\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = +(2 - 0) = 2$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} 10 & 0 & 0 \\ -10 & 5 & 0 \\ 2 & -4 & 2 \end{bmatrix}' = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj. A} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}.$$

8. $\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{3} & \mathbf{3} & \mathbf{0} \\ \mathbf{5} & \mathbf{2} & \mathbf{-1} \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$ $\quad \therefore \quad |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

Expanding along first row $|A| = 1(-3 - 0) - 0 + 0 = -3 \neq 0$

$$A_{11} = +\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = (-3 - 0) = -3,$$

$$A_{12} = -\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = -(-3 - 0) = 3,$$

$$A_{13} = +\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = +(6 - 15) = -9,$$

$$A_{21} = -\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = -(0 - 0) = 0,$$

$$A_{22} = +\begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = +(-1 - 0) = -1,$$

$$A_{23} = -\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -(2-0) = -2,$$

$$A_{31} = +\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = (0-0) = 0,$$

$$A_{32} = -\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = -(0-0) = 0,$$

$$A_{33} = +\begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = +(3-0) = 3$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} -3 & 3 & -9 \\ 0 & -1 & -2 \\ 0 & 0 & 3 \end{bmatrix}' = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj. A} = \frac{-1}{3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ -9 & -2 & 3 \end{bmatrix}.$$

9. $\begin{bmatrix} \mathbf{2} & \mathbf{1} & \mathbf{3} \\ \mathbf{4} & \mathbf{-1} & \mathbf{0} \\ \mathbf{-7} & \mathbf{2} & \mathbf{1} \end{bmatrix}$.

Sol. Let $|A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix}$

Expanding by first row,

$= 2(-1) - 1(4) + 3(8-7) = -2 - 4 + 3 = -3 \neq 0$

$\Rightarrow$ A is non-singular $\quad \therefore \; A^{-1}$ exists.

$$A_{11} = +\begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = -1, \qquad A_{12} = -\begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} = -4,$$

$$A_{13} = +\begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = 8 - 7 = 1, \qquad A_{21} = -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 5,$$

$$A_{22} = +\begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = 23, \qquad A_{23} = -\begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -11,$$

$$A_{31} = \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = 3, \qquad A_{32} = -\begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = 12,$$

$$A_{33} = \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -6$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} -1 & -4 & 1 \\ 5 & 23 & -11 \\ 3 & 12 & -6 \end{bmatrix}' = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj. A} = -\frac{1}{3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}.$$

10. $\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{0} & \mathbf{2} & \mathbf{-3} \\ \mathbf{3} & \mathbf{-2} & \mathbf{4} \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$ $\quad \therefore \quad |A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$

Expanding along first row,

$$= 1(8 - 6) - (-1)(0 + 9) + 2(0 - 6)$$
$$= 2 + 9 - 12 = -1 \neq 0$$

$\therefore$ A^{-1} exists.

$$A_{11} = + \begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = (8 - 6) = 2,$$

$$A_{12} = - \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -(0 + 9) = -9$$

$$A_{13} = + \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = +(0 - 6) = -6,$$

$$A_{21} = - \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = -(-4 + 4) = 0$$

$$A_{22} = + \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (4 - 6) = -2,$$

$$A_{23} = - \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -(-2 + 3) = -1$$

$$A_{31} = + \begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = 3 - 4 = -1,$$

$$A_{32} = - \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = -(-3 - 0) = 3$$

$$A_{33} = + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = (2 - 0) = 2$$

$$\therefore \quad \text{adj. A} = \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}' = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj. } A = \frac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix} \qquad \left(\because \frac{1}{-1} = \frac{-1}{1} = -1\right)$$

11. $\begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{\cos\alpha} & \mathbf{\sin\alpha} \\ \mathbf{0} & \mathbf{\sin\alpha} & \mathbf{-\cos\alpha} \end{bmatrix}$.

Sol. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$ $\quad \therefore |A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{vmatrix}$

Expanding along first row

$$= 1(-\cos^2\alpha - \sin^2\alpha) - 0 + 0 = -(\cos^2\alpha + \sin^2\alpha)$$

or $\quad |A| = -1 \neq 0$

$\therefore \quad A^{-1}$ exists.

$$A_{11} = +\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} = (-\cos^2\alpha - \sin^2\alpha)$$

$$= -(\cos^2\alpha + \sin^2\alpha) = -1$$

$$A_{12} = -\begin{vmatrix} 0 & \sin\alpha \\ 0 & -\cos\alpha \end{vmatrix} = -(0-0) = 0,$$

$$A_{13} = +\begin{vmatrix} 0 & \cos\alpha \\ 0 & \sin\alpha \end{vmatrix} = 0$$

$$A_{21} = -\begin{vmatrix} 0 & 0 \\ \sin\alpha & -\cos\alpha \end{vmatrix} = -(0-0) = 0,$$

$$A_{22} = +\begin{vmatrix} 1 & 0 \\ 0 & -\cos\alpha \end{vmatrix} = (-\cos\alpha - 0) = -\cos\alpha$$

$$A_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -(\sin\alpha - 0) = -\sin\alpha,$$

$$A_{31} = +\begin{vmatrix} 0 & 0 \\ \cos\alpha & \sin\alpha \end{vmatrix} = 0 - 0 = 0$$

$$A_{32} = -\begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -\sin\alpha,$$

$$A_{33} = + \begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix} = \cos\alpha.$$

$$\therefore \text{adj. } A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj. } A = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$(\because \ |A| = -1$, obtained above)

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}.$$

12. Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1}A^{-1}$.

Sol. Given: Matrix $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$.

Therefore $|A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$

$$\therefore A^{-1} = \frac{1}{|A|} \text{ adj. } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \left(\because \ \text{adj.} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\right)$$

Given: Matrix $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$$\therefore \quad |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$$

$$\therefore \quad B^{-1} = \frac{1}{|B|} \text{ adj. } B = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} = \frac{1}{-2} \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$$

$$\text{Now} AB = \left[\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\right] \left[\begin{bmatrix} 6 \\ 7 \end{bmatrix} \begin{bmatrix} 8 \\ 9 \end{bmatrix}\right] = \begin{bmatrix} 18 + 49 & 24 + 63 \\ 12 + 35 & 16 + 45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$$

$$\therefore \quad |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix} = 67(61) - 87(47) = 4087 - 4089$$

$$= -2 \neq 0$$

$$\therefore \ \text{L.H.S.} = (AB)^{-1} = \frac{1}{|AB|} \text{ adj. } (AB)$$

$$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \quad ...(i)$$

$$\text{R.H.S.} = B^{-1}A^{-1} = \frac{-1}{2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$= \frac{-1}{2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix} \quad ...(ii)$$

From (i) and (ii) we have L.H.S. = R.H.S. *i.e.,* $(AB)^{-1} = B^{-1}A^{-1}$.

13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$. Hence find A^{-1}.

Sol. Given: $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\therefore A^2 = A \,.\, A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

L.H.S. $= A^2 - 5A + 7I = A^2 - 5A + 7I_2$

(I is I_2 here because A is 2×2)

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 8-15 & 5-5 \\ -5+5 & 3-10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = \text{R.H.S.}$$

$$\Rightarrow \quad A^2 - 5A + 7I_2 = O \quad ...(i)$$

Hence to find A^{-1}. Multiplying both sides of eqn. (i) by A^{-1},

$$A^2A^{-1} - 5AA^{-1} + 7I_2A^{-1} = O.A^{-1}$$

$$\Rightarrow \quad A - 5I_2 + 7A^{-1} = O$$

[$\because A^2A^{-1} = A.A.A^{-1} = AI_2 = A$ and $AA^{-1} = I_2$ and $IB = B$]

$$\Rightarrow \quad 7A^{-1} = -A + 5I_2$$

$$= -\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$\Rightarrow \quad 7A^{-1} = \begin{bmatrix} -3+5 & -1+0 \\ 1+0 & -2+5 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}.$$

Caution. Because we were to find; **Hence** A^{-1} *i.e.,* A^{-1} from $A^2 - 5A + 7I = O$, so don't use

$$A^{-1} = \frac{\text{adj. A}}{|A|} \text{ to find } A^{-1} \text{ here.}$$

14. For the matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find numbers *a* and *b* such that $A^2 + aA + bI = O$.

Sol. Given: Matrix A = $\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$$\therefore \quad A^2 = A.A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$

Putting values of A^2 and A in $A^2 + aA + bI_2 = O$,
(Here I is I_2 because A is 2 × 2), we have

$$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + a\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = O$$

$$\Rightarrow \quad \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3a & 2a \\ a & a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 11+3a+b & 8+2a+0 \\ 4+a+0 & 3+a+b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Equating corresponding entries, we have

$11 + 3a + b = 0$ ...(*i*)

$8 + 2a = 0 \quad (\Rightarrow \; 2a = -8 \; \Rightarrow \; a = -4)$

$4 + a = 0 \quad (\Rightarrow \; a = -4), \; 3 + a + b = 0$ ...(*ii*)

Value of $a = -4$ is same from both equations.
Therefore, $a = -4$ is correct.
Putting $a = -4$ in (*i*), $11 - 12 + b = 0$ or $b - 1 = 0$ *i.e.,* $b = 1$
Again putting $a = -4$ in (*ii*), $3 - 4 + b = 0$
i.e., $-1 + b = 0$ or $b = 1$
The two values of $b = 1$ are same from both equations.
$\therefore \quad A^2 + aA + bI = 0$ holds true when $a = -4$ and $b = 1$.

15. For the matrix A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$, show that $A^3 - 6A^2 + 5A + 11I = O$. Hence find A^{-1}.

Sol. $A^2 = A \,.\, A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Performing row by column multiplication,

$$= \begin{bmatrix} 1+1+2 & 1+2-1 & 1-3+3 \\ 1+2-6 & 1+4+3 & 1-6-9 \\ 2-1+6 & 2-2-3 & 2+3+9 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \qquad ...(i)$$

$$\therefore \quad A^3 = A^2 \,.\, A = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2+2 & 4+4-1 & 4-6+3 \\ -3+8-28 & -3+16+14 & -3-24-42 \\ 7-3+28 & 7-6-14 & 7+9+42 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

Now, putting values of A^3, A^2, A and I_3 in $A^3 - 6A^2 + 5A + 11I_3$

(Here I is I_3 because matrix A is of order 3×3)

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6 \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5 \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 8-24+5 & 7-12+5 & 1-6+5 \\ -23+18+5 & 27-48+10 & -69+84-15 \\ 32-42+10 & -13+18-5 & 58-84+15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O_{3 \times 3}.$$

$\therefore \quad A^3 - 6A^2 + 5A + 11I_3 = O_{3 \times 3}$ **(proved above)**

Hence find A^{-1}.

(See caution at the end of solution of Q. No. 13)

Now multiplying both sides by A^{-1}.

$(A^{-1}A)\, A^2 - 6(A^{-1}A)\, A + 5(A^{-1}A) + 11A^{-1}I_3 = A^{-1} \,.\, O_{3 \times 3}$

$\Rightarrow \; A^2 - 6IA + 5I + 11A^{-1} = 0 \qquad (\because \; A^{-1}A = I \text{ and } A^{-1}0 = 0)$

$\Rightarrow \; A^2 - 6A + 5I + 11A^{-1} = 0$

$\Rightarrow \; 11A^{-1} = 6A - 5I - A^2$

or $\quad 11A^{-1} = 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$- \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \quad \text{(From } (i))$$

or $\quad 11A^{-1} = \begin{bmatrix} 6-5-4 & 6-2 & 6-1 \\ 6+3 & 12-5-8 & -18+14 \\ 12-7 & -6+3 & 18-5-14 \end{bmatrix}$

or $\quad 11A^{-1} = \begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$ or $A^{-1} = \dfrac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$.

16. If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$, verify that $A^3 - 6A^2 + 9A - 4I = O$ and hence find A^{-1}.

Sol. Given: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\therefore \quad A^2 = A.A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} 4+1+1 & -2-2-1 & 2+1+2 \\ -2-2-1 & 1+4+1 & -1-2-2 \\ 2+1+2 & -1-2-2 & 1+1+4 \end{bmatrix}$

$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$

$\therefore \quad A^3 = A^2 . A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 12+5+5 & -6-10-5 & 6+5+10 \\ -10-6-5 & 5+12+5 & -5-6-10 \\ 10+5+6 & -5-10-6 & 5+5+12 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$

L.H.S. $= A^3 - 6A^2 + 9A - 4I = A^3 - 6A^2 + 9A - 4I_3$
(Here I is I_3 because A is 3×3)

Putting values

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - 6 \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$+ 9 \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix}$$

$$+ \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 22-36 & -21+30 & 21-30 \\ -21+30 & 22-36 & -21+30 \\ 21-30 & -21+30 & 22-36 \end{bmatrix} + \begin{bmatrix} 18-4 & -9-0 & 9-0 \\ -9-0 & 18-4 & -9-0 \\ 9-0 & -9-0 & 18-4 \end{bmatrix}$$

$$= \begin{bmatrix} -14 & 9 & -9 \\ 9 & -14 & 9 \\ -9 & 9 & -14 \end{bmatrix} + \begin{bmatrix} 14 & -9 & 9 \\ -9 & 14 & -9 \\ 9 & -9 & 14 \end{bmatrix}$$

$$= \begin{bmatrix} -14+14 & 9-9 & -9+9 \\ 9-9 & -14+14 & 9-9 \\ -9+9 & 9-9 & -14+14 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{R.H.S.}$$

$\therefore \quad A^3 - 6A^2 + 9A - 4I_3 = O$ ...(*i*)

Hence to find A^{-1}

(See **caution** at the end of solution of Q. No. 13)

Multiplying both sides of (*i*) by A^{-1},

$$A^3A^{-1} - 6A^2\,A^{-1} + 9A\,A^{-1} - 4I_3A^{-1} = O.A^{-1}$$

or $\quad A^2 - 6A + 9I_3 - 4A^{-1} = 0$

$[\because \quad A^3A^{-1} = A^2AA^{-1} = A^2I = A^2$ etc. and also $IB = B]$

$\Rightarrow \quad -4A^{-1} = -A^2 + 6A - 9I_3$

Dividing by -4, $A^{-1} = \frac{1}{4}A^2 - \frac{6}{4}A + \frac{9}{4}I_3 \qquad = \frac{1}{4}[A^2 - 6A + 9I_3]$

$$\Rightarrow A^{-1} = \frac{1}{4}\left[\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right]$$

$$\Rightarrow A^{-1} = \frac{1}{4}\left[\begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\right]$$

$$= \frac{1}{4}\begin{bmatrix} 6-12+9 & -5+6+0 & 5-6+0 \\ -5+6+0 & 6-12+9 & -5+6+0 \\ 5-6+0 & -5+6+0 & 6-12+9 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

17. Let A be a non-singular matrix of order 3 × 3. Then |adj. A| is equal to

(A) | A | **(B) $|A|^2$** **(C) $|A|^3$** **(D) 3 | A |.**

Sol. If A is a non-singular matrix of order $n \times n$,

then **| adj. A | = $|A|^{n-1}$.**

Putting $n = 3$, $|\text{adj. } A| = |A|^2$

$\therefore$ Option (B) is the correct answer.

18. If A is an invertible matrix of order 2, then det (A^{-1}) is equal to

(A) det A **(B) $\frac{1}{\det A}$** **(C) 1** **(D) 0.**

Sol. We know that $AA^{-1} = I$ for every invertible matrix A.

Taking determinants on both sides, we have

$|AA^{-1}| = |I| \Rightarrow |A|\,|A^{-1}| = 1$

Dividing by | A |, $|A^{-1}| = \frac{1}{|A|}$ *i.e.,* $\det(A^{-1}) = \frac{1}{\det A}$

$\therefore$ Option (B) is the correct answer.