# Exercise 3.2

Question 1: Rationalise the denominators of each of the following (i - vii):

(i) 3/ √5 (ii) 3/(2 √5) (iii) 1/ √12 (iv) √2/ √5

(v)  $(\sqrt{3} + 1)/\sqrt{2}$  (vi)  $(\sqrt{2} + \sqrt{5})/\sqrt{3}$  (vii)  $3\sqrt{2}/\sqrt{5}$ 

Solution:

(i) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$= \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \frac{3 \times \sqrt{5}}{5}$$

(ii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{3}{2\sqrt{5}} = \frac{3 \times \sqrt{5}}{2 \times \sqrt{5} \times \sqrt{5}}$$

$$=\frac{3\sqrt{5}}{2\times 5}=\frac{3\sqrt{5}}{10}=\frac{3}{10}\sqrt{5}$$

(iii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{1}{\sqrt{12}} = \frac{1}{\sqrt{4 \times 3}} = \frac{1}{2\sqrt{3}}$$

$$=\frac{1\times\sqrt{3}}{2\sqrt{3}\times\sqrt{3}}=\frac{\sqrt{3}}{2\times3}=\frac{\sqrt{3}}{6}$$

(iv) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{2}}{\sqrt{5}} = \frac{\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{10}}{5} = \frac{1}{5} \sqrt{10}$$

(v) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{3}+1}{\sqrt{2}} = \frac{(\sqrt{3}+1)\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$$

(vi) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}=\frac{(\sqrt{2}+\sqrt{5})\times\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$=\frac{\sqrt{6}+\sqrt{15}}{3}$$

(vii) Multiply both numerator and denominator to with same number to rationalise the denominator.

$$\frac{3\sqrt{2}}{\sqrt{5}} = \frac{3\sqrt{2} \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{3 \times \sqrt{10}}{5}$$

$$=\frac{3}{5}\sqrt{10}$$

Question 2: Find the value to three places of decimals of each of the following. It is given that  $\sqrt{2}$  = 1.414,  $\sqrt{3}$  = 1.732,  $\sqrt{5}$  = 2.236 and  $\sqrt{10}$  = 3.162

(i) 
$$\frac{2}{\sqrt{3}}$$

(i) 
$$\frac{2}{\sqrt{3}}$$
 (ii)  $\frac{3}{\sqrt{10}}$ 

(iii) 
$$\frac{\sqrt{5}+1}{\sqrt{2}}$$

(iii) 
$$\frac{\sqrt{5}+1}{\sqrt{2}}$$
 (iv)  $\frac{\sqrt{10}+\sqrt{15}}{\sqrt{2}}$ 

(v) 
$$\frac{2+\sqrt{3}}{3}$$
 (vi)  $\frac{\sqrt{2}-1}{\sqrt{5}}$ 

(vi) 
$$\frac{\sqrt{2}-1}{\sqrt{5}}$$

**Solution:** 

(i) 
$$\frac{2}{\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

$$=\frac{2\sqrt{3}}{3}=\frac{2\times1.732}{3}=\frac{3.464}{3}=1.154$$

(ii) 
$$\frac{3}{\sqrt{10}} = \frac{3 \times \sqrt{10}}{\sqrt{10} \times \sqrt{10}} = \frac{3\sqrt{10}}{10}$$

$$=\frac{3(3.162)}{10}=\frac{9.486}{10}=0.9486$$

(iii) 
$$\frac{\sqrt{5}+1}{\sqrt{2}} = \frac{(\sqrt{5}+1)\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}$$

$$=\frac{\sqrt{10}+\sqrt{2}}{2}=\frac{3.162+1.414}{2}$$
$$=\frac{4.576}{2}=2.288$$

(iv) 
$$\frac{\sqrt{10} + \sqrt{15}}{\sqrt{2}} = \frac{(\sqrt{10} + \sqrt{15})\sqrt{2}}{\sqrt{2} \times \sqrt{2}}$$
$$= \frac{\sqrt{20} + \sqrt{30}}{2} = \frac{2\sqrt{5} + \sqrt{10} \times \sqrt{3}}{2}$$
$$= \frac{2(2.236) + 3.162 \times 1.732}{2} = 4.974$$

(v) 
$$\frac{2+\sqrt{3}}{3} = \frac{2+1.732}{3} = \frac{3.732}{3} = 1.244$$

(vi) 
$$\frac{\sqrt{2}-1}{\sqrt{5}} = \frac{(\sqrt{2}-1) \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$
$$= \frac{\sqrt{10}-\sqrt{5}}{5} = \frac{3.162-2.236}{5}$$
$$= \frac{0.926}{5} = 0.185$$

Question 3: Express each one of the following with rational denominator:

(i) 
$$\frac{1}{3+\sqrt{2}}$$
 (ii)  $\frac{1}{\sqrt{6}-\sqrt{5}}$  (iii)  $\frac{16}{\sqrt{41}-5}$ 

(iv) 
$$\frac{30}{5\sqrt{3}-3\sqrt{5}}$$
 (v)  $\frac{1}{2\sqrt{5}-\sqrt{3}}$  (vi)  $\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$ 

(vii) 
$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$
 (viii)  $\frac{3\sqrt{2}+1}{2\sqrt{5}-3}$  (ix)  $\frac{b^2}{\sqrt{a^2+b^2}+a}$ 

#### Solution:

Using identity:  $(a + b) (a - b) = a^2 - b^2$ 

(i) Multiply and divide given number by 3-V2

$$\frac{1}{3+\sqrt{2}}$$
=  $\frac{3-\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})}$ 
=  $\frac{3-\sqrt{2}}{9-2}$ 
=  $\frac{3-\sqrt{2}}{7}$ 

(ii) Multiply and divide given number by  $\sqrt{6} + \sqrt{5}$ 

$$\frac{1}{\sqrt{6} - \sqrt{5}}$$
=  $\frac{\sqrt{6} + \sqrt{5}}{(\sqrt{6} - \sqrt{5})(\sqrt{6} + \sqrt{5})}$ 
=  $\frac{\sqrt{6} + \sqrt{5}}{6 - 5}$ 
=  $\sqrt{6} + \sqrt{5}$ 

(iii) Multiply and divide given number by  $\sqrt{41} + 5$ 

$$\frac{\frac{16}{\sqrt{41}-5}}{\frac{16\times(\sqrt{41}+5)}{(\sqrt{41}-5)(\sqrt{41}+5)}}$$

$$=\frac{\frac{16\sqrt{41}+80}{41-25}}{\frac{16\sqrt{41}+80}{16}}$$

$$=\frac{\frac{16(\sqrt{41}+80}{16}}{\frac{16}{16}}$$

$$=\sqrt{41}+5$$

(iv) Multiply and divide given number by 5 $\sqrt{3}$  + 3 $\sqrt{5}$ 

$$\frac{30}{5\sqrt{3}-3\sqrt{5}}$$
=\frac{30\times (5\sqrt{3}+3\sqrt{5})}{(5\sqrt{3}-3\sqrt{5})(5\sqrt{3}+3\sqrt{5})}
=\frac{30\times (5\sqrt{3}+3\sqrt{5})}{75-45}
=\frac{30\times (5\sqrt{3}+3\sqrt{5})}{30}
=5\sqrt{3}+3\sqrt{5}

(v) Multiply and divide given number by 2V5 + V3

$$\frac{1}{2\sqrt{5} - \sqrt{3}}$$

$$= \frac{2\sqrt{5} + \sqrt{3}}{(2\sqrt{5} - \sqrt{3})(2\sqrt{5} + \sqrt{3})}$$

$$= \frac{2\sqrt{5} + \sqrt{3}}{20 - 3}$$

$$= \frac{2\sqrt{5} + \sqrt{3}}{17}$$

(vi) Multiply and divide given number by 2v2 + v3

$$\frac{\sqrt{3}+1}{2\sqrt{2}-\sqrt{3}}$$

$$=\frac{(\sqrt{3}+1)(2\sqrt{2}+\sqrt{3})}{(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})}$$

$$=\frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{8-3}$$

$$=\frac{(2\sqrt{6}+3+2\sqrt{2}+\sqrt{3})}{5}$$

(vii) Multiply and divide given number by 6 - 4V2

$$\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$$

$$=\frac{(6-4\sqrt{2})(6-4\sqrt{2})}{(6+4\sqrt{2})(6-4\sqrt{2})}$$

$$=\frac{(6-4\sqrt{2})^2}{36-32}$$

$$=\frac{36-48\sqrt{2}+32}{4}$$

$$=\frac{68-48\sqrt{2}}{4}$$

$$=\frac{4(17-12\sqrt{2})}{4}$$

$$=17-12\sqrt{2}$$

(viii) Multiply and divide given number by 2√5 + 3

$$\begin{split} &\frac{3\sqrt{2}+1}{2\sqrt{5}-3} \\ &= \frac{(3\sqrt{2}+1)\times(2\sqrt{5}+3)}{(2\sqrt{5}-3)(2\sqrt{5}+3)} \\ &= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{(20-9)} \\ &= \frac{6\sqrt{10}+9\sqrt{2}+2\sqrt{5}+3}{11} \end{split}$$

(ix) Multiply and divide given number by  $\sqrt{(a^2+b^2)}$  - a

$$=\frac{b^2}{\sqrt{(a^2+b^2)}+a}$$

$$=\frac{b^2(\sqrt{(a^2+b^2)}-a)}{(\sqrt{(a^2+b^2)}+a)(\sqrt{(a^2+b^2)}-a)}$$

$$=\frac{b^2(\sqrt{(a^2+b^2)}-a)}{(a^2+b^2)-a^2)}$$

$$=\frac{b^2(\sqrt{(a^2+b^2)}-a)}{b^2}$$

Question 4: Rationales the denominator and simplify:

(i) 
$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$
 (ii)  $\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$  (iii)  $\frac{1+\sqrt{2}}{3-2\sqrt{2}}$ 

(ii) 
$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

(iii) 
$$\frac{1+\sqrt{2}}{3-2\sqrt{2}}$$

(iv) 
$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$

(iv) 
$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$
 (v)  $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$  (vi)  $\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$ 

(vi) 
$$\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}$$

Solution:

[Use identities:  $(a + b) (a - b) = a^2 - b^2$ ;  $(a + b)^2 = (a^2 + 2ab + b^2)$  and  $(a - b)^2 = (a^2 - 2ab + b^2)$ 

(i) Multiply both numerator and denominator by  $\sqrt{3}$ – $\sqrt{2}$  to rationalise the denominator.

$$\begin{split} &\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\ &= \frac{(\sqrt{3} - \sqrt{2})(\sqrt{3} - \sqrt{2})}{(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})} \\ &= \frac{(\sqrt{3} - \sqrt{2})^2}{3 - 2} \\ &= \frac{3 - 2\sqrt{3}\sqrt{2} + 2}{1} \\ &= 5 - 2\sqrt{6} \end{split}$$

(ii) Multiply both numerator and denominator by 7–4v3 to rationalise the denominator.

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}}$$

$$=\frac{(5+2\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})}$$

$$=\frac{(5+2\sqrt{3})(7-4\sqrt{3})}{49-48}$$

$$=35-20\sqrt{3}+14\sqrt{3}-24$$

$$=11-6\sqrt{3}$$

(iii) Multiply both numerator and denominator by 3+2v2 to rationalise the denominator.

$$\frac{1+\sqrt{2}}{3-2\sqrt{2}}$$
= 
$$\frac{(1+\sqrt{2})(3+2\sqrt{2})}{(3-2\sqrt{2})(3+2\sqrt{2})}$$
= 
$$\frac{(1+\sqrt{2})(3+2\sqrt{2})}{9-8}$$
= 
$$3+2\sqrt{2}+3\sqrt{2}+4$$
= 
$$7+5\sqrt{2}$$

(iv) Multiply both numerator and denominator by 3√5+2√6 to rationalise the denominator.

$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$$
$$(2\sqrt{6}-\sqrt{5})$$

$$=\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$$

$$=\frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{45-24}$$

$$= \frac{(2\sqrt{6} - \sqrt{5})(3\sqrt{5} + 2\sqrt{6})}{21}$$

$$=\frac{6\sqrt{30}{+}24{-}15{-}2\sqrt{30}}{21}$$

$$= \frac{4\sqrt{30} + 9}{21}$$

(v) Multiply both numerator and denominator by  $\sqrt{48}$ – $\sqrt{18}$  to rationalise the denominator.

$$\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$$

$$= \frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{(\sqrt{48}+\sqrt{18})(\sqrt{48}-\sqrt{18})}$$

$$=\frac{(4\sqrt{3}+5\sqrt{2})(\sqrt{48}-\sqrt{18})}{48-18}$$

$$= \frac{48 - 12\sqrt{6} + 20\sqrt{6} - 30}{30}$$

$$= \frac{18 + 8\sqrt{6}}{30}$$

$$= \frac{9+4\sqrt{6}}{15}$$

(vi) Multiply both numerator and denominator by  $2\sqrt{2} - 3\sqrt{3}$  to rationalise the denominator.

$$\begin{split} &\frac{2\sqrt{3}-\sqrt{5}}{2\sqrt{2}+3\sqrt{3}}\\ &=\frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{(2\sqrt{2}+3\sqrt{3})(2\sqrt{2}-3\sqrt{3})}\\ &=\frac{(2\sqrt{3}-\sqrt{5})(2\sqrt{2}-3\sqrt{3})}{8-27}\\ &=\frac{(4\sqrt{6}-2\sqrt{10})-18+3\sqrt{15})}{-19}\\ &=\frac{(18-4\sqrt{6}+2\sqrt{10}-3\sqrt{15})}{19} \end{split}$$