

NCERT Class 12 Maths

Solutions

Chapter - 4

Exercise 4.1

1. $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Sol. Determinant $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix} = 2(-1) - 4(-5)$
 $= -2 + 20 = 18.$

2. (i) $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix}$ (ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix}$

Sol. (i) Determinant $\begin{vmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{vmatrix} = \cos \theta (\cos \theta) - (-\sin \theta) (\sin \theta)$
 $= \cos^2 \theta + \sin^2 \theta = 1.$

(ii) $\begin{vmatrix} x^2 - x + 1 & x - 1 \\ x + 1 & x + 1 \end{vmatrix} = (x^2 - x + 1)(x + 1) - (x + 1)(x - 1)$
 $= (x^3 + 1) - (x^2 - 1) = x^3 - x^2 + 2.$

3. If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$.

Sol. Given: Matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$

$$\therefore 2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 \times 1 & 2 \times 2 \\ 2 \times 4 & 2 \times 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\therefore \text{L.H.S.} = |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 2(4) - 4(8) = 8 - 32 = -24 \quad \dots(i)$$

$$\text{R.H.S.} = 4|A| = 4 \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 4(2 - 8) = 4(-6) = -24 \quad \dots(ii)$$

From (i) and (ii), we have L.H.S. = R.H.S.

4. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that $|3A| = 27|A|$.

Sol. $3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$

$$\therefore \text{L.H.S.} = |3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$$

Expanding along first column = $3[36 - 0] = 3 \times 36 = 108$.

$$\text{Also R.H.S.} = 27|A| = 27 \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$$

Expanding along first column

$$\begin{aligned} &= 27 [1(4 - 0)] = 27 \times 4 \\ &= 108 = \text{L.H.S.} \end{aligned} \quad \therefore |3A| = 27|A|$$

5. Evaluate the determinants:

$$(i) \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

$$(ii) \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

$$(iii) \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$$

$$(iv) \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

Sol. (i) Given determinant is $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ and is of order 3.

Expanding along first row

$$\begin{aligned} &= 3 \begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2) \begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix} \\ &= 3(0 - 5) + 1(0 - (-3)) - 2(0 - 0) \\ &= -15 + 3 - 0 = -12. \end{aligned}$$

(ii) Given determinant is $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$ and is of order 3.

Expanding along first row

$$\begin{aligned} &= 3 \begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} - (-4) \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + 5 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} \\ &= 3(1 + 6) + 4(1 - (-4)) + 5(3 - 2) \\ &= 3(7) + 4(5) + 5(1) = 21 + 20 + 5 = 46. \end{aligned}$$

(iii) Given determinant is $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ and is of order 3.

Expanding along first row

$$\begin{aligned} &= 0 \begin{vmatrix} 0 & -3 \\ 3 & 0 \end{vmatrix} - 1 \begin{vmatrix} -1 & -3 \\ -2 & 0 \end{vmatrix} + 2 \begin{vmatrix} -1 & 0 \\ -2 & 3 \end{vmatrix} \\ &= 0(0 + 9) - (0 - 6) + 2(-3 - 0) = 0 + 6 - 6 = 0. \end{aligned}$$

(iv) Given determinant is $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ and is of order 3.

Expanding along first row

$$\begin{aligned} &= 2 \begin{vmatrix} 2 & -1 \\ -5 & 0 \end{vmatrix} - (-1) \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2) \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} \\ &= 2(0 - 5) + (0 + 3) - 2(0 - 6) = -10 + 3 + 12 = 5. \end{aligned}$$

6. If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$, find $|A|$.

Sol. Matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$ $\therefore \text{Det } A \text{ i.e., } |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$

Expanding along first row

$$\begin{aligned}
 &= 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2) \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix} \\
 &= (-9 - (-12)) - (-18 - (-15)) - 2(8 - 5) \\
 &= -9 + 12 - (-18 + 15) - 2(3) = 3 - (-3) - 6 \\
 &= 3 + 3 - 6 = 0
 \end{aligned}$$

Note. Such a matrix A for which $|A| = 0$ is called a **singular matrix**.

7. Find values of x , if

$$(i) \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix} \qquad (ii) \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$$

Sol. (i) **Given:** $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$$\Rightarrow 2 - 20 = 2x^2 - 24 \Rightarrow -18 = 2x^2 - 24$$

$$\Rightarrow -2x^2 = -24 + 18 = -6$$

$$\text{Dividing by } -2, x^2 = 3$$

$$\text{Taking square roots, } x = \pm \sqrt{3}.$$

(ii) **Given:** $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$$\Rightarrow 10 - 12 = 5x - 6x \Rightarrow -2 = -x$$

$$\text{Dividing by } -1, 2 = x \text{ i.e., } x = 2.$$

8. If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, then x is equal to

(A) 6 (B) ± 6 (C) -6 (D) 0.

Sol. Given: $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$$\Rightarrow x^2 - 36 = 36 - 36 \Rightarrow x^2 - 36 = 0 \Rightarrow x^2 = 36$$

Taking square roots, $x = \pm 6$. $\therefore$ Option (B) is the correct answer.