Exercise 10.2

Question 1: In figure, it is given that RT = TS, ∠ 1 = 2 ∠ 2 and ∠4 = 2(∠3). Prove that ΔRBT ≅ ΔSAT.
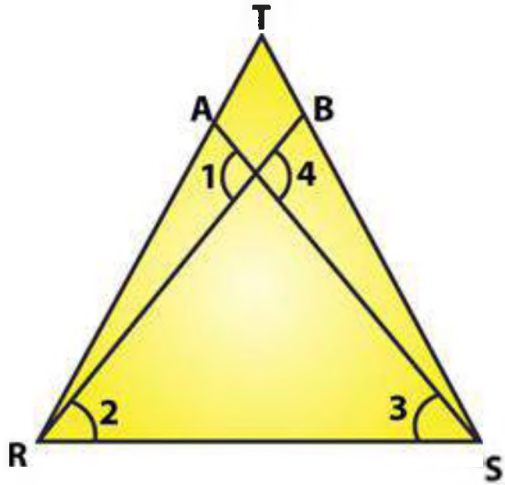
Solution:

In the figure,

RT = TS   ......(i)

∠ 1 = 2 ∠ 2   ......(ii)

And ∠ 4 = 2 ∠ 3   ......(iii)

To prove: ΔRBT ≅ ΔSAT

Let the point of intersection RB and SA be denoted by O

∠ AOR = ∠ BOS   [Vertically opposite angles]

or ∠ 1 = ∠ 4

2 ∠ 2 = 2 ∠ 3   [From (ii) and (iii)]

or ∠ 2 = ∠ 3   ......(iv)

Now in Δ TRS, we have RT = TS

=> Δ TRS is an isosceles triangle

∠ TRS = ∠ TSR   ......(v)

But, ∠ TRS = ∠ TRB + ∠ 2   ......(vi)

∠ TSR = ∠ TSA + ∠ 3   ......(vii)

Putting (vi) and (vii) in (v) we get

∠ TRB + ∠ 2 = ∠ TSA + ∠ B

=> ∠ TRB = ∠ TSA   [From (iv)]

Consider Δ RBT and Δ SAT

RT = ST   [From (i)]

∠ TRB = ∠ TSA [From (iv)]

By ASA criterion of congruence, we have

Δ RBT ≅ Δ SAT

Question 2: Two lines AB and CD intersect at O such that BC is equal and parallel to AD. Prove that the lines AB and CD bisect at O.

Solution: Lines AB and CD Intersect at O
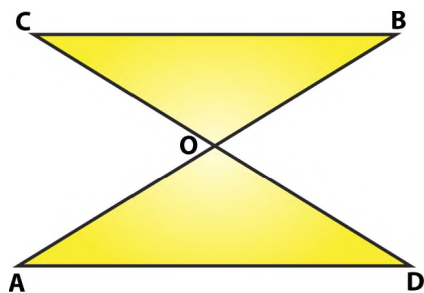
Such that BC ‖ AD and

BC = AD ....(i)

To prove : AB and CD bisect at O.

First we have to prove that Δ AOD ≅ Δ BOC

∠OCB =∠ODA   [AD||BC and CD is transversal]
AD = BC   [from (i)]
∠OBC = ∠OAD   [AD||BC and AB is transversal]
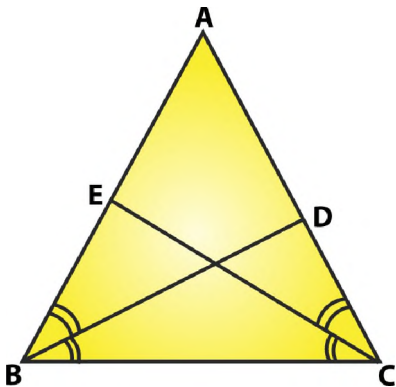By ASA Criterion:
Δ AOD ≅ Δ BOC

OA = OB and OD = OC (By c.p.c.t.)

Therefore, AB and CD bisect each other at O.

Hence Proved.

Question 3: BD and CE are bisectors of ∠ B and ∠ C of an isosceles Δ ABC with AB = AC. Prove that BD = CE.

Solution:

Δ ABC is isosceles with AB = AC and BD and CE are bisectors of ∠ B and ∠ C We have to prove BD = CE. (Given)

Since AB = AC

=> ∠ABC = ∠ACB   ……(i)
[Angles opposite to equal sides are equal]

Since BD and CE are bisectors of ∠ B and ∠ C

∠ ABD = ∠ DBC = ∠ BCE = ECA = ∠B/2 = ∠C/2 …(ii)

Now, Consider Δ EBC = Δ DCB

∠ EBC = ∠ DCB   [From (i)]

BC = BC [Common side]

∠ BCE = ∠ CBD   [From (ii)]

By ASA congruence criterion, Δ EBC ≅ Δ DCB

Since corresponding parts of congruent triangles are equal.

=> CE = BD

or, BD = CE

Hence proved.