Exercise 4.3

1. Find the area of the triangle with vertices at the points given in each of the following:

(*i*) (1, 0), (6, 0), (4, 3) **(*ii*) (2, 7), (1, 1), (10, 8)**

(*iii*) (– 2, – 3), (3, 2), (– 1, – 8).

Sol. (*i*) Area of the triangle having vertices at (1, 0), (6, 0), (4, 3)

$$= \text{Modulus of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along first row,

$$= \frac{1}{2}[1(0 - 3) - 0(6 - 4) + 1(18 - 0)]$$

i.e., Area of triangle = modulus of $\frac{1}{2}(-3 + 18)$

$$= \left|\frac{15}{2}\right| = \frac{15}{2} \text{ sq. units}$$

($\because$ Modulus of a positive number is number itself)

(*ii*) Area of the triangle having vertices at (2, 7), (1, 1), (10, 8).

$$= \text{Modulus of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

Expanding along first row

$$= \frac{1}{2}[2(1 - 8) - 7(1 - 10) + 1(8 - 10)]$$

$$= \frac{1}{2}[2(-7) - 7(-9) - 2] = \frac{1}{2}(-14 + 63 - 2)$$

$$= \frac{1}{2}(63 - 16)$$

i.e., Area of triangle $= \left|\frac{47}{2}\right| = \frac{47}{2}$ sq. units.

($\because$ Modulus of a positive real number is number itself)

(*iii*) Area of the triangle having vertices at

(– 2, – 3), (3, 2), (– 1, – 8) is

$$\text{Modulus of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}[-2(2 + 8) - (-3)(3 + 1) + 1(-24 + 2)]$$

$$= \frac{1}{2}[-2(10) + 3(4) - 22] = \frac{1}{2}(-20 + 12 - 22)$$

$= \frac{1}{2}(-42 + 12) = \frac{1}{2}(-30) = -15$

$\therefore$ Area of triangle = Modulus of -15 *i.e.,* = $|-15|$

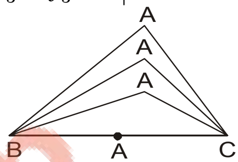
= 15 sq. units

($\because$ Modulus of a negative real number is negative of itself)

2. Show that the points A(a, $b + c$), B(b, $c + a$), C(c, $a + b$) are collinear.

Sol. The given points are A(a, $b + c$), B(b, $c + a$), C(c, $a + b$).

$\therefore$ Area of triangle ABC is modulus of $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$



$$= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

Expanding along first row,

$$= \frac{1}{2}[a(c + a - a - b) - (b + c)(b - c) + 1(b(a + b) - c(c + a))]$$

$$= \frac{1}{2}[a(c - b) - (b^2 - c^2) + (ab + b^2 - c^2 - ac)]$$

$$= \frac{1}{2}(ac - ab - b^2 + c^2 + ab + b^2 - c^2 - ac) = \frac{1}{2}(0) = 0$$

i.e., Area of ΔABC = 0

$\therefore$ Points A, B, C are collinear (See above figure).

3. Find values of k if area of triangle is 4 sq. units and vertices are:

(*i*) (k, 0), (4, 0), (0, 2) **(*ii*) (– 2, 0), (0, 4), (0, k).**

Sol. (*i*) **Given:** Area of the triangle whose vertices are (k, 0), (4, 0), (0, 2) is 4 sq. units.

$$\Rightarrow \text{ Modulus of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$$

$$\Rightarrow \text{ Modulus of } \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix} = 4$$

Expanding along first row, $\left|\frac{1}{2}\{k(0 - 2) - 0 + 1(8 - 0)\}\right| = 4$

$\Rightarrow \left|\frac{1}{2}(-2k + 8)\right| = 4 \Rightarrow |-k + 4| = 4$

$\Rightarrow -k + 4 = \pm 4$

[$\because$ If $x \in$ R and $|x| = a$ where $a \geq 0$, then $x = \pm a$]

Taking positive sign, $-k + 4 = 4$

$\Rightarrow -k = 0 \qquad \Rightarrow k = 0$

Taking negative sign, $-k + 4 = -4$

$\Rightarrow \quad -k = -8 \qquad \Rightarrow \quad k = 8 \qquad$ Hence $\qquad k = 0, k = 8.$

(*ii*) **Given:** Area of the triangle whose vertices are $(-2, 0)$, $(0, 4)$, $(0, k)$ is 4 sq. units.

$$\Rightarrow \quad \text{Modulus of } \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 4$$

$$\Rightarrow \quad \text{Modulus of } \frac{1}{2}\begin{vmatrix} -2 & 0 & 1 \\ 0 & 4 & 1 \\ 0 & k & 1 \end{vmatrix} = 4$$

Expanding along first row, $\left|\frac{1}{2}\{-2(4-k) - 0 + 1(0-0)\}\right| = 4$

$$\Rightarrow \quad \left|\frac{1}{2}(-8+2k)\right| = 4 \quad \Rightarrow \quad |-4+k| = 4$$

$\Rightarrow \qquad -4 + k = \pm 4$

($\because$ If $|x| = a$ where $a \geq 0$, then $x = \pm a$)

Taking positive sign, $-4 + k = 4 \quad \Rightarrow \quad k = 4 + 4 = 8$

Taking negative sign, $-4 + k = -4 \quad \Rightarrow \quad k = 0$

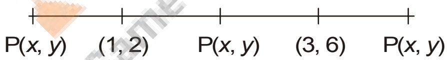
Hence, $\qquad k = 0, k = 8.$

4. (*i*) Find the equation of the line joining (1, 2) and (3, 6) using determinants.

(*ii*) Find the equation of the line joining (3, 1) and (9, 3) using determinants.

Sol. (*i*) Let P(x, y) be **any** point on the line joining the points (1, 2) and (3, 6).

$\therefore$ Three points are collinear.



$\therefore$ Area of triangle that could be formed by them is zero.

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0 \qquad \left[\left|\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}\right|\right]$$

Multiplying both sides by 2, and expanding the determinant on left hand side along first row,

$$x(2-6) - y(1-3) + 1(6-6) = 0$$

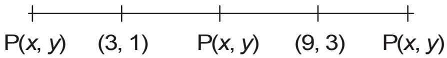
$\Rightarrow \quad -4x + 2y = 0.$ Dividing by -2, $2x - y = 0$

or $\quad -y = -2x \quad$ *i.e.,* $\quad y = 2x$ which is the required equation of the line.

(*ii*) Let P(x, y) be any point on the line joining the points (3, 1) and (9, 3).

$\therefore$ Three points are collinear.

Aapkitab
Same textbooks, lock away



$\therefore$ Area of triangle that could be formed by them is zero.

$$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Multiplying both sides by 2 and expanding the determinant on left hand side along first row,

$$x(1 - 3) - y(3 - 9) + 1(9 - 9) = 0$$

$$\Rightarrow \quad - 2x + 6y = 0$$

Dividing by -2, $x - 3y = 0$ which is the required equation of the line.

5. If area of triangle is 35 sq. units with vertices (2, – 6), (5, 4) and (k, 4). Then k is

(A) 12 (B) – 2 (C) – 12, – 2 (D) 12, – 2.

Sol. Given: Area of triangle having vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq. units.

$$\therefore \text{ Modulus of } \left(\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}\right) = 35 \quad \text{(Given)}$$

Expanding along first row,

$$\left|\frac{1}{2}\{2(4 - 4) - (-6)(5 - k) + 1(20 - 4k)\}\right| = 35$$

$$\Rightarrow \quad \left|\frac{1}{2}\{0 + 30 - 6k + 20 - 4k\}\right| = 35$$

$$\Rightarrow \quad \left|\frac{1}{2}(50 - 10k)\right| = 35 \Rightarrow |25 - 5k| = 35$$

$$\Rightarrow \quad 25 - 5k = \pm 35$$

[$\because$ If $|x| = a$ where $a \geq 0$, then $x = \pm a$]

Taking positive sign, $25 - 5k = 35 \Rightarrow -5k = 10$

$$\Rightarrow \quad k = \frac{-10}{5} = -2$$

Taking negative sign, $25 - 5k = -35$

$\Rightarrow \quad -5k = -60 \quad \Rightarrow \quad k = 12$

Thus, $k = 12, -2$ $\quad\therefore$ Option (D) is the correct answer.

copykitab
same textbooks, klick away