Exercise 1.1

Question 1: Determine whether each of the following relations are reflexive, symmetric and transitive.

(i) Relation R in the set A = {1, 2, 3...13, 14} defined as R = {(x, y): 3x - y = 0}

(ii) Relation R in the set N of natural numbers defined as R = {(x, y): y = x + 5 and x < 4}

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6) as R= {(x, y): y is divisible by x}

(iv) Relation R in the set Z of all integers defined as R = {(x, y): x - y is as integer}

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) R = {(x, y): x and y work at the same place}

(b) R = {(x, y): x and y live in the same locality}

(c) R = {(x, y): x is exactly 7 cm taller than y}

(d) R = {(x, y): x is wife of y}

(e) R = {(x, y): x is father of y}

Solution:

(i)

R = {(1, 3), (2, 6), (3, 9), (4, 12)}

R is not reflexive because (1, 1), (2, 2) … and (14, 14) ∉ R.

R is not symmetric because (1, 3) ∈ R, but (3, 1) ∉ R. [since 3(3) − 1 ≠ 0]

R is not transitive because (1, 3) and (3, 9) ∈ R, but (1, 9) ∉ R. [3(1) − 9 ≠ 0]

So, R is not reflexive, symmetric or transitive.

(ii)

R = {(1, 6), (2, 7), (3, 8)}

(1, 1) ∉ R.

So R is not reflexive.

(1, 6) ∈ R but (6, 1) ∉ R.

So R is not symmetric.

There isn't any ordered pair in R such that (x, y) and (y, z) both ∈ R, so (x, z) cannot belong to R.

So R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii)

R = {(x, y) : y is divisible by x}

We know that any number other than 0 is divisible by itself.

Thus, (x, x) ∈ R

So, R is reflexive.

(2, 4) ∈ R [because 4 is divisible by 2]

But (4, 2) ∉ R [since 2 is not divisible by 4]

So, R is not symmetric.

Let (x, y) and (y, z) ∈ R. So, y is divisible by x and z is divisible by y.

So, z is divisible by x ⇒ (x, z) ∈ R

So, R is transitive.

So, R is reflexive and transitive but not symmetric.

(iv)

R = {(x, y): x − y is an integer}

For x ∈ **Z**, (x, x) ∈ R because x − x = 0 is an integer.

So, R is reflexive.

For x, y ∈ **Z**, if (x, y) ∈ R, then x − y is an integer ⇒ (y − x) is an integer.

So, (y, x) ∈ R

So, R is symmetric.

Let (x, y) and (y, z) ∈ R, where x, y, z ∈ **Z**.

⇒ (x − y) and (y − z) are integers.

⇒ x − z = (x − y) + (y − z) is an integer.

So, (x, z) ∈ R

So, R is transitive.

So, R is reflexive, symmetric, and transitive.

(v)

(a)

R = {(x, y): x and y work at the same place}

⇒ (x, x) ∈ R

So, R is reflexive.

If (x, y) ∈ R, then x and y work at the same place ⇒ y and x also work at the same place.

⇒ (y, x) ∈ R.

So, R is symmetric.

Let (x, y), (y, z) ∈ R

⇒ x and y work at the same place and y and z work at the same place.

⇒ x and z also work at the same place.

⇒ (x, z) ∈ R

So, R is transitive.

So, R is reflexive, symmetric and transitive.

(b)

R = {(x, y): x and y live in the same locality}

(x, x) ∈ R

So, R is reflexive.

If (x, y) ∈ R, then x and y live in the same locality.

⇒ y and x also live in the same locality.

⇒ (y, x) ∈ R

So, R is symmetric.

Let (x, y) ∈ R and (y, z) ∈ R.

⇒ x and y live in the same locality and y and z live in the same locality.

⇒ x and z also live in the same locality.

⇒ (x, z) ∈ R

So, R is transitive.

So, R is reflexive, symmetric and transitive.

(c)

R = {(x, y): x is exactly 7 cm taller than y}

Clearly, (x, x) ∉ R

So, R is not reflexive.

Let (x, y) ∈ R ⇒ x is exactly 7 cm taller than y.

Then, y is clearly not taller than x.

∴ (y, x) ∉ R

So, R is not symmetric.

Let (x, y), (y, z) ∈ R.

⇒ x is exactly 7 cm taller than y and y is exactly 7 cm taller than z.

⇒ x is exactly 14 cm taller than z.

So, (x, z) ∉ R

So, R is not transitive.

So, R is not reflexive, symmetric or transitive.

(d)

R = {(x, y): x is the wife of y}

Clearly, (x, x) ∉ R

So, R is not reflexive.

Let (x, y) ∈ R

⇒ x is the wife of y.

So y is not the wife of x.

∴ (y, x) ∉ R

So, R is not transitive.

Let (x, y), (y, z) ∈ R

⇒ x is the wife of y and y is the wife of z, which is not possible.

∴ (x, z) ∉ R

So, R is not transitive.

So, R is not reflexive, symmetric or transitive.

(e)

R = {(x, y): x is the father of y}

Clearly (x, x) ∉ R

So, R is not reflexive.

Let (x, y) ∈ R

⇒ x is the father of y.

⇒ y is not the father of x.

∴ (y, x) ∉ R

So, R is not symmetric.

Let (x, y) ∈ R and (y, z) ∈ R.

⇒ x is the father of y and y is the father of z.

⇒ x is not the father of z.

∴ (x, z) ∉ R

∴So, R is not transitive.

So, R is not reflexive, symmetric or transitive.

Question 2: Show that the relation R in the set **R** of real numbers, defined as R = {(a, b): $a \le b^2$} is neither reflexive nor symmetric nor transitive.

Solution:

(i) R = {(a, b): $a \le b^2$}

$\left(\frac{1}{2}, \frac{1}{2}\right) \notin R$,

Because, $\frac{1}{2} > \left(\frac{1}{2}\right)^2$

R is not reflexive.

(1, 4) ∈ R as 1 < 4

But, 4 is not less than 1^2.

∴ (4, 1) ∉ R

∴ R is not symmetric.

(3, 2), (2, 1.5) ∈ R [Because $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$]

$3 > (1.5)^2 = 2.25$

∴ (3, 1.5) ∉ R

∴ R is not transitive.

R is neither reflexive, nor symmetric, nor transitive.

Question 3: Check whether the relation R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b): b = a + 1} is reflexive, symmetric or transitive.

Solution:

A = {1, 2, 3, 4, 5, 6}.

R = {(a, b): b = a + 1}

R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)} (a, a) ∉ R, a ∈ A.

(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) ∉ R

∴ R is not reflexive.

(1, 2) ∈ R, but (2, 1) ∉ R.

∴ R is not symmetric.

(1, 2), (2, 3) ∈ R

(1, 3) ∉ R

∴ R is not transitive

R is neither reflexive, nor symmetric, nor transitive.

Question 4: Show that the relation R in **R** defined as R = {(a, b): a ≤ b}, is reflexive and transitive but not symmetric.

Solution:

R = {(a, b): a ≤ b}

(a, a) ∈ R

∴ R is reflexive.

(2, 4) ∈ R (as 2 < 4)

(4, 2) ∉ R as 4 > 2.

∴ R is not symmetric.

(a, b), (b, c) ∈ R.

a ≤ b and b ≤ c

⇒ a ≤ c

⇒ (a, c) ∈ R

∴ R is transitive.

R is reflexive and transitive but not symmetric

Question 5: Check whether the relation R in **R** defined as R = {(a, b): $a \le b^3$} is reflexive, symmetric or transitive.

Solution:

R = {(a, b): $a \le b^3$}

$\left(\frac{1}{2}, \frac{1}{2}\right) \notin$ R, since, $\frac{1}{2} > \left(\frac{1}{2}\right)^3$

∴ R is not reflexive.

(1, 2) ∈ R (as $1 < 2^3 = 8$)

(2, 1) ∉ R (as $2^3 > 1$)

∴ R is not symmetric.

$\left(3, \frac{3}{2}\right), \left(\frac{3}{2}, \frac{6}{5}\right) \in$ R, since $3 < \left(\frac{3}{2}\right)^2$ and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$

$\left(3, \frac{6}{5}\right) \notin$ R as $3 > \left(\frac{6}{5}\right)^3$

∴ R is not transitive.

R is neither reflexive, nor symmetric, nor transitive.

Question 6: Show that the relation R in the set {1, 2, 3} given by R = {(1, 2), (2, 1)} is symmetric but neither reflexive nor transitive.

Solution:

A = {1, 2, 3}.

R = {(1, 2), (2, 1)}.

(1, 1), (2, 2), (3, 3) ∉ R.

∴ R is not reflexive.

(1, 2) ∈ R and (2, 1) ∈ R, then R is symmetric.

(1, 2) and (2, 1) ∈ R

(1, 1) ∉ R

∴ R is not transitive.

R is symmetric but neither reflexive nor transitive.

Question 7: Show that the relation R in the set A of all the books in a library of a college, given by R = {(x, y): x and y have same number of pages} is an equivalence relation.

Solution:

R = {x, y): x and y have the same number of pages}

R is reflexive since (x, x) ∈ R as x and x have same number of pages.

(x, y) ∈ R ⇒ x and y have the same number of pages.

⇒ y and x have the same number of pages.

⇒ (y, x) ∈ R

∴ R is symmetric.

(x, y) ∈R and (y, z) ∈ R.

⇒ x and y and have same number of pages and y and z have same number of pages.

⇒ x and z have same number of pages.

⇒ (x, z) ∈ R

R is transitive.

R is an equivalence relation.

Question 8: Show that the relation R in the set A = {1, 2, 3, 4, 5} given by $R = \{(a, b): |a - b|\ is\ even\}$, is an equivalence relation. Show that all the elements of {1, 3, 5} are related to each other and all the elements of {2, 4} are related to each other. But no element of {1, 3, 5} is related to any element of 2, 4}.

Solution:

a ∈ A,

|a-a| = 0 (which is even).

∴ R is reflexive.

(a, b) ∈ R.

⇒ $|a - b|\ is\ even$

⇒|-(a-b)|= |b - a| is also even

⇒ (b, a) ∈ R

∴ R is symmetric.

(a, b) ∈ R and (b, c) ∈ R.

⇒|a-b| is even and |b-c| is even

⇒(a-b) is even and (b-c) is even

⇒(a-c) = (a-b) + (b-c) is even

⇒ $|a - b|\ is\ even.$

⇒ (a, c) ∈ R

∴ R is transitive.

R is an equivalence relation.

all elements of {1, 3, 5} are related to each other because they are all odd. So, the modulus of the difference between any two elements is even.

Similarly, all elements of {2, 4} are related to each other because they are all even.

no element of {1, 3, 5} can be related to any element of {2, 4} as all elements of {1, 3, 5} are odd and all elements of {2, 4} are even.So, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Question 9: Show that each of the relation R in the set A = {x ∈ Z: 0 ≤ x ≤ 12}, given by

(i) R = {(a, b) : |a – b| is a multiple of 4}

(ii) R = {(a, b) : a = b}

is an equivalence relation. Find the set of all elements related to 1 in each case.

Solution:

$A = \{x \in \boldsymbol{Z}: 0 \le x \le 12\}$ = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}

(i) R = {(a, b): |a – b| is a multiple of 4}

a ∈ A, (a, a) ∈ R as |a – a| = 0 is a multiple of 4.

∴ R is reflexive.

(a, b) ∈ R ⇒ |a – b| is a multiple of 4.

⇒| – (a – b) | = |b – a| is a multiple of 4.

⇒ (b, a) ∈ R

∴ R is symmetric.

(a, b), (b, c) ∈ R.

⇒ |a – b| is a multiple of 4 and |b – c| is a multiple of 4.

⇒ (a – b) is a multiple of 4 and (b – c) is a multiple of 4.

⇒ (a – c) = (a – b) + (b – c) is a multiple of 4.

⇒ |a – c| is a multiple of 4.

⇒ (a, c) ∈ R

∴ R is transitive.

R is an equivalence relation.

The set of elements related to 1 is {1, 5, 9} as

$|1 - 1| = 0$ is a multiple of 4.

$|5 - 1| = 4$ is a multiple of 4.

$|9 - 1| = 8$ is a multiple of 4.

(ii) R = {(a, b): a = b}

a ∈ A, (a, a) ∈ R, since a = a.

∴ R is reflexive.

(a, b) ∈ R.

⇒ a = b

⇒ b = a ⇒ (b, a) ∈ R

∴ R is symmetric.

(a, b) ∈ R and (b, c) ∈ R.

⇒ a = b and b = c

⇒ a = c

⇒ (a, c) ∈ R

∴ R is transitive.

R is an equivalence relation.

the set of elements related to 1 is {1}.

Question 10: Given an example of a relation. Which is

(i) Symmetric but neither reflexive nor transitive.

(ii) Transitive but neither reflexive nor symmetric.

(iii)Reflexive and symmetric but not transitive.

(iv) Reflexive and transitive but not symmetric.

(v) Symmetric and transitive but not reflexive

Solution:

(i) A = {5, 6, 7}.

R = {(5, 6), (6, 5)}.

R is not reflexive as (5, 5), (6, 6), (7, 7) ∉ R.

(5, 6) ∈ R and (6, 5) ∈ R, R is symmetric.

⇒ (5, 6), (6, 5) ∈ R, but (5, 5) ∉ R

∴ R is not transitive.

relation R is symmetric but not reflexive or transitive.

(ii)

R = {(a, b): a < b}

a ∈ R, (a, a) ∉ R since a cannot be strictly less than itself.

∴ R is not reflexive.

(1, 2) ∈ R (as 1 < 2)

But, 2 is not less than 1.

∴ (2, 1) ∉ R

∴ R is not symmetric.

(a, b), (b, c) ∈ R.

⇒ a < b and b < c

⇒ a < c

⇒ (a, c) ∈ R

∴ R is transitive.

relation R is transitive but not reflexive and symmetric.

(iii) A = {4, 6, 8}.

A = {(4, 4), (6, 6), (8, 8), (4, 6), (6, 4), (6, 8), (8, 6)}

R is reflexive since for a ∈ A, (a, a) ∈ R

R is symmetric since (a, b) ∈ R ⇒ (b, a) ∈ R for a, b ∈ R.

R is not transitive since (4, 6), (6, 8) ∈ R, but (4, 8) ∉ R.

R is reflexive and symmetric but not transitive.

(iv)

R = {a, b): $a^3 \geq b^3$}

Clearly (a, a) ∈ R

∴ R is reflexive.

(2, 1) ∈ R

But, (1, 2) ∉ R

∴ R is not symmetric.

(a, b), (b, c) ∈ R.

⇒ $a^3 \geq b^3$ and $b^3 \geq c^3$

⇒ $a^3 \geq c^3$

⇒ (a, c) ∈ R

∴ R is transitive.

R is reflexive and transitive but not symmetric.

(v) Let A = {−5, −6}.

R = {(−5, −6), (−6, −5), (−5, −5)}

R is not reflexive as (−6, −6) ∉ R.

(−5, −6), (−6, −5) ∈ R.

(−5, −5) ∈ R.

R is transitive.

R is symmetric and transitive but not reflexive.

Question 11: Show that the relation R in the set A of points in a plane given by R = {(P, Q): Distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all point related to a point P ≠ (0, 0) is the circle passing through P with origin as centre.

Solution: R = {(P, Q): Distance of P from the origin is the same as the distance of Q from the origin}

Clearly, (P, P) ∈ R

∴ R is reflexive.

(P, Q) ∈ R.

Clearly R is symmetric.

(P, Q), (Q, S) ∈ R.

⇒ The distance of P and Q from the origin is the same and also, the distance of Q and S from the origin is the same.

⇒ The distance of P and S from the origin is the same.

⇒ (P, S) ∈ R

∴ R is transitive.

R is an equivalence relation.

The set of points related to P ≠ (0, 0) will be those points whose distance from origin is same as distance of P from the origin.

set of points forms a circle with the centre as origin and this circle passes through P.

Question 12: Show that the relation R defined in the set A of all triangles as R = {(T_1, T_2): T_1 is similar to T_2}, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and T_3 with sides 6, 8, 10. Which triangles among T_1, T_2 and T_3 are related?

Solution: R = {(T_1, T_2): T_1 is similar to T_2}

R is reflexive since every triangle is similar to itself.

If (T_1, T_2) ∈ R, then T_1 is similar to T_2.

⇒ T_2 is similar to T_1.

⇒ $(T_2, T_1) \in R$

∴ R is symmetric.

$(T_1, T_2), (T_2, T_3) \in R$.

⇒ T_1 is similar to T_2 and T_2 is similar to T_3.

⇒ T_1 is similar to T3.

⇒ $(T_1, T_3) \in R$

∴ R is transitive.

R is an equivalence relation.

$$\frac{3}{6}=\frac{4}{8}=\frac{5}{10}\left(=\frac{1}{2}\right)$$

∴The corresponding sides of triangles T_1 and T_3 are in the same ratio.

triangle T_1 is similar to triangle T_3.

Hence, T_1 is related to T_3.

Question 13: Show that the relation R defined in the set A of all polygons as R = {(P_1, P_2): P_1 and P_2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Solution:

R = {(P_1, P_2): P_1 and P_2 have same number of sides}

R is reflexive,

$(P_1, P_1) \in R$, as same polygon has same number of sides.

$(P_1, P_2) \in R$.

⇒ P_1 and P_2 have same number of sides.

⇒ P_2 and P_1 have same number of sides.

⇒ $(P_2, P_1) \in R$

∴ R is symmetric.

$(P_1, P_2), (P_2, P_3) \in R$.

⇒ P_1 and P_2 have same number of sides.

P_2 and P_3 have same number of sides.

⇒ P_1 and P_3 have same number of sides.

⇒ $(P_1, P_3) \in R$

∴ R is transitive.

R is an equivalence relation.

The elements in A related to right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides

set of all elements in A related to triangle T is the set of all triangles.

Question 14: Let L be the set of all lines in XY plane and R be the relation in L defined as $R = \{(L_1, L_2): L_1$ is parallel to $L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Solution:

$R = \{(L_1, L_2)$: L1 is parallel to $L_2\}$

R is reflexive as any line L1 is parallel to itself i.e., $(L_1, L_1) \in R$.

$(L_1, L_2) \in R$.

⇒ L_1 is parallel to L_2 ⇒ L_2 is parallel to L_1.

⇒ $(L_2, L_1) \in R$

∴ R is symmetric.

$(L_1, L_2), (L_2, L_3) \in R$.

⇒ L_1 is parallel to L_2.

L_2 is parallel to L_3.

⇒ L_1 is parallel to L_3.

∴ R is transitive.

R is an equivalence relation.

set of all lines related to line y = 2x + 4 is set of all lines that are parallel to the line y = 2x + 4.

Slope of line y = 2x + 4 is m = 2

line parallel to the given line is of the form y = 2x + c, where c ∈ R.

set of all lines related to the given line is given by y = 2x + c, where c ∈ R.

Question 15: Let R be the relation in the set {1, 2, 3, 4} given by

R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}. Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation

Solution: R = {(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)}

(a, a) ∈ R, for every a ∈ {1, 2, 3, 4}.

∴ R is reflexive.

(1, 2) ∈ R, but (2, 1) ∉ R.

∴ R is not symmetric.

(a, b), (b, c) ∈ R ⇒ (a, c) ∈ R for all a, b, c ∈ {1, 2, 3, 4}.

∴ R is transitive.

R is reflexive and transitive but not symmetric.

The correct answer is B.

Question 16: Let R be the relation in the set N given by R = {(a, b): a = b − 2, b > 6}. Choose the correct answer. (A) (2, 4) ∈ R (B) (3, 8) ∈ R (C) (6, 8) ∈ R (D) (8, 7) ∈ R

Solution: R = {(a, b): a = b − 2, b > 6}

Now,

b > 6, (2, 4) ∉ R

3 ≠ 8 − 2,

∴ (3, 8) ∉ R And, as 8 ≠ 7 − 2

∴ (8, 7) ∉ R

consider (6, 8).

8 > 6 and, 6 = 8 − 2.

∴ (6, 8) ∈ R

The correct answer is C.