Exercise 5.1

Question 1: Factorize $x^3 + x - 3x^2 - 3$ Solution: $x^3 + x - 3x^2 - 3$ Here x is common factor in $x^3 + x$ and -3 is common factor in $-3x^2 - 3x^3 - 3x^2 + x - 3x^2 - 3x^2 + x - 3x^2 + x - 3x^2 + x - 3x^2 + x - 3x^2 - 3x^2 + x - 3x^2 + 3x^2 - 3x^2 + x - 3x^2 + x - 3x^2 + x - 3x^2 + 3x^2 + x - 3x^2 + 3x^2$

Question 2: Factorize $a(a + b)^3 - 3a^2b(a + b)$ Solution: $a(a + b)^3 - 3a^2b(a + b)$ $= a(a + b) \{(a + b)^2 - 3ab\}$

= $a(a + b) \{a^2 + b^2 + 2ab - 3ab\}$ = $a(a + b) (a^2 + b^2 - ab)$

Question 3: Factorize $x(x^3 - y^3) + 3xy(x - y)$ Solution: $x(x^3 - y^3) + 3xy(x - y)$ = $x(x - y) (x^2 + xy + y^2) + 3xy(x - y)$ Taking x(x - y) as a common factor = $x(x - y) (x^2 + xy + y^2 + 3y)$ = $x(x - y) (x^2 + xy + y^2 + 3y)$

Question 4: Factorize $a^2x^2 + (ax^2 + 1)x + a$ Solution: $a^2x^2 + (ax^2 + 1)x + a$ $= a^2x^2 + a + (ax^2 + 1)x$ $= a(ax^2 + 1) + x(ax^2 + 1)$ $= (ax^2 + 1) (a + x)$

Question 5: Factorize $x^2 + y - xy - x$ Solution: $x^2 + y - xy - x$ $= x^2 - x - xy + y$ = x(x - 1) - y(x - 1)= (x - 1) (x - y)

Question 6: Factorize $x^3 - 2x^2y + 3xy^2 - 6y^3$ Solution: $x^3 - 2x^2y + 3xy^2 - 6y^3$ = $x^2(x - 2y) + 3y^2(x - 2y)$ = $(x - 2y) (x^2 + 3y^2)$

Question 7: Factorize $6ab - b^2 + 12ac - 2bc$ Solution: $6ab - b^2 + 12ac - 2bc$ $= 6ab + 12ac - b^2 - 2bc$ Taking 6a common from first two terms and -b from last two terms = 6a(b + 2c) - b(b + 2c)Taking (b + 2c) common factor = (b + 2c) (6a - b)

Question 8: Factorize $(x^2 + 1/x^2) - 4(x + 1/x) + 6$

Solution:

 $(x^{2} + 1/x^{2}) - 4(x + 1/x) + 6$ = $x^{2} + 1/x^{2} - 4x - 4/x + 4 + 2$ = $x^{2} + 1/x^{2} + 4 + 2 - 4/x - 4x$ = $(x^{2}) + (1/x)^{2} + (-2)^{2} + 2x(1/x) + 2(1/x)(-2) + 2(-2)x$ As we know, $x^{2} + y^{2} + z^{2} + 2xy + 2yz + 2zx = (x+y+z)^{2}$ So, we can write; = $(x + 1/x + (-2))^{2}$ or $(x + 1/x - 2)^{2}$ Therefore, $x^{2} + 1/x^{2}$) - $4(x + 1/x) + 6 = (x + 1/x - 2)^{2}$

Question 9: Factorize x(x - 2) (x - 4) + 4x - 8Solution: x(x - 2) (x - 4) + 4x - 8= x(x - 2) (x - 4) + 4(x - 2)= (x - 2) [x(x - 4) + 4]

= (x - 2) (x² - 4x + 4)= (x - 2) [x² - 2 (x)(2) + (2)²] = (x - 2) (x - 2)² = (x - 2)³

Question 10: Factorize (x + 2) ($x^2 + 25$) - $10x^2$ - 20x

Solution :

 $(x + 2) (x^{2} + 25) - 10x (x + 2)$

Take (x + 2) as common factor;

$$= (x + 2)(x^{2} + 25 - 10x)$$

 $=(x+2)(x^2-10x+25)$

Expanding the middle term of $(x^2 - 10x + 25)$

$$=(x+2)(x^2-5x-5x+25)$$

 $=(x+2){x(x-5)-5(x-5)}$

$$=(x+2)(x-5)(x-5)$$

=(x + 2)(x − 5)²

Therefore, $(x + 2) (x^2 + 25) - 10x (x + 2) = (x + 2)(x - 5)^2$

Question 11: Factorize $2a^2 + 2\sqrt{6} ab + 3b^2$

Solution:

 $2a^2 + 2\sqrt{6} ab + 3b^2$

Above expression can be written as $(\sqrt{2a})^2 + 2 \times \sqrt{2a} \times \sqrt{3b} + (\sqrt{3b})^2$

As we know, $(p+q)^2 = p^2 + q^2 + 2pq$ Here $p = \sqrt{2}a$ and $q = \sqrt{3}b$

 $= (\sqrt{2a} + \sqrt{3b})^2$

Therefore, $2a^2 + 2\sqrt{6} ab + 3b^2 = (\sqrt{2}a + \sqrt{3}b)^2$

Question 12: Factorize $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c) (b - c + a)$ Solution: $(a - b + c)^2 + (b - c + a)^2 + 2(a - b + c) (b - c + a)$ {Because $p^2 + q^2 + 2pq = (p + q)^2$ } Here p = a - b + c and q = b - c + a $= [a - b + c + b - c + a]^2$ $= (2a)^2$

= (2a) = 4a²

Question 13: Factorize $a^2 + b^2 + 2(ab+bc+ca)$ Solution: $a^2 + b^2 + 2ab + 2bc + 2ca$

As we know, $p^2 + q^2 + 2pq = (p + q)^2$

We get,

 $= (a+b)^2 + 2bc + 2ca$

 $= (a+b)^2 + 2c(b+a)$

Or $(a+b)^2 + 2c(a+b)$

Take (a + b) as common factor;

= (a + b)(a + b + 2c)

Therefore, $a^2 + b^2 + 2ab + 2bc + 2ca = (a + b)(a + b + 2c)$

Question 14: Factorize $4(x-y)^2 - 12(x - y)(x + y) + 9(x + y)^2$

Solution :

Consider (x - y) = p, (x + y) = q

 $= 4p^2 - 12pq + 9q^2$

Expanding the middle term, -12 = -6 - 6 also $4 \times 9 = -6 \times -6$ $= 4p^2 - 6pq - 6pq + 9q^2$ = 2p(2p - 3q) - 3q(2p - 3q) = (2p - 3q)(2p - 3q) $= (2p - 3q)^2$ Substituting back p = x - y and q = x + y; $= [2(x-y) - 3(x+y)]^2 = [2x - 2y - 3x - 3y]^2$ $= (2x-3x-2y-3y)^2$ $= [-x - 5y]^2$ $= [(-1)(x+5y)]^2$ $= (x+5y)^2$ Therefore, $4(x-y)^2 - 12(x-y)(x+y) + 9(x+y)^2 = (x+5y)^2$

Question 15: Factorize $a^2 - b^2 + 2bc - c^2$

Solution :

 $a^2 - b^2 + 2bc - c^2$

As we know, $(a-b)^2 = a^2 + b^2 - 2ab$

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= a^2 - (b - c)^2
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Also we know, $a^2 - b^2 = (a+b)(a-b)$

= (a + b - c)(a - (b - c))

= (a + b - c)(a - b + c)

Therefore, $a^2 - b^2 + 2bc - c^2 = (a + b - c)(a - b + c)$

Question 16: Factorize $a^2 + 2ab + b^2 - c^2$ Solution: $a^2 + 2ab + b^2 - c^2$ = $(a^2 + 2ab + b^2) - c^2$ = $(a + b)^2 - (c)^2$ We know, $a^2 - b^2 = (a + b) (a - b)$ = (a + b + c) (a + b - c)

Therefore $a^2 + 2ab + b^2 - c^2 = (a + b + c) (a + b - c)$