Book Name: NCERT Solutions

Exercise 2.1

Question 1:

Find The principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

Solution 1:

Let $\sin^{-1}\left(-\frac{1}{2}\right)=y,$

Then Sin y= $\left(-\frac{1}{2}\right)$

$=-\sin\left(\frac{\pi}{6}\right)=\sin\left(-\frac{\pi}{6}\right).$

Range of the principal value of $\sin^{-1}x$ is

$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ *and* $\sin\left(-\frac{\pi}{6}\right)=-\frac{1}{2},$

Thus, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ *is* $-\frac{\pi}{6},$

Question 2:

Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Solution 2:

Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y.$

$\Rightarrow \cos y=\frac{\sqrt{3}}{2}=\cos\left(\frac{\pi}{6}\right)$

Range of the principal value of $\cos^{-1}x$ is

$[0,\pi]$ and $\cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}.$

Thus, the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) is \frac{\pi}{6}$.

Question 3:

Find the principal value of $\text{cosec}^{-1}(2)$

Solution 3:

Let $\text{cosec}^{-1}(2) = y$.

$\Rightarrow \text{cosec } y = 2 = \text{cosec}\left(\frac{\pi}{6}\right)$.

Range of the principal value of $\text{cosec}^{-1}x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

Thus, the principal value of $\text{cosec}^{-1}(2)$ is $\frac{\pi}{6}$.

Question 4:

Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

Solution 4:

Let $\tan^{-1}\left(-\sqrt{3}\right) = y$.

$\Rightarrow \tan y = -\sqrt{3} = -\tan\frac{\pi}{3} = \tan\left(-\frac{\pi}{3}\right)$.

Range of the principal value of $\tan^{-1}x$ is

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{3}\right) is -\sqrt{3}$.

Thus, known that the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $-\frac{\pi}{3}$.

Question 5:

Find the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$

Solution 5:

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y.$

$\Rightarrow \cos y = -\frac{1}{2} = -\cos\left(\frac{\pi}{3}\right) = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right).$

Range of the principal value of $\cos^{-1} x$ is

$[0, \pi]$ and $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}.$

Thus, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\left(\frac{2\pi}{3}\right).$

Question 6:

Find the principal value of $\tan^{-1}(-1)$

Solution 6:

Let $\tan^{-1}(-1) = y.$

$\Rightarrow \tan y = -1 = -\tan\left(\frac{\pi}{4}\right)$

$= \tan\left(-\frac{\pi}{4}\right).$

Range of the principal value of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ and $\tan\left(-\frac{\pi}{4}\right) = -1.$

Thus, the principal value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}.$

Question 7:

Find the principal value of

Solution 7:

Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right) = y.$

$\Rightarrow \sec y = \frac{2}{\sqrt{3}} = \sec\left(\frac{\pi}{6}\right).$

Range of the principal value of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$ and $\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}.$

Thus, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is $\frac{\pi}{6}$.

Question 8:

Find the principal value of $\cot^{-1}\left(\sqrt{3}\right)$

Solution 8:

Let $\cot^{-1}\left(\sqrt{3}\right) = y$.

$\Rightarrow \cot y = \sqrt{3} = \cot\left(\frac{\pi}{6}\right)$.

Range of the principal value of $\cot^{-1} x$ is $(0, n)$ and $\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$.

Thus, the principal value of $\cot^{-1}\left(\sqrt{3}\right)$ is $\frac{\pi}{6}$.

Question 9:

Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Solution 9:

Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) = y$.

$\Rightarrow \cos y = -\frac{1}{\sqrt{2}}$

$= -\cos\left(\frac{\pi}{4}\right)$

$= \cos\left(\pi - \frac{\pi}{4}\right)$

$= \cos\left(\frac{3\pi}{4}\right)$.

Range of the principal value of $\cos^{-1}x$ is $[0,n]$ and $\cos\left(\frac{3\pi}{4}\right)=-\frac{1}{\sqrt{2}}$.

Thus, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

Question 10:

Find the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$

Solution 10:

Let $\text{cosec}^{-1}\left(-\sqrt{2}\right)=y.$

$\Rightarrow \text{cosec}\, y=-\sqrt{2}$

$=-\text{cosec}\left(\frac{\pi}{4}\right)$

$=\text{cosec}\left(-\frac{\pi}{4}\right)$

Range of the principal value of $\text{cosec}^{-1}x$ is

$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and $\text{cosec}\left(-\frac{\pi}{4}\right)=-\sqrt{2}$.

Thus, the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$.

Question 11:

Find the value of $\tan^{-1}(1)+\cos^{-1}\left(-\frac{1}{2}\right)+\sin^{-1}\left(-\frac{1}{2}\right)$

Solution 11:

Let $\tan^{-1}(1)=x.$

$\Rightarrow \tan\ x=1=\tan\frac{\pi}{4}.$

$\therefore \tan^{-1}(1)=\frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y.$

$\Rightarrow \cos y = -\frac{1}{2}$

$= -\cos\left(\frac{\pi}{3}\right)$

$= \cos\left(\pi - \frac{\pi}{3}\right)$

$= \cos\left(\frac{2\pi}{3}\right).$

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z.$

$\Rightarrow \sin z = -\frac{1}{2}$

$= -\sin\left(\frac{\pi}{6}\right)$

$= \sin\left(-\frac{\pi}{6}\right).$

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\therefore \tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$

$= \frac{3\pi + 8\pi - 2\pi}{12}$

$= \frac{9\pi}{12}$

$= \frac{3\pi}{4}$

Question 12:

Find the value of $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

Solution 12:

Let $\cos^{-1}\left(\frac{1}{2}\right)=x.$

$\Rightarrow \cos x=\frac{1}{2}=\cos\left(\frac{\pi}{3}\right).$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right)=y.$

$\Rightarrow \sin y=\frac{1}{2}=\sin\left(\frac{\pi}{6}\right).$

$\therefore \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$

$\therefore \cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}+\frac{2\pi}{6}+\frac{\pi}{3}+\frac{\pi}{3}$

$=\frac{2\pi}{3}$

Question 13:

Find the value of if $\sin^{-1}x=y$, then

(A) $0\le y\le\pi$ (B) $-\frac{\pi}{2}\le y\le\frac{\pi}{2}$ (C) $0<y<\pi$ (D) $-\frac{\pi}{2}<y<\frac{\pi}{2}$

Solution 13:

It is given that $\sin^{-1}x=y$.

Range of the principal value of $\sin^{-1}x$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right].$

Thus, $-\frac{\pi}{2}\le y\le\frac{\pi}{2}.$

Question 14:

Find the value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to

(A) 0 (B) $-\frac{\pi}{3}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

Solution 14:

Let $\tan^{-1}\sqrt{3}=x$.

$\Rightarrow \tan x=\sqrt{3}=\tan\frac{\pi}{3}$

Range of the principal value of $\tan^{-1}x$ is $\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$.

$\therefore \tan^{-1}\sqrt{3}=\frac{\pi}{3}$

Let $\sec^{-1}(-2)=y$.

$\Rightarrow \sec y=-2$

$=-\sec\left(\frac{\pi}{3}\right)$

$=\sec\left(\pi-\frac{\pi}{3}\right)$

$=\sec\frac{2\pi}{3}$.

Range of the principal value of $\sec^{-1}x$ is $[0,\pi]-\left\{\frac{\pi}{2}\right\}$.

$\therefore \sec^{-1}(-2)=\frac{2\pi}{3}$

Thus, $\tan^{-1}\left(\sqrt{3}\right)-\sec^{-1}(-2)=\frac{\pi}{3}-\frac{2\pi}{3}=-\frac{\pi}{3}$