RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.1

Derivatives EX 30.1 Q1

We have,

$f(x) = 3x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(a) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h}$$

$$= \lim_{h\to 0} \frac{3(2+h) - 6}{h}$$

$$= \lim_{h\to 0} \frac{3h}{h}$$

$$= \lim_{h\to 0} 3$$

$$\therefore f'(2) = 3$$

Derivatives EX 30.1 Q2

We have,

$f(x) = x^2 - 2$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(10) = \lim_{h\to 0} \frac{f(10+h) - f(10)}{h}$$

$$= \lim_{h\to 0} \frac{(10+h)^2 - 2 - 98}{h}$$

$$= \lim_{h\to 0} \frac{100 + 20h + h^2 - 100}{h}$$

$$= \lim_{h\to 0} \frac{h(20+h)}{h}$$

$$= \lim_{h\to 0} (20+h)$$

$$\therefore f'(10) = 20$$

Derivatives EX 30.1 Q3

We have,

$f(x) = 99x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(100) = \lim_{h\to 0} \frac{f(100+h) - f(100)}{h}$$

$$= \lim_{h\to 0} \frac{99(100+h) - 9900}{h}$$

$$= \lim_{h\to 0} \frac{9900 + 99h - 9900)}{h}$$

$$= \lim_{h\to 0} 99$$

$$\therefore f'(100) = 99$$

Derivatives EX 30.1 Q4

We have,

$f(x) = x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(1) = \lim_{h\to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h\to 0} \frac{(1+h) - 1}{h}$$

$$= \lim_{h\to 0} 1$$

$$\therefore f'(1) = 1$$

Derivatives EX 30.1 Q5

We have,
$f(x) = \cos x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(0+h) - \cos 0}{h}$$

$$= \lim_{h \to 0} \frac{\cos h - \cos 0}{h}$$

$$= \lim_{h \to 0} \frac{(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - --) - 1}{h} \qquad \left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots\right]$$

$$= \lim_{h \to 0} \frac{h(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} - \ldots)}{h}$$

$$= \lim_{h \to 0} h(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} - \ldots)$$

$$= 0$$

$$\therefore f'(0) = 0$$

Derivatives EX 30.1 Q6

We have,
$f(x) = \tan x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{\tan h - \tan 0}{h}$$

$$= \lim_{h \to 0} \frac{\tan h}{h}$$

$$= 1 \qquad \left[\because \lim_{\theta \to 0} \frac{\tan \theta}{\theta} = 1\right]$$

$\therefore f'(0) = 1$

Derivatives EX 30.1 Q7(i)

We have,

$f(x) = \sin x$

$$\because f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'\left(\frac{\pi}{2}\right) = \lim_{h\to 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\sin\left(\frac{\pi}{2}+h\right) - \sin\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{\cos h - 1}{h}$$

$$= \lim_{h\to 0} \frac{\left(1 - \frac{h^2}{2!} + \frac{h^4}{4!} - \dots\right) - 1}{h} \quad \left[\because \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots\right]$$

$$= \lim_{h\to 0} \frac{h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \dots\right)}{h}$$

$$= \lim_{h\to 0} h\left(-\frac{h}{2!} + \frac{h^3}{4!} - \frac{h^5}{6!} + \dots\right)$$

$$= 0$$

$$\therefore f'\left(\frac{\pi}{2}\right) = 0$$

Derivatives EX 30.1 Q7(ii)

We have,

$f(x) = x$

$$\because f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\therefore f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$= \lim_{h \to 0} \frac{1+h-1}{h}$$

$$= \lim_{h \to 0} 1$$

$$\therefore f'(1) = 1$$

Derivatives EX 30.1 Q7(iii)

We have,

$\because f(x) = 2\cos x$

$$\therefore f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{f\left(\frac{\pi}{2} + h\right) - f\left(\frac{\pi}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{\pi}{2} + h\right) - 2\cos\frac{\pi}{2}}{h}$$

$$= \lim_{h \to 0} \frac{-2\sin h - 0}{h}$$

$$= -2 \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$\therefore f'\left(\frac{\pi}{2}\right) = -2$$

Derivatives EX 30.1 Q7(iv)

We have, $f(x)=\sin 2x$

Therefore,

$$
\begin{aligned}
f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}\\
&= \lim_{h\to 0}\frac{f\left(\frac{\pi}{2}+h\right)-f\left(\frac{\pi}{2}\right)}{h}\\
&= \lim_{h\to 0}\frac{\sin 2\left(\frac{\pi}{2}+h\right)-\sin 2\left(\frac{\pi}{2}\right)}{h}\\
&= \lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2}\times 2+2h\right)-\sin(\pi)}{h}\\
&= \lim_{h\to 0}\frac{-\cos 2h-0}{h}\\
&= -2
\end{aligned}
$$

Therefore $f'\left(\frac{\pi}{2}\right)=-2$

RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.2

Derivatives Ex 30.2 Q1(i)

We have,

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{f\,\frac{2}{x+h} - \frac{2}{x}}{h}$$

$$= \lim_{h\to 0} \frac{(2x - 2x - 2h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{2(x - x - h)}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{-2h}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{-2}{x(x+h)}$$

$$= \frac{-2}{x^2}$$

Derivatives Ex 30.2 Q1(ii)

We have,

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}.h}$$

$$= \lim_{h\to 0} \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}.h} \times \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}\,h}$$

$$= \lim_{h\to 0} \frac{x - (x+h)}{\sqrt{x}\sqrt{x+h}.h\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-h}{\sqrt{x}\sqrt{x+h}.h\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{x\sqrt{x+h} + \sqrt{x}\left(\sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{-1}{2x\sqrt{x}}$$

$$= \frac{-1}{2}x^{-\frac{3}{2}}$$

Derivatives Ex 30.2 Q1(iii)

We have,

$$f(x) = \frac{1}{x^3}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{(x+h)^3} - \frac{1}{x^3}}{h}$$

$$= \lim_{h\to 0} \frac{x^3 - (x+h)^3}{x^3 h (x+h)^3}$$

$$= \lim_{h\to 0} \frac{x^3 - \left(x^3 + 3x^2h + 3xh^2 + h^3\right)}{x^3 h (x+h)^3}$$

$$= \lim_{h\to 0} \frac{x^3 - x^3 - 3x^2 - 3xh - h^2}{x^3 (x+h)^3}$$

$$= \lim_{h\to 0} \frac{-3x^2 - 3xh - h^2}{x^3 (x+h)^3}$$

$$= \frac{-3x^2}{x^6}$$

$$= \frac{-3}{x^4}$$

Derivatives Ex 30.2 Q1(iv)

We have,

$$f(x) = \frac{x^2+1}{x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h)^2+1}{(x+h)} - \frac{x^2+1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\left[x^2 + h^2 + 2xh + 1\right] - \left(x^2+1\right)(x+h)}{hx(x+h)}$$

$$= \lim_{h\to 0} \frac{x^3 + xh^2 + 2x^2h + x - x^3 - x - x^2h - h}{h.x(x+h)}$$

$$= \lim_{h\to 0} \frac{xh + 2x^2 - x^2 - 1}{x(x+h)}$$

$$= \frac{x^2-1}{x^2}$$

$$= 1 - \frac{1}{x^2}$$

Derivatives Ex 30.2 Q1(v)

We have,

$$f(x) = \frac{x^2 - 1}{x}$$

$$\therefore\ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)^2 - 1}{(x+h)} - \frac{x^2 - 1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{x\left(x^2 + h^2 + 2xh - 1\right) - (x+h)\left(x^2 - 1\right)}{x(x+h)h}$$

$$= \lim_{h \to 0} \frac{xh + 2x^2 - x^2 + 1}{x(x+h)}$$

$$= \frac{x^2 + 1}{x^2}$$

$$= 1 + \frac{1}{x^2}$$

Derivatives Ex 30.2 Q1(vi)

We have,

$$f(x) = \frac{x+1}{x+2}$$

$$\because\ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)+1}{(x+h)+2} - \frac{x+1}{x+2}}{h}$$

$$= \lim_{h \to 0} \frac{(x+2)(x+h+1) - (x+1)(x+h+2)}{(x+h+2)(x+2)h}$$

$$= \lim_{h \to 0} \frac{\left(x^2 + 2x + xh + 2h + 2 + x\right) - \left(x^2 + xh + 2x + x + h + 2\right)}{(x+h+2)(x+2)h}$$

$$= \lim_{h \to 0} \frac{h}{(x+h+2)(x+2)h}$$

$$= \frac{1}{(x+2)^2}$$

Derivatives Ex 30.2 Q1(vii)

We have,

$$f(x) = \frac{x+2}{3x+5}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{(x+h+2)}{3(x+h)+5} - \frac{x+2}{3x+5}}{h}$$

$$= \lim_{h\to 0} \frac{(3x+5)(x+h+2) - (x+2)(3x+3h+5)}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{(3x^2+5x+3xh+5h+6x+10) - (3x^2+3xh+5x+6x+6h+10)}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{-h}{(3x+5)(3x+3h+5)h}$$

$$= \lim_{h\to 0} \frac{-1}{(3x+5)(3x+3h+5)}$$

$$= \frac{-1}{(3x+5)^2}$$

Derivatives Ex 30.2 Q1(viii)

We have,

$$f(x) = kx^n$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{k(x+h)^n - kx^n}{h}$$

$$= k \lim_{h\to 0} \frac{\left(x^n + nx^{n-1}h + \frac{n(n-1)}{2}x^{n-2}h^2 + \ldots\right) - x^n}{h} \qquad \left[\because (x+y)^n = x^n + nx^{n-1}y\ldots\right]$$

$$= k \lim_{h\to 0} nx^{n-1} + \frac{n(n-1)}{2!}x^{n-2}h + \frac{n(n-1)(n-2)}{3!}x^{n-3}h^2\ldots$$

$$= k\, nx^{n-1} + 0 + 0\ldots$$

$$= k\, nx^{n-1}$$

Derivatives Ex 30.2 Q1(ix)

We have,

$$f(x)=\frac{1}{\sqrt{3-x}}$$

$$\because f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\frac{1}{\sqrt{3-(x+h)}}-\frac{1}{\sqrt{3-x}}}{h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3-x}-\sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)}\times h}$$

$$=\lim_{h\to 0}\frac{\sqrt{3-x}-\sqrt{3-(x+h)}}{\sqrt{3-x}\sqrt{3-(x+h)}h}\times\frac{\sqrt{3-x}+\sqrt{3-(x+h)}}{\sqrt{3-x}+\sqrt{3-(x+h)}}$$ [Rationalising the numerator by $\sqrt{3-x}+\sqrt{3-(x+h)}$]

$$=\lim_{h\to 0}\frac{(3-x)-(3-(x+h))}{\sqrt{3-x}\sqrt{3-(x+h)}\times h\left(\sqrt{3-x}+\sqrt{3-(x+h)}\right)}$$

$$=\lim_{h\to 0}\frac{h}{\sqrt{3-x}\sqrt{3-(x+h)}\times h\left(\sqrt{3-x}+\sqrt{3-(x+h)}\right)}$$

$$=\frac{1}{(3-x)\times 2\sqrt{3-x}}$$

$$=\frac{1}{2(3-x)^{\frac{3}{2}}}$$

Derivatives Ex 30.2 Q1(x)

We have,

$$f(x)=x^2+x+3$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\left\{(x+h)^2+(x+h)+3\right\}-x^2+x+3}{h}$$

$$=\lim_{h\to 0}\frac{x^2+h^2+2xh+x+h+3-x^2-x-3}{h}$$

$$=\lim_{h\to 0}\frac{2xh+h^2+h}{h}$$

$$=\lim_{h\to 0}\frac{h(2x+h+1)}{h}$$

$$=2x+0+1$$

$$=2x+1$$

Derivatives Ex 30.2 Q1(xi)

We have,

$$f(x)=(x+2)^3$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{(x+h+2)^3-(x+2)^3}{h}$$

$$=\lim_{h\to 0}\frac{\left\{(x+2)+h\right\}^3-(x+2)^3}{h}$$

$$=\lim_{h\to 0}\frac{(x+2)^3+h^3+3h(x+2)^2+3(x+2)h^2-(x+2)^3}{h}$$

$$=\lim_{h\to 0}3(x+2)^2+3(x+2)h+h^2$$

$$=3(x+2)^2$$

Derivatives Ex 30.2 Q1(xii)

We have,

$f(x) = x^3 + 4x^2 + 3x + 2$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 + 4(x+h)^2 + 3(x+h) + 2 - (x^3 + 4x^2 + 3x + 2)}{h}$$

On solving we get,

$$= \lim_{h \to 0} \frac{x^3 + h^3 + 3x^2h + 3h^2x + 4x^2 + 4h^2 + 8hx + 3x + 3h + 2 - x^3 - 4x^2 - 3x - 2}{h}$$

$$= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 + 4h^2 + 8hx + 3h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 4h + 8x + 3$$

$$= 3x^2 + 8x + 3$$

Derivatives Ex 30.2 Q1(xiii)

We have,

$f(x) = x^3 - 5x^2 + x - 5$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\{(x+h)^3 + (x+h) - 5(x+h)^2 - 5\} - (x^3 - 5x^2 + x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{(x^3 + h^3 + 3x^2h + 3h^2x + x + h - 5x^2 - 5h^2 - 10xh - 5) - (x^3 - 5x^2 + x - 5)}{h}$$

$$= \lim_{h \to 0} \frac{(3x^2h + 3h^2x + h^3 + h - 5h^2 - 10xh)}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3xh + h^2 + 1 - 5h - 10x$$

$$= 3x^2 - 10x + 1$$

Derivatives Ex 30.2 Q1(xiv)

We have,

$$f(x) = \sqrt{2x^2+1}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{2(x+h)^2+1} - \sqrt{2x^2+1}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)^2+1} + \sqrt{2x^2+1}$

$$= \lim_{h\to 0} \frac{\left\{2(x+h)^2+1-\left(2x^2+1\right)\right\}}{h\left(\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{2x^2+2h^2+4xh+1-2x^2-1}{h\left(\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}\right)}$$

$$= \lim_{h\to 0} \frac{4xh+2h^2}{h\sqrt{2(x+h)^2+1}+\sqrt{2x^2+1}}$$

$$= \frac{4x}{2\sqrt{2x^2+1}}$$

$$= \frac{2x}{\sqrt{2x^2+1}}$$

Derivatives Ex 30.2 Q1(xv)

We have, $f(x) = \frac{2x+3}{x-2}$

Therefore,

$$f'(x) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

$$= \lim_{h\to 0} \frac{\left(\frac{2x+2h+3}{x+h-2}\right)-\left(\frac{2x+3}{x-2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{2x^2+2hx+3x-4x-4h-6-2x^2-2hx+4x-3x-3h+6}{h(x+h-2)(x-2)}$$

$$= \lim_{h\to 0} \frac{-7}{(x+h-2)(x-2)}$$

$$= \boxed{\frac{-7}{(x-2)^2}}$$

Derivatives Ex 30.2 Q2(i)

We have,

$$f(x) = e^{-x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{-(x+h)}-e^{-x}}{h}$$

$$= \lim_{h\to 0} \frac{e^{-x}\left(e^{-h}-1\right)}{h}$$

$$= \lim_{h\to 0} \frac{-e^{-x}\left(e^{-h}-1\right)}{-h}$$

$$= -e^{-x} \qquad \left[\because \lim_{\theta\to 0} \frac{e^{\theta}-1}{\theta} = 1\right]$$

Derivatives Ex 30.2 Q2(ii)

We have,

$f(x) = e^{3x}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x+h)} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x)}.e^{3h} - e^{3x}}{h}$$

$$= \lim_{h \to 0} \frac{e^{3(x)}\left(e^{3h} - 1\right)}{h}$$

Multiplying Numerator and Denominator by 3

$$= \lim_{h \to 0} e^{3(x)} \frac{\left(e^{3h} - 1\right)}{3h} \qquad \left[\lim_{h \to 0} \frac{e^{3h} - 1}{3} = 1\right]$$

$$= 3e^{3x}$$

Derivatives Ex 30.2 Q2(iii)

We have,

$f(x) = e^{ax+b}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{a(x+h)+b} - e^{ax+b}}{h}$$

$$= \lim_{h \to 0} \frac{e^{ax} \times e^{ah} \times e^{b} - e^{ax} \times e^{b}}{h}$$

$$= \lim_{h \to 0} \frac{e^{b} \times e^{ax}\left(e^{ah} - 1\right)}{h}$$

$$= \lim_{h \to 0} e^{ax+b} \times \frac{a\left(e^{ah} - 1\right)}{a.h}$$

Multiplying Numerator and denominator by a $\qquad \left[\lim_{h \to 0} \frac{\left(e^{ah} - 1\right)}{ah} = 1\right]$

$$= ae^{ax+b}$$

Derivatives Ex 30.2 Q2(iv)

We have,

$f(x) = xe^x$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)e^{(x+h)} - xe^x}{h}$$

$$= \lim_{h\to 0} \frac{xe^x.e^h + he^x.e^h - xe^x}{h}$$

$$= \lim_{h\to 0} \quad xe^x\left(\frac{e^h-1}{h}\right) + \frac{he^{x+h}}{h}$$

$$= xe^x + e^x$$

$$= e^x(x+1)$$

Derivatives Ex 30.2 Q2(v)

Let $f(x) = -x$. Then, $f(x+h) = -(x+h)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{-(x+h)+(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{-h}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} -1$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -1$$

Derivatives Ex 30.2 Q2(vi)

Let $f(x) = (-x)^{-1}$. Then, $f(x+h) = (-(x+h))^{-1}$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{(-(x+h))^{-1} - (-x)^{-1}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{\frac{-x+x+h}{x(x+h)}}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{h}{hx(x+h)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0} \frac{1}{x(x+h)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x(x+0)}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \frac{1}{x^2}$$

Derivatives Ex 30.2 Q2(vii)

Let $f(x) = \sin(x+1)$. Then, $f(x+h) = \sin((x+h)+1)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\sin((x+h)+1)-\sin(x+1)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{2\sin\left[\frac{((x+h)+1)-(x+1)}{2}\right]\cos\left[\frac{((x+h)+1)+(x+1)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{2\sin\left[\frac{h}{2}\right]\cos\left[\frac{2x+2+h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}}\times\lim_{h\to 0}\cos\left[\frac{2x+2+h}{2}\right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = 1\times\cos\left[\frac{2x+2+0}{2}\right]$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \cos(x+1)$$

Derivatives Ex 30.2 Q2(viii)

Let $f(x) = \cos\left(x-\frac{\pi}{8}\right)$. Then, $f(x+h) = \cos\left(x+h-\frac{\pi}{8}\right)$

$$\therefore \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{\cos\left(x+h-\frac{\pi}{8}\right)-\cos\left(x-\frac{\pi}{8}\right)}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-2\sin\left[\frac{\left(x+h-\frac{\pi}{8}\right)+\left(x-\frac{\pi}{8}\right)}{2}\right]\sin\left[\frac{\left(x+h-\frac{\pi}{8}\right)-\left(x-\frac{\pi}{8}\right)}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}\frac{-2\sin\left[\frac{2x+h-\frac{2\pi}{8}}{2}\right]\sin\left[\frac{h}{2}\right]}{h}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = \lim_{h\to 0}-\sin\left[\frac{2x+h-\frac{2\pi}{8}}{2}\right]\times\lim_{h\to 0}\frac{\sin\left[\frac{h}{2}\right]}{\frac{h}{2}}$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left[\frac{2x+0-\frac{2\pi}{8}}{2}\right]\times 1$$

$$\Rightarrow \frac{d}{dx}(f(x)) = -\sin\left(x-\frac{\pi}{8}\right)$$

Derivatives Ex 30.2 Q2(ix)

We have,

$f(x) = x \sin x$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)\sin(x+h) - x\sin x}{h}$$

$$= \lim_{h \to 0} \frac{x\left(\sin(x+h) - \sin x\right)}{h} + \sin(x+h) \qquad \left[\sin c - \sin d = 2\cos\frac{c+d}{2}\sin\frac{c-d}{2}\right]$$

$$= \lim_{h \to 0} \frac{x \times 2\cos\left(x + \frac{h}{2}\right)\sin\frac{h}{2}}{h} + \sin(x+h) \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= 2x \times \cos x \times \frac{1}{2} + \sin x$$

$$= x \times \cos x + \sin x$$

$$= \sin x + x\cos x$$

Derivatives Ex 30.2 Q2(x)

We have,

$f(x) = x \cos x$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)\cos(x+h) - x\cos x}{h}$$

$$= \lim_{h \to 0} \frac{x\cos(x+h) h\cos(x+h) - x\cos x}{h}$$

$$= \lim_{h \to 0} \frac{x\left\{\cos(x+h) - \cos x\right\}}{h} + \cos(x+h)$$

$$= \lim_{h \to 0} x.2\sin\left(x - x - \frac{h}{2}\right)\sin\left(x + \frac{h}{2}\right) + \cos(x+h) \qquad \left[\because \cos A - \cos B = 2\sin\frac{B-A}{2}\sin\frac{B+A}{2}\right]$$

$$= \lim_{h \to 0} 2x.\sin\left(\frac{-h}{2}\right)\sin\left(x + \frac{h}{2}\right) + \cos(x+h)$$

$$= -x\sin x + \cos x$$

Derivatives Ex 30.2 Q2(xi)

We have,

$f(x) = \sin(2x - 3)$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin\left\{2(x+h) - 3\right\} - \sin(2x - 3)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\frac{(2x + 2h - 3 + 2x - 3)}{2} \times \frac{\sin(2x + 2h - 3 - 2 - x + 3)}{2}}{h} \qquad \left[\because \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h \to 0} 2\cos(2x - 3 + h).\frac{\sinh}{2} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1\right]$$

$$= 2\cos(2x - 3)$$

Derivatives Ex 30.2 Q3(i)

We have,

$$f(x) = \sqrt{\sin 2x}$$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h}$$

Multiplying Numerator and Denominator by $\left(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}\right)$

$$= \lim_{h \to 0} \frac{\sqrt{\sin 2(x+h)} - \sqrt{\sin 2x}}{h} \times \frac{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}}{\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}}$$

$$= \lim_{h \to 0} \frac{\sin(2x+2h) - \sin 2x}{h\left(\sqrt{\sin 2(x+h)} + \sqrt{\sin 2x}\right)}$$

$$= \lim_{h \to 0} \frac{\sin(2x+2h) - \sin 2x}{h\left(\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}\right)} \qquad \left[\sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h \to 0} \frac{2\cos(2x+h) \times \sin h}{h} \times \frac{1}{\sqrt{\sin(2x+2h)} + \sqrt{\sin 2x}}$$

$$= \frac{2\cos 2x}{2\sqrt{\sin 2x}}$$

$$= \frac{\cos 2x}{\sqrt{\sin 2x}}$$

Derivatives Ex 30.2 Q3(ii)

We have,

$$f(x) = \frac{\sin x}{x}$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h)}{x+h} - \frac{\sin x}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x\sin(x+h) - (x+h)\sin x}{xh(x+h)}$$

$$= \lim_{h\to 0} \frac{x(\sin x.\cos h + \cos x.\sin h) - x.\sin x - h.\sin x}{xh(x+h)} \quad [\because \sin(A+B = \sin A.\cos B + \cos A.\sin B)]$$

$$= \lim_{h\to 0} \frac{x.\sin x(\cos h - 1)}{xh(x+h)} + \frac{x.\cos x.\sin h}{(x+h)xh} - \frac{h\sin x}{(x+h)xh} \quad \left[\because 1 - \cos h = 2\sin^2\frac{h}{2}\right]$$

$$= \frac{-x\sin x}{x(x+h)} \times \frac{2\sin^2\frac{h}{2}}{\frac{h^2}{4}} \times \frac{h}{4} + \frac{x\cos x}{x^2} - \frac{\sin x}{x^2}$$

$$\because h \to 0 \Rightarrow \frac{h}{2} \to 0 \quad and \quad \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

$$= 0 + \frac{x\cos x - \sin x}{x^2}$$

$$= \frac{x\cos x - \sin x}{x^2}$$

Derivatives Ex 30.2 Q3(iii)

We have,

$$f(x) - \frac{\cos x}{x}$$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\cos(x+h)}{x+h} - \frac{\cos x}{x}}{h}$$

$$= \lim_{h\to 0} \frac{x.\cos(x+h) - (x+h)\cos x}{(x+h)xh} \quad [\because \cos(A+B) = \cos A.\cos B - \sin A.\sin B]$$

$$= \lim_{h\to 0} \frac{x[\cos x.\cos h - \sin x.\sin h] - x.\cos x - h.\cos x}{(x+h)x.h}$$

$$= \lim_{h\to 0} \frac{x\cos x(\cos h - 1)}{(x+h)x.h} - \frac{x.\cos x.\sin h}{(x+h)xh} - \frac{h.\cos x}{(x+h)xh}$$

$$= \lim_{h\to 0} \frac{-x\cos x.2\sin^2\frac{h}{2}}{(x+h)x\frac{h^2}{4}} \times \frac{h^2}{4} - \frac{x.\sin x}{x(x+h)} - \frac{\cos x}{x(x+h)}$$

$$= 0 - \frac{x\sin x}{x^2} - \frac{\cos x}{x^2}$$

$$= -\frac{x\sin x - \cos x}{x^2}$$

Derivatives Ex 30.2 Q3(iv)

We have,

$f(x) = x^2 \sin x$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 \sin(x+h) - x^2 \sin x}{h}$$

$$= \lim_{h\to 0} \frac{(x^2 + h^2 + 2hx)(\sin x.\cos h + \cos x.\sin h) - x^2 \sin x}{h} \quad [\because \sin(A+B) = \sin A.\cos B + \cos B.\sin A]$$

$$= \lim_{h\to 0} \frac{x^2 \sin x(\cos h - 1)}{h} + \frac{h(h+2x)\sin x.\cos h}{h} + (x+h)^2 \cos x \frac{\sin h}{h}$$

$$= \lim_{h\to 0} -x^2 \sin x \times \frac{2\sin^2 \frac{h}{2}}{\left(\frac{h}{2}\right)^2} \times \frac{h^2}{4} + (h+2x)\sin x.\cos h + (x+h)^2 \cos x$$

$$= 0 + (2x \sin x + x^2 \cos x)$$

$$= 2x \sin x + x^2 \cos x$$

Derivatives Ex 30.2 Q3(v)

We have,

$f(x) = \sqrt{\sin(3x+1)}$

$$\therefore f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\sin 3(x+h)+1} - \sqrt{\sin(3x+1)}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{\sin(3x+3h)+1} - \sqrt{\sin(3x+1)}}{h} \times \frac{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}$$

$$= \lim_{h\to 0} \frac{\sin(3x+3h+1) - \sin(3x+1)}{h\left(\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}\right)}$$

$$= \lim_{h\to 0} 2\cos\left(3x+1+\frac{3h}{2}\right) \times \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} \times \frac{3}{2} \times \frac{1}{\sqrt{\sin(3x+3h)+1} + \sqrt{\sin(3x+1)}}$$

$$= \frac{3\cos(3x+1)}{2\sqrt{\sin(3x+1)}} \qquad \left[\lim_{h\to 0} \frac{\sin\frac{3h}{2}}{\frac{3h}{2}} = 1\right]$$

Derivatives Ex 30.2 Q3(vi)

We have,

$f(x) = \sin x + \cos x$

$$\therefore\ f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\{\sin(x+h)+\cos(x+h)\} - \sin x + \cos x}{h}$$

$$= \lim_{h\to 0}\frac{\{\sin(x+h)+\cos(x+h) - \sin x - \cos x\}}{h}$$

$$= \lim_{h\to 0}\frac{\{\sin(x+h)-\sin x\} + \{\cos(x+h)-\cos x\}}{h}$$

$$= \lim_{h\to 0}\frac{\left\{2\sin\frac{(x+h-x)}{2}\cos\frac{(x+h+x)}{2}\right\} + \left\{-2\sin\frac{x+h+x}{2}\sin\frac{x+h-x}{2}\right\}}{h}$$

$$\left[\because \sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2} \text{ and } \cos A - \cos B = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \lim_{h\to 0}\frac{\sinh.\cos\frac{2x+h}{2} - 2\sin\left(x+\frac{h}{2}\right)\sinh}{h}$$

$$= \lim_{h\to 0}\frac{\sinh}{h}\left\{\cos\frac{x+h}{2} - \sin\left(x+\frac{h}{2}\right)\right\} \qquad \left[\because \lim_{h\to 0}\frac{\sinh}{h} = 1\right]$$

$$= \cos x - \sin x$$

Derivatives Ex 30.2 Q3(vii)

We have,

$f(x) = x^2e^x$

$$\because\ f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{x^2e^xe^h + h^2.e^x.e^h + 2xhe^xe^h - x^2e^x}{h}$$

$$= \lim_{h\to 0} x^2e^x\frac{(e^h-1)}{h} + e^xe^h\frac{(h^2+2xh)}{h} \qquad \left[\because \frac{e^h-1}{h} - 1\right]$$

$$\therefore x^2e^x + e^x(0+2x)$$

$$= x^2e^x + 2xe^x$$

$$= e^x(x^2+2x)$$

Derivatives Ex 30.2 Q3(viii)

We have,

$f(x) = e^{x^2+1}$

$$\therefore f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e^{(x+h)^2} + 1 - e^{x^2+1}}{h}$$

$$= \lim_{h \to 0} \frac{e^{x^2+h^2+2xh+1} - e^{x^2+1}}{h}$$

$$= \lim_{h \to 0} \frac{e^{x^2+1}\left(e^{2xh}.e^{h^2} - 1\right)}{h}$$

$$= \lim_{h \to 0} \frac{e^{x^2+1}\left(e^{2xh+h^2} - 1\right)}{2xh + h^2} \times \frac{2xh + h^2}{h}$$

$\because h \to 0$

$\Rightarrow 2xh + h^2 = 0$

and $\lim_{\theta \to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$= \lim_{h \to 0} \quad e^{x^2+1}.1 \times 2x + h$

$= 2xe^{x^2+1}$

Derivatives Ex 30.2 Q3(ix)

We have,

$f(x) = e\sqrt{2x}$

$$\because f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{e\sqrt{2(x+h)} - e\sqrt{2x}}{h}$$

$$= \lim_{h \to 0} \frac{e\sqrt{2x}\left(e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1\right)}{h}$$

$$= \lim_{h \to 0} \quad e\sqrt{2x} \frac{\left(e^{\sqrt{2(x+h)} - \sqrt{2x}} - 1\right)}{\sqrt{2(x+h)} - \sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{2(x+h)} - \sqrt{2x}$

$\because h \to 0 \Rightarrow \sqrt{2(x+h)} - \sqrt{2x} \Rightarrow 0$

and $\lim_{\theta \to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$$\therefore \lim_{h \to 0} \quad e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h}$$

Again Multiplying Numerator and Denominator by $\sqrt{2(x+h)} + \sqrt{2x}$

$$\therefore \lim_{h \to 0} \quad e^{\sqrt{2x}} \times \frac{\sqrt{2(x+h)} - \sqrt{2x}}{h} \times \frac{\sqrt{2(x+h)} + \sqrt{2x}}{\sqrt{2(x+h)} + \sqrt{2x}}$$

$$= e^{\sqrt{2x}} \times \frac{1}{2\sqrt{2x}}$$

Derivatives Ex 30.2 Q3(x)

We have,

$f(x) = e^{\sqrt{ax+b}}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{e^{\sqrt{a(x+h)+b}} - e^{\sqrt{ax+b}}}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{a(x+h)+b}} \frac{\left(e^{\sqrt{a(x+h)+b}-\sqrt{ax+b}} - 1\right)}{h}$$

$$= \lim_{h\to 0} e^{\sqrt{ax+b}} \times \frac{e^{\sqrt{a(x+h)+b}-\sqrt{ax+b}} - 1}{\sqrt{a(x+h)+b} - \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{a(x+h)+b} - \sqrt{ax+b}$

$\because h \to 0$

$\therefore \sqrt{a(x+h)+b} - \sqrt{ax+b} = 0$

and $\lim_{\theta\to 0} \frac{e^{\theta} - 1}{\theta} = 1$

$$= \lim_{h\to 0} e^{\sqrt{ax+b}} \times 1 \times \frac{\sqrt{a(x+h)+b} - \sqrt{ax+b}}{\sqrt{a(x+h)+b} + \sqrt{ax+b}} \times \frac{\sqrt{a(x+h)+b} + \sqrt{ax+b}}{h}$$

Again multiplied Numerator and Denominator by $\sqrt{a(x+h)+b} + \sqrt{ax+b}$

$$= \lim_{h\to 0} e^{\sqrt{ax+b}} \times \frac{a(x+h)+b-(ax+b)}{h} \times \frac{1}{\left(\sqrt{a(x+h)+b} + \sqrt{ax+b}\right)}$$

$$= \frac{e^{\sqrt{ax+b}} \times a}{2\sqrt{ax+b}}$$

$$= \frac{ae^{\sqrt{ax+b}}}{2\sqrt{ax+b}}$$

Derivatives Ex 30.2 Q3(xi)

$$f(x)=a^{\sqrt{x}}=e^{\sqrt{x}\log a}$$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{e^{\sqrt{x+h}\log a}-e^{\sqrt{x}\log a}}{h}$$

$$=\lim_{h\to 0}e^{\sqrt{x}\log a}\frac{e^{\sqrt{x+h}\log a-\sqrt{x}\log a}-1}{h}$$

$$=\lim_{h\to 0}e^{\sqrt{x}\log a}\frac{e^{\left(\sqrt{x+h}-\sqrt{x}\right)\log a}-1}{h}$$

Multiply numerator and denominator by $\left(\sqrt{x+h}-\sqrt{x}\right)\log a$

$$f'(x)=\lim_{h\to 0}e^{\sqrt{x}\log a}\frac{e^{\left(\sqrt{x+h}-\sqrt{x}\right)\log a}-1}{h\left(\sqrt{x+h}-\sqrt{x}\right)\log a}\left(\sqrt{x+h}-\sqrt{x}\right)\log a$$

$$=e^{\sqrt{x}\log a}\lim_{h\to 0}\frac{e^{\left(\sqrt{x+h}-\sqrt{x}\right)\log a}-1}{\left(\sqrt{x+h}-\sqrt{x}\right)\log a}\lim_{h\to 0}\log a\frac{\left(\sqrt{x+h}-\sqrt{x}\right)}{h}$$

$$=e^{\sqrt{x}\log a}\lim_{h\to 0}\log a\frac{\left(\sqrt{x+h}-\sqrt{x}\right)}{h}$$

Multiply numerator and denominator by $\left(\sqrt{x+h}+\sqrt{x}\right)$

$$f'(x)=e^{\sqrt{x}\log a}\lim_{h\to 0}\log a\frac{\left(\sqrt{x+h}-\sqrt{x}\right)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\left(\sqrt{x+h}+\sqrt{x}\right)$$

$$=e^{\sqrt{x}\log a}\lim_{h\to 0}\log a\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$=e^{\sqrt{x}\log a}\frac{\log a}{2\sqrt{x}}$$

$$=\frac{a^{\sqrt{x}}}{2\sqrt{x}}\log_e a$$

Derivatives Ex 30.2 Q3(xii)

We have,

$$f(x)=3^{x^2}=e^{x^2\log 3}$$

$$\because f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{e^{(x+h)^2\log 3}-e^{x^2\log 3}}{h}$$

$$=\lim_{h\to 0}e^{x^2\log 3}\frac{\left[e^{\left((x+h)^2-x^2\right)^{\log 3}}-1\right]}{h}$$

$$=\lim_{h\to 0}e^{x^2\log 3}\frac{\left[e^{(x+h)^2-x^2}\right]^{\log 3}-1}{(x+h)^2-x^2}\times\frac{(x+h)^2-x^2}{h}$$

Multiplying Numerator and Denominator by $(x+h)^2-x^2$

$$\therefore\lim_{h\to 0}e^{x^2\log 3}\times\frac{(x+h+x)(x+h-x)}{h}$$

$$=e^{x^2}\log 3\times 2x$$

$$=2x\,e^{x^2}\log 3$$

$$=2x\,3^{x^2}\log 3$$

Derivatives Ex 30.2 Q4(i)

We have,

$f(x) = \tan^2 x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan^2(x+h) - \tan^2 x}{h}$$

$$= \lim_{h\to 0} \frac{\{\tan(x+h) + \tan x\}\{\tan(x+h) - \tan x\}}{h} \quad \left[\because \tan^2 A - \tan^2 B = (\tan A + \tan B)(\tan A - \tan B)\right]$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h+x)}{\cos(x+h)\cos x} \times \frac{\sin(x+h-x)}{\cos(x+h)\cos x}}{h}$$

$$= \lim_{h\to 0} \frac{\sin(2x+h)}{h.\cos(x+h)\cos x} \times \frac{\sin h}{\cos(x+h)\cos x}$$

$$= \lim_{h\to 0} \frac{\sin h}{h} \times \frac{\sin 2x}{\cos^2 x.\cos^2(x+h)}$$

$$= \lim_{h\to 0} \frac{\sin 2x}{\cos^2 x.\cos^2 x} \quad \left[\because \lim_{h\to 0} \frac{\sin h}{h} = 1\right]$$

$$= \lim_{h\to 0} \frac{2\sin x.\cos x}{\cos^2 x} \times \frac{1}{\cos^2 x} \quad [\sin 2x = 2\sin x \cos x]$$

$$= 2\tan x.\sec^2 x$$

Derivatives Ex 30.2 Q4(ii)

We have,

$f(x) = \tan(2x+1)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan\{2(x+h)+1\} - \tan(2x+1)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(2x+2h+1-2x-1)}{h.\cos\{2(x+h)+1\}\cos(2x+1)} \quad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0} \frac{2.\sin 2h}{2h.\cos(2x+2h+1)\cos(2x+1)}$$

Multiplying both, Numerator and Denominator by 2.

$$\therefore \lim_{h\to 0} 2\left(\frac{\sin 2h}{2}\right) \times \frac{1}{\cos(2x+2h+1)\cos(2x+1)}$$

$$= \frac{2}{\cos^2(2x+1)} \quad \left[\because \lim_{h\to 0} \frac{\sin 2h}{2} = 1\right]$$

$$= 2\sec^2(2x+1) \quad \left[\because \sec^2 x = \frac{1}{\cos^2 x}\right]$$

$$= 2\sec^2(2x+1)$$

Derivatives Ex 30.2 Q4(iii)

We have,

$f(x) = \tan 2x$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\tan 2(x+h) - \tan 2x}{h}$$

$$= \lim_{h \to 0} \frac{\sin(2x + 2h - 2x)}{h.\cos(2x+2h)\cos 2x} \qquad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h \to 0} \frac{\sin 2h}{h.\cos(2x+2h)\cos 2x}$$

$$= \lim_{h \to 0} \left(\frac{\sin 2h}{2h}\right) \times \frac{1 \times 2}{\cos(2h+2x)\cos 2x}$$

$$= \frac{2}{\cos 2x.\cos 2x} \qquad \left[\because \lim_{h \to 0} \frac{\sin 2h}{2h} = 1\right]$$

$$= 2\sec^2 2x \qquad \left[\because \frac{1}{\cos^2 x} = \sec^2 x\right]$$

Derivatives Ex 30.2 Q4(iv)

We have,

$f(x) = \sqrt{\tan x}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{\tan(x+h)} - \sqrt{\tan x}}{h}$$

Multiplying Numerator and Denominator by $\sqrt{\tan(x+h)} + \sqrt{\tan x}$

$$= \lim_{h \to 0} \frac{\tan(x+h) - \tan x}{h\left(\sqrt{\tan(x+h)} + \sqrt{\tan x}\right)}$$

$$= \lim_{h \to 0} \frac{\sin(x+h-x)}{h.\cos(x+h)\cos x\left\{\sqrt{\tan(x+h)} + \sqrt{\tan x}\right\}}$$

$$= \lim_{h \to 0} \frac{\sinh}{h} \times \frac{1}{\cos(x+h)\cos x\left\{\sqrt{\tan(x+h)} + \sqrt{\tan x}\right\}}$$

$$= \lim_{h \to 0} \frac{1}{\cos^2 x.2\sqrt{\tan x}} \qquad \left[\lim_{h \to 0} \frac{\sinh}{h} = 1\right]$$

$$= \frac{1}{2}\frac{\sec^2 x}{\sqrt{\tan x}} \qquad \left[\because \frac{1}{\cos^2 x} = \sec^2 x\right]$$

Derivatives Ex 30.2 Q5(i)

let $f(x) = \sin\sqrt{2x}$. Then $f(x+h) = \sin\sqrt{2(x+h)}$

$$\frac{d(f(x))}{dx} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\sqrt{2(x+h)} - \sin\sqrt{2x}}{h}$$

$$= \lim_{h\to 0}\frac{2\sin\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)\cos\left(\frac{\sqrt{2(x+h)}+\sqrt{2x}}{2}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\sin\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}\frac{\left(\sqrt{2(x+h)}-\sqrt{2x}\right)\left(\sqrt{2(x+h)}+\sqrt{2x}\right)}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)h}\cos\left(\frac{\sqrt{2(x+h)+2x}}{2}\right)$$

$$= \lim_{h\to 0}\frac{\sin\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}{\left(\frac{\sqrt{2(x+h)}-\sqrt{2x}}{2}\right)}\lim_{h\to 0}\frac{2(x+h)-2x}{\left(\sqrt{2(x+h)}+\sqrt{2x}\right)h}\lim_{h\to 0}\cos\left(\frac{\sqrt{2(x+h)+2x}}{2}\right)$$

$$= 1\times\frac{2}{2\sqrt{2x}}\cos\left(\sqrt{2x}\right)$$

$$= \frac{\cos\left(\sqrt{2x}\right)}{\sqrt{2x}}$$

Derivatives Ex 30.2 Q5(ii)

We have,

$f(x) = \cos\sqrt{x}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos\sqrt{x+h} - \cos\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{h}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\left(\sqrt{x+h}-\sqrt{x}\right)\left(\sqrt{x+h}+\sqrt{x}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)\left(\sqrt{x+h}+\sqrt{x}\right)h} \times \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)$$

Multiplied Numerator and Denominator by $\left(\sqrt{x+h}-\sqrt{x}\right)$

$$= \lim_{h\to 0} \frac{-\sin\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)}{\left(\frac{\sqrt{x+h}-\sqrt{x}}{2}\right)} \times \frac{x+h-x}{\left(\sqrt{x+h}+\sqrt{x}\right)h} \times \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right)$$

$$= \lim_{h\to 0} -1 \frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)} \times \sin\left(\frac{\sqrt{x+h}+\sqrt{x}}{2}\right) \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \lim_{h\to 0} \frac{-1}{\left(\sqrt{x+h}+\sqrt{x}\right)} \sin\frac{\sqrt{x+h}+\sqrt{x}}{2}$$

$$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$$

Derivatives Ex 30.2 Q5(iii)

We have,

$f(x) = \tan\sqrt{x}$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\therefore f'(x) = \lim_{h\to 0} \frac{\tan\sqrt{(x+h)} - \tan\sqrt{x}}{h}$$

$$= \lim_{h\to 0} \frac{\sin\sqrt{x+h} - \sqrt{x}}{h.\cos\sqrt{x+h}\cos\sqrt{x}} \qquad \left[\because \tan A - \tan B = \frac{\sin(A-B)}{\cos A.\cos B}\right]$$

$$= \lim_{h\to 0} \frac{\sin\sqrt{x+h} - \sqrt{x}}{(x+h-x)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\sin\left(\sqrt{x+h} - \sqrt{x}\right)}{\left(\sqrt{x+h} - \sqrt{x}\right)\left(\sqrt{x+h} + \sqrt{x}\right)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{\sin\left(\sqrt{x+h} - \sqrt{x}\right)}{\left(\sqrt{x+h} - \sqrt{x}\right)} \times \frac{1}{\left(\sqrt{x+h} + \sqrt{x}\right)\cos\sqrt{x}.\cos\sqrt{x+h}}$$

$$= 1 \times \frac{1}{2\sqrt{x}\cos\sqrt{x}\cos\sqrt{x+}} \qquad \left[\because \lim_{h\to 0} = \frac{\sin\sqrt{x+h} - \sqrt{x}}{\sqrt{x+h} - \sqrt{x}} = 1\right]$$

$$= \frac{1}{2\sqrt{x}\cos^2 x}$$

$$= \frac{\sec^2 x}{2\sqrt{x}}$$

Derivatives Ex 30.2 Q5(iv)

We have,

$f(x) = \tan x^2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\tan(x+h)^2 - \tan x^2}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h)^2}{\cos(x+h)^2} - \frac{\sin x^2}{\cos x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{\sin(x+h)^2 \cos x^2 - \cos(x+h)^2 \sin x^2}{\cos(x+h)^2 \cos x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\sin\left((x+h)^2 - x^2\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h\to 0} \frac{\sin\left(x^2 + h^2 + 2hx - x^2\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h\to 0} \frac{\sin\left(h^2 + 2hx\right)}{h.\cos(x+h)^2.\cos x^2}$$

$$= \lim_{h\to 0} \frac{\sinh}{h} \times \frac{(h + 2x)}{\cos(x+h)^2.\cos x^2}$$

$$= 1.\frac{2x}{\cos^2(x)^2} \qquad \left[\because \lim_{h\to 0} \frac{\sinh}{h} = 1\right]$$

$$= 2x \sec^2 x^2$$

Derivatives Ex 30.2 Q6(i)

We have,

$f(x) = (-x)$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{-(x+h) + x}{h}$$

$$= \lim_{h\to 0} \frac{-x - h + x}{h}$$

$$= -1$$

Derivatives Ex 30.2 Q6(ii)

We have,

$f(x) = (-x)^{-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{-1}{x+h} + \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{-x + x + h}{hx(x+h)}$$

$$= \lim_{h \to 0} \frac{1}{x^2 + xh}$$

$$= \frac{1}{x^2}$$

Derivatives Ex 30.2 Q6(iii)

We have,

$f(x) = \sin(x+1)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sin(x+h+1) - \sin(x+1)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{x+h+1+x+1}{2}\right)\sin\left(\frac{x+h+1-x-1}{2}\right)}{h}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{2x+2+h}{2}\right)\sin\frac{h}{2}}{h} \quad \left[\because \sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}\right]$$

$$= \lim_{h \to 0} \cos\left(\frac{2x+2+h}{2}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)$$

$$= \cos\left(\frac{2(x+1)}{2}\right) \quad \left[\because \lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

$$= \cos(x+1)$$

Derivatives Ex 30.2 Q6(iv)

We have,

$$f(x) = \cos\left(x - \frac{\pi}{8}\right)$$

$$\because f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\cos\left(x + h - \frac{\pi}{8}\right) - \cos\left(x - \frac{\pi}{8}\right)}{h}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right)\sin\left(\frac{h + x - \frac{\pi}{8} - x + \frac{\pi}{8}}{2}\right)}{h} \quad \left[\because \cos C - \cos D = -2\sin\frac{C+D}{2}\sin\frac{C-D}{2}\right]$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right) \times \sin\left(\frac{h}{2}\right)}{2.\frac{h}{2}}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right)}{2.} \times \lim_{h\to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}$$

$$= \lim_{h\to 0} \sin\left(\frac{2x + h - \frac{2\pi}{8}}{2}\right) \quad \left[\because \lim_{h\to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} = 1\right]$$

$$= \sin\left(\frac{2x - \frac{2\pi}{8}}{2}\right)$$

$$= -\sin\left(x - \frac{\pi}{8}\right)$$

RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.3

Derivatives 30 EX 30.3 Q1

We have to differentiate $f(x)$ with respect to x

$$\frac{d}{dx}\left(x^4 - 2\sin x + 3\cos x\right)$$

$$= \frac{d(x)^4}{dx} - 2\frac{d}{dx}(\sin x) + 3\frac{d}{dx}(\cos x)$$

$$= 4x^3 - 2\cos x - 3\sin x$$

Derivatives 30 EX 30.3 Q2

We have to differentiate $f[x]$ with respect to x

$$\frac{d}{dx}\left(3^x + x^3 + 3^3\right)$$

$$= \frac{d}{dx}(3^x) + \frac{d}{dx}(x^3) + \frac{d}{dx}(3^3)$$

$$= 3^x \log 3 + 3x^2 + 0 \qquad \left[\because \frac{d}{dx}(a^x) = a^x \log a\right]$$

$$= 3^x \log 3 + 3x^2$$

Derivatives 30 EX 30.3 Q3

We have to differentiate $f(x)$ with respect to x

$$\frac{d}{dx}\left(\frac{x^3}{3} - 2\sqrt{x} + \frac{5}{x^2}\right)$$

$$= \frac{1d}{3dx}(x^3) - 2\frac{d}{dx}(\sqrt{x}) + 5\frac{d}{dx}(x^{-2})$$

$$= \frac{1}{3}.3x^2 - 2.\frac{1.1}{2\sqrt{x}} + 5.(-2)x^{-3}$$

$$= x^2 - x^{\frac{-1}{2}} - 10x^{-3}$$

$$= x^2 - \frac{1}{\sqrt{x}} - \frac{10}{x^3}$$

Derivatives 30 EX 30.3 Q4

We have,

$$\frac{d}{dx}\left(e^{x\log a}+e^{a\log x}+e^{a\log a}\right)$$

$$=\frac{d}{dx}\left(e^{x\log a}\right)+\frac{d}{dx}\left(e^{a\log x}\right)+\frac{d}{dx}\left(e^{a\log a}\right)$$

$$=e^{x\log a}.\log a+e^{a\log x}.\frac{a}{x}+0 \qquad \left[\because e^{a\log a}\text{is constant}\right]$$

$$=\log a.e^{x\log a}+\frac{a}{x}e^{a\log x}$$

$$=\log a.a^{x}+\frac{a}{x}x^{a} \qquad \left[a^{x}\text{ can be written as } e^{x\log a}\right]$$

$$=a^{x}\log a+ax^{a-1}$$

Derivatives 30 EX 30.3 Q5

We have,

$$\frac{d}{dx}\left(2x^2+1\right)\left(3x+2\right)$$

$$=\left(3x+2\right)\frac{d}{dx}\left(2x^2+1\right)+\left(2x^2+1\right)\frac{d}{dx}\left(3x+2\right) \qquad \left[\text{Using product rule}\right]$$

$$=\left(3x+2\right)\left(4x+0\right)+\left(2x^2+1\right)\left(3+0\right)$$

$$=\left(12x^2+8x+6x^2+3\right)$$

$$=18x^2+8x+3$$

Derivatives 30 EX 30.3 Q6

We have,

$$\frac{d}{dx}f\left(x\right)=\frac{d}{dx}\left(\log_3 x+3\log_e x+2\tan x\right)$$

$$=\frac{1}{\log 3}\frac{d}{dx}\left(\log x\right)+3\frac{d}{dx}\left(\log_e x\right)+2\frac{d}{dx}\left(\tan x\right) \qquad \left[\because \log_3 x=\frac{\log x}{\log 3}\right]$$

$$=\frac{1}{\log 3}\times\frac{1}{x}+\frac{3}{x}+2\sec^2 x$$

$$=\frac{1}{x\log 3}+\frac{3}{x}+2\sec^2 x$$

Derivatives 30 EX 30.3 Q7

We have,

$$\frac{d}{dx}\left(x+\frac{1}{x}\right)\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$$

$$=\left(x+\frac{1}{x}\right)\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)+\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\frac{d}{dx}\left(x+\frac{1}{x}\right) \quad \text{[Using product rule]}$$

$$=\left(x+\frac{1}{x}\right)\left(\frac{1}{2\sqrt{x}}-\frac{1}{2x^{\frac{3}{2}}}\right)+\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)\left(1-\frac{1}{x^2}\right)$$

$$=\left(\frac{x}{2\sqrt{x}}-\frac{x}{2x^{\frac{3}{2}}}+\frac{1}{2x^{\frac{3}{2}}}-\frac{1}{2x^{\frac{5}{2}}}\right)+\left(\sqrt{x}-\frac{\sqrt{x}}{x^2}+\frac{1}{\sqrt{x}}-\frac{1}{x^{\frac{5}{2}}}\right)$$

$$=\left(\frac{1}{2}\sqrt{x}-\frac{1}{2\sqrt{x}}+\frac{1}{2x^{\frac{3}{2}}}-\frac{1}{2x^{\frac{5}{2}}}+\sqrt{x}-\frac{1}{x^{\frac{3}{2}}}+\frac{1}{\sqrt{x}}-\frac{1}{x^{\frac{5}{2}}}\right)$$

$$=\left(\frac{3}{2}\sqrt{x}+\frac{1}{2}\sqrt{x}-\frac{1}{2x^{\frac{3}{2}}}-\frac{3}{2x^{\frac{5}{2}}}\right)$$

$$=\frac{3}{2}x^{\frac{1}{2}}+\frac{1}{2}x^{\frac{-1}{2}}-\frac{1}{2}x^{\frac{-3}{2}}-\frac{3}{2}x^{\frac{-5}{2}}$$

Derivatives 30 EX 30.3 Q8

We have,

$$\frac{d}{dx}\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^3$$

$$=\frac{d}{dx}\left(x^{3/2}+3x.\frac{1}{\sqrt{x}}+3\sqrt{x}.\frac{1}{x}+\frac{1}{x^{3/2}}\right) \quad \left[(a+b)^3=a^2+3a^2b+3ab^2+b^3\right]$$

$$=\frac{d}{dx}\left(x^{3/2}+3x^{1/2}+3x^{-1/2}+x^{-3/2}\right)$$

$$=\frac{3}{2}x^{1/2}+\frac{3}{2}x^{-1/2}+3.\left(\frac{-1}{2}\right)x^{-3/2}-\frac{3}{2}x^{-5/2}$$

$$=\frac{3}{2}x^{1/2}-\frac{3}{2}x^{-5/2}+\frac{3}{2}x^{-1/2}-\frac{3}{2}x^{-3/2}$$

Derivatives 30 EX 30.3 Q9

We have,

$$\frac{d}{dx}\left(\frac{2x^2+3x+4}{x}\right)$$

$$= \frac{d}{dx}\left(\frac{2x^2}{x}+\frac{3x}{x}+\frac{4}{x}\right)$$

$$= \frac{d}{dx}\left(2x+3+4x^{-1}\right)$$

$$= 2-\frac{4}{x^2}$$

Derivatives 30 EX 30.3 Q10

We have,

$$\frac{d}{dx}\frac{\left\{\left(x^3+1\right)\left(x-2\right)\right\}}{x^2}$$

$$= \frac{d}{dx}\frac{\left\{\left(x^4-2x^3+x-2\right)\right\}}{x^2}$$

$$= \frac{d}{dx}\left(x^2-2x+x^{-1}-2x^{-2}\right)$$

$$= \frac{d}{dx}\left(x^2\right)-2\frac{dx}{dx}+\frac{dx^{-1}}{dx}-2\frac{dx^{-2}}{dx}$$

$$= 2x-2-\frac{1}{x^2}+2.\frac{2}{x^3}$$

$$= 2x-2-\frac{1}{x^2}+\frac{4}{x^3}$$

Derivatives 30 EX 30.3 Q11

We have,

$$\frac{d}{dx}\left(\frac{a\cos x + b\sin x + c}{\sin x}\right)$$

$$= a\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) + b\frac{d}{dx}(1) + c\frac{d}{dx}\left(\frac{1}{\sin x}\right)$$

$$= a\left(-\operatorname{cosec}^2 x\right) + 0 + c\left(-\operatorname{cosec} x.\cot x\right)$$

$$= -a\operatorname{cosec}^2 x - c\operatorname{cosec} x.\cot x$$

Derivatives 30 EX 30.3 Q12

We have,

$$\frac{d}{dx}(2\sec x + 3\cot x - 4\tan x)$$

$$= 2\frac{d}{dx}(2\sec x) + 3\frac{d}{dx}(\cot x) - 4\frac{d}{dx}(\tan x)$$

$$= 2\sec x\tan x - 3\operatorname{cosec}^2 x - 4\sec^2 x$$

Derivatives 30 EX 30.3 Q13

We have,

$$\frac{d}{dx}\left(a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots + a_{n-1}x + a_n\right)$$

$$= a_0\frac{d(x)^n}{dx} + a_1\frac{d(x)^{n-1}}{dx} + a_2\frac{d(x)^{n-2}}{dx} + \ldots + a_{n-1}\frac{d(x)}{dx} + a_n\frac{d(1)}{dx}$$

$$= na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots + a_{n-1} + 0$$

$$= na_0x^{n-1} + (n-1)a_1x^{n-2} + \ldots + a_{n-1}$$

Derivatives 30 EX 30.3 Q14

We have,

$$\frac{d}{dx}\left(\frac{1}{\sin x}+2^{x+3}+\frac{4}{\log x^3}\right)$$

$$= \frac{d}{dx}\operatorname{cosec}x + 2^3\frac{d}{dx}\left(2^x\right)+\frac{4}{\log 3}\times\frac{d}{dx}(\log x)\quad\left[\because\ \log_b a = \frac{\log a}{\log b}\right]$$

$$= -\operatorname{cosec}x.\cot x + 8.2^x\log 2+\frac{4}{\log 3}\times\frac{1}{x}\qquad\left[\because \frac{d}{dx}\left(a^x\right)=a^x\log a\right]$$

$$= -\operatorname{cosec}x\cot x + 2^{x+3}\log 2 + \frac{4}{x\log 3}$$

Derivatives 30 EX 30.3 Q15

We have,

$$\frac{d}{dx}\left\{\frac{(x+5)\left(2x^2-1\right)}{x}\right\}$$

$$= \frac{d}{dx}\left(\frac{2x^3+10x^2-x-5}{x}\right)$$

$$= \frac{d}{dx}\left(2x^2+10x-1-5x^{-1}\right)$$

$$= 2\frac{d}{dx}\left(x^2\right)+10\frac{d}{dx}(x)-\frac{d}{dx}(1)-5\frac{d}{dx}\left(x^{-1}\right)$$

$$= 2\times 2x+10-0+\frac{5}{x^2}$$

$$= 4x+10+\frac{5}{x^2}$$

Derivatives 30 EX 30.3 Q16

$$\frac{d}{ax}\left\{\log\left(\frac{1}{\sqrt{x}}\right)+5x^{a}-3a^{x}+\sqrt[3]{x^{2}}+6\sqrt[4]{x^{-3}}\right\}$$

$$=\frac{d}{ax}\log\left(\frac{1}{\sqrt{x}}\right)+5\frac{d}{ax}\left(x^{a}\right)-3\left(a^{x}\right)+\frac{d}{ax}\left(\sqrt[3]{x^{2}}\right)+6\frac{d}{ax}\left(\sqrt[4]{x^{-3}}\right)$$

$$=\frac{-1}{2}\frac{1}{x}+5ax^{a-1}-3a^{x}\log a+\frac{2x^{-1/3}}{3}+6x^{-7/4}\left(-3/4\right)$$

$$=\frac{-1}{2x}+5ax^{a-1}-3a^{x}\log a+\frac{2x^{-1/3}}{3}-\frac{9}{2}x^{-7/4}$$

Derivatives 30 EX 30.3 Q17

We have,

$$\frac{d}{dx}\{\cos(x+a)\}$$

$$=\frac{d}{dx}(\cos x.\cos a-\sin x.\sin a) \qquad [\because \cos(x+a)=\cos x\cos a-\sin x\sin a]$$

$$=\cos a\frac{d}{dx}(\cos x)-\sin a\frac{d}{dx}(\sin x)$$

$$=\cos a(-\sin x)-\sin a(\cos x)$$

$$=\cos x\sin a+\sin x\cos a$$

$$=-(\sin x\cos a+\cos x\sin a)$$

$$=-\sin(x+a)$$

RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.4

Derivatives Ex 30.4 Q1

We have,

$$\frac{d}{dx}\left(x^3 \sin x\right)$$

$$= \sin x \frac{d\left(x^3\right)}{dx} + x^3 \frac{d(\sin x)}{dx} \quad \text{[Using product rule]}$$

$$= \sin x . 3x^2 + x^3 . \cos x$$

$$= x^2 (3 \sin x + x \cos x)$$

Derivatives Ex 30.4 Q2

We have,

$$\frac{d}{dx}\left(x^3 e^x\right)$$

$$= e^x \frac{d}{dx}\left(x^3\right) + x^3 \frac{d}{dx}\left(e^x\right) \quad \text{[Using product rule]}$$

$$= e^x 3x^2 + x^3 e^x$$

$$= x^2 e^x (3 + x)$$

Derivatives Ex 30.4 Q3

We have,

$$\frac{d}{dx}\left(x^2 e^x \log x\right)$$

$$= e^x \log x \frac{d}{dx}\left(x^2\right) + x^2 \log x \frac{d}{dx}\left(e^x\right) + x^2 e^x \frac{d}{dx}(\log x) \quad \text{[Usingproductrule]}$$

$$= e^x \log x . 2x + x^2 \log x . e^x + x^2 e^x . \frac{1}{x}$$

$$= x e^x (2 . \log x + x \log x + 1)$$

Derivatives Ex 30.4 Q4

We have,

$$\frac{d}{dx}\left(x^n \tan x\right)$$

$$= \tan x \frac{d}{dx}\left(x^n\right) + x^n \frac{d}{dx}(\tan x) \qquad \text{[Using product rule]}$$

$$= \tan x . nx^{n-1} + x^n \sec^2 x$$

$$= x^{n-1}\left(n . \tan x + x . \sec^2 x\right) \qquad \left[x^n = x^{n-1}x^1 = x^{n-1+1}\right]$$

Derivatives Ex 30.4 Q5

We have,

$$\frac{d}{dx}\left(x^n \log_a x\right)$$

$$= \log_a x \frac{d}{dx}\left(x^n\right) + x^n \frac{d}{dx}(\log_a x) \qquad \text{[Using product rule]}$$

$$= nx^{n-1} . \log_a x + \frac{x^n}{\log a} . \frac{1}{x} \qquad \left[\because \log_a x = \frac{\log x}{\log a}\right]$$

$$= x^{n-1}\left[n . \log_a x + \frac{1}{\log a}\right]$$

Derivatives Ex 30.4 Q6

We have,

$$\frac{d}{dx}\left(x^3 + x^2 + 1\right)\sin x$$

$$= \sin x \frac{d}{dx}\left(x^3 + x^2 + 1\right) + \left(x^3 + x^2 + 1\right)\frac{d}{dx}(\sin x) \qquad \text{[Using product rule]}$$

$$= \sin x\left(3x^2 + 2x\right) + \left(x^3 + x^2 + 1\right)\cos x$$

$$\therefore \left(x^3 + x^2 + 1\right)\cos x + \left(3x^2 + 2x\right)\sin x$$

Derivatives Ex 30.4 Q7

We have,

$$\frac{d}{dx}(\sin x \times \cos x)$$

$$\cos x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(\cos x) \qquad [\text{using product rule}]$$

$$= \cos x(\cos x) + \sin x(-\sin x)$$

$$= \cos^2 x - \sin^2 x \qquad [\because \cos 2x = \cos^2 x - \sin^2 x]$$

$$= \cos 2x$$

Derivatives Ex 30.4 Q8

We have,

$$\frac{d}{dx}\left(2^x \times \cot x \times x^{-1/2}\right)$$

$$= \cot x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}\left(2^x\right) + 2^x \times \frac{1}{\sqrt{x}} \times \frac{d}{dx}(\cot x) + 2^x \times \cot x \times \frac{d}{dx}\left(x^{-1/2}\right) \qquad [\text{Using product rule}]$$

$$= \frac{\cot x}{\sqrt{x}} \times 2^x \times \log 2 + \frac{2^x}{\sqrt{x}}\left(-\operatorname{cosec}^2 x\right) + 2^x \times \cot x\left(-\frac{1}{2}\right)\frac{1}{2^{3/2}}$$

$$= \frac{2x}{\sqrt{x}}\left(\cot x \times \log 2 - \operatorname{cosec}^2 x - \frac{\cot x}{2x}\right)$$

Derivatives Ex 30.4 Q9

$$\frac{d}{dx}\left(x^2 \sin x \log x\right)$$

$$= \sin x \log x \frac{d}{dx}\left(x^2\right) + x^2 \log x \frac{d}{dx}(\sin x) + x^2 \sin x \frac{d}{dx}(\log x) \qquad [\text{Using product rule}]$$

$$= \sin x \log x \times 2x + x^2 \log x \times \cos x + x^2 \sin x \times \frac{1}{x}$$

$$= 2x \times \sin x \times \log x + x^2 \times \cos x \times \log x + x \sin x$$

Derivatives Ex 30.4 Q10

We have,

$$\frac{d}{dx}\left(x^5 e^x + x^6 \log x\right)$$

$$= \frac{d}{dx}\left(x^5 e^x\right) + \frac{d}{dx}\left(x^6 \log x\right)$$

$$= e^x \frac{dx^5}{dx} + x^5 \frac{de^5}{dx} + \log x \frac{d}{dx}\left(x^6\right) + x^6 \frac{d}{dx}\left(\log x\right) \qquad \text{[Using product rule]}$$

$$= e^x \times 5x^4 + x^5 \times e^x + \log x \times 6x^5 + x^6 \times \frac{1}{x}$$

$$= 5x^4 \times e^x + x^5 \times e^x + 6x^5 \times \log x + x^5$$

$$= x^4\left(5e^x + ex^x + 6x \log x + x\right)$$

Derivatives Ex 30.4 Q11

We have,

$$\frac{d}{dx}\left\{\left(x \sin x + \cos x\right)\left(x \cos x - \sin x\right)\right\}$$

We will apply product rule,

$$= \left(x \cos x - \sin x\right)\frac{d}{dx}\left(x \sin x + \cos x\right) + \left(x \sin x + \cos x\right)\frac{d}{dx}\left(x \cos x - \sin x\right)$$

$$= \left(x \cos x - \sin x\right)\left\{\frac{d}{dx}\left(x \sin x\right) + \frac{d}{dx}\left(\cos x\right)\right\} + \left(x \sin x + \cos x\right)\left\{\frac{d}{dx}\left(x \cos x\right) - \frac{d}{dx}\left(\sin x\right)\right\}$$

Again apply product rule,

$$= \left(x \cos x - \sin x\right)\left\{\left(\sin x \frac{dx}{dx} + x \frac{d \sin x}{dx}\right)\right\} + \left(-\sin x\right) + \left(x \cos x + \sin x\right)\left\{\left(\sin x \frac{dx}{dx} + x \frac{d \cos x}{dx} - \cos x\right)\right\}$$

$$= \left(x \cos x - \sin x\right)\left\{\left(\sin x + x \cos x\right) - \sin x\right\} + \left(x \sin x + \cos x\right)\left\{\left(\cos x - x \sin x - \cos x\right)\right\}$$

$$= \left(x \cos x - \sin x\right) \times x \cos x + \left(x \sin x + \cos x\right)\left(-x \sin x\right)$$

$$= \left(x^2 \cos^2 x - x \sin x \times \cos x\right) + \left(-x^2 \sin^2 x - x \sin x \times \cos x\right)$$

$$= x^2\left(\cos^2 x - \sin^2 x\right) - x\left(\sin x \cos x + \sin x \cos x\right)$$

$$= x^2 - \cos 2x - x \times 2 \sin x \cos x$$

$= x^2 \cos x 2x - x \sin 2x$

$= x\{x \times \cos 2x - \sin 2x\}$

Derivatives Ex 30.4 Q12

We have,

$\frac{d}{dx}\{(x \sin x + \cos x)(e^x + x^2 \log x)\}$

We will apply product rule,

$$= (e^x + x^2 \log x)\frac{d}{dx}(x \sin x + \cos x) + (x \sin x + \cos x)\frac{d}{dx}(e^x + x^2 \log x)$$

$$= (e^x + x^2 \log x)\left(\frac{d}{dx}(x \sin x) + \frac{d}{dx}\cos x\right) + (x \sin x + \cos x) \times \left\{\frac{d}{dx}(e^x) + \frac{d}{dx}(x^2 \log x)\right\}$$

Again apply product rule,

$$= (e^x + x^2 \log x)\left(\sin x \frac{d}{dx}(x) + x\frac{d}{dx}(\sin x)\right) - \sin x + (x \sin x + \cos x)\left\{e^x + \left(\log x \frac{d}{dx}(x^2) + x^2 \frac{d}{dx}(\log x)\right)\right\}$$

$$= (e^x + x^2 \log x)(\sin x + x \cos x - \sin x) + (x \sin x + \cos x)\left(e^x + \log x \times 2x + x^2 \frac{1}{x}\right)$$

$$= (e^x + x^2 \log x)x \cos x + (x \sin x + \cos x)(e^x + 2x \times \log x + x)$$

$$= x \cos x e^x + e^3 \cos x \log x + x e^x \sin x + e^x \cos x + 2x^2 \sin x \times \log x + 2x \cos x \log x + x^2 \sin x + x \cos x$$

$$= x \cos x (e^x + x^2 \log x) + (x \sin x + \cos x)(e^x + x + 2x \log x)$$

Derivatives Ex 30.4 Q13

We have,

$\frac{d}{dx}\{(1 - 2\tan x)(5 + 4\sin x)\}$

$= (5 + 4\sin x)\frac{d}{dx}(1 - 2\tan x) + (1 - 2\tan x)\frac{d}{dx}(5 + 4\sin x)$ [Using product rule,]

$= (5 + 4\sin x)(0 - 2\sec^2 x) + (1 - 2\tan x)(0 + 4\cos x)$

$= -10\sec^2 x - 8\sin x \times \sec^2 x + 4\cos x - 8\cos x \times \tan x$

$= 4\left(\frac{-5}{2}\sec^2 x - 2\sin x \times \frac{1}{\cos^2 x} + \cos x - 2\cos x \times \frac{\sin x}{\cos x}\right)$

$= 4\left(\frac{-5}{2}\sec^2 x - 2\tan x \sec x + \cos x - 2\sin x\right)$

$$= 4\left(\cos x - 2\sin x - 2\tan x \sec x - \frac{5}{2}\sec^2 x\right)$$

Derivatives Ex 30.4 Q14

We have,

$$\frac{d}{dx}\left\{\left(1+x^2\right)\cos x\right\}$$

$$= \cos x \frac{d}{dx}\left(1+x^2\right) + \left(1+x^2\right)\frac{d}{dx}(\cos x) \quad \text{(using product rule)}$$

$$= \cos x \times 2x + \left(1+x^2\right)(-\sin x)$$

$$= 2x\cos x - \left(1+x^2\right)\sin x$$

Derivatives Ex 30.4 Q15

We have,

$$\frac{d}{dx}\left(\sin^2 x\right)$$

$$= \frac{d}{dx}(\sin x)(\sin x)$$

$$= \sin x \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(\sin x) \quad \text{[Using product rule]}$$

$$= \sin x \times \cos x + \sin x \times \cos x$$

$$= 2\sin x \cos x$$

$$= \sin 2x \quad [\because \sin 2A = 2\sin A\cos A]$$

Derivatives Ex 30.4 Q16

We have,

$$\frac{d}{dx}\left(\log_{x^2} x\right)$$

$$\log_{x^2} x = \frac{\log x}{\log x^2}$$

$$= \frac{\log x}{2\log x}$$

$$= \frac{1}{2}$$

$$\frac{d}{dx}\left(\frac{1}{2}\right) = 0$$

$$\therefore \frac{d}{dx}\left(\log_{x^2} x\right) = 0$$

Derivatives Ex 30.4 Q17

$$\frac{d}{dx}\left(e^x \log\sqrt{x}\ \tan x\right)$$

Apply product rule,

$$= \log\sqrt{x} \times \tan x \frac{d}{dx}\left(e^x\right) + e^x \times \tan x \frac{d}{dx}\left(\log\sqrt{x}\right) + e^x \log\sqrt{x} \frac{d}{dx}\left(\tan x\right)$$

$$= \log\sqrt{x} \times \tan e^x + e^x \tan x \frac{1}{2x} + e^x \log\sqrt{x} \times \sec^2 x$$

$$= \frac{1}{2}\log x \times \tan x \times e^x + \frac{1 \tan x}{2x} e^x + e^x \frac{1}{2} \log x \sec^2 x \qquad \left[\because \log\sqrt{x} = \frac{1}{2}\log x\right]$$

$$= \frac{1}{2} e^x \left(\log x \times \tan x + \frac{\tan x}{x} + \log x \sec^2 x\right)$$

RD Sharma Solutions Class 11 Maths Chapter 30 Ex 30.5

Derivatives Ex 30.5 Q1

Using quotient rule, we have

$$\frac{d}{dx}\left(\frac{x^2+1}{x+1}\right) = \frac{(x+1)\frac{d}{dx}(x^2+1)-(x^2+1)\frac{d}{dx}(x+1)}{(x+1)^2}$$

$$= \frac{(x+1)\times 2x-(x^2+1)\times 1}{(x+1)^2}$$

$$= \frac{2x^2+2x-x^2-1}{(x+1)^2}$$

$$= \frac{x^2+2x-1}{(x+1)^2}$$

Derivatives Ex 30.5 Q2

Using quotient rule, we have get,

$$\frac{d}{dx}\left(\frac{2x-1}{x^2+1}\right)$$

$$= \frac{(x^2+1)\frac{d}{dx}(2x-1)-(2x-1)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\times 2-(2x-1)\times 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2+2x}{(x^2+1)^2}$$

$$= \frac{-2x^2+2x+2}{(x^2+1)^2}$$

$$= \frac{2(-x^2+x+1)}{(x^2+1)^2}$$

$$= \frac{2(1+x-x^2)}{(1+x^2)^2}$$

Derivatives Ex 30.5 Q3

By using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{x+e^x}{1+\log x}\right)$$

$$=\frac{(1+\log x)\frac{d}{dx}\left(x+e^x\right)-\left(x+e^x\right)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$=\frac{(1+\log x)\left(1+e^x\right)-\left(x+e^x\right)\times\frac{d}{dx}}{(1+\log x)^2}$$

$$=\frac{x\left(1+\log x+e^x+e^x\log x\right)-x-e^x}{x(1+\log x)^2}$$

$$=\frac{x+x\log x+xe^x+xe^x\log x-x-e^x}{x(1+\log x)^2}$$

$$=\frac{x\log x\left(1+e^x\right)-e^x(1-x)}{x(1+\log x)^2}$$

Derivatives Ex 30.5 Q4

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{e^x-\tan x}{\cot x-x^n}\right)$$

$$=\frac{\left(\cot x-x^n\right)\frac{d}{dx}\left(e^x-\tan x\right)-\left(e^x-\tan x\right)\frac{d}{dx}\left(\cot x-x^n\right)}{\left(\cot x-x^n\right)^2}$$

$$=\frac{\left(\cot x-x^n\right)\left(e^x-\sec^2 x\right)-\left(e^x-\tan x\right)\left(-\operatorname{cosec}^2 x-nx^{n-1}\right)}{\left(\cot x-x^n\right)^2}$$

$$=\frac{\left(\cot x-x^n\right)\left(e^x-\sec^2 x\right)+\left(e^x-\tan x\right)\left(\operatorname{cosec}^2 x+nx^{n-1}\right)}{\left(\cot x-x^n\right)^2}$$

Derivatives Ex 30.5 Q5

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{ax^2+bx+c}{px^2+qx+r}\right)$$

$$=\frac{\left(px^2+qx+r\right)\frac{d}{dx}\left(ax^2+bx+c\right)-\left(ax^2+bx+c\right)\frac{d}{dx}\left(px^2+qx+r\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{\left(px^2+qx+r\right)\left(2ax+b\right)-\left(ax^2+bx+c\right)\left(2px+q\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{2apx^3+2aqx^2+2axr+bpx^2+bqx+br-\left(2apx^3+2pbx^2+2pcx+qax^2+bqx+cq\right)}{\left(px^2+qx+r\right)^2}$$

$$=\frac{2apx^3-2apx^3+2aqx^2+bpx^2-2pbx^2-qax^2+2arx+bqx-2pcx-bqx+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{aqx^2-bpx^2+2arx-2cpx+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{x^2\left(aq-bp\right)+2\left(ar-cp\right)x+br-cq}{\left(px^2+qx+r\right)^2}$$

$$=\frac{\left(aq-bp\right)x^2+2\left(ar-cp\right)x+br-cq}{\left(px^2+qx+r\right)^2}$$

Derivatives Ex 30.5 Q6

Using quotient rule, we have,

$$\frac{d}{dx}\left(\frac{x}{1+\tan x}\right)$$

$$=\frac{\left(1+\tan x\right)\frac{d}{dx}\left(x\right)-x\frac{d}{dx}\left(1+\tan x\right)}{\left(1+\tan x\right)^2}$$

$$=\frac{\left(1+\tan x\right)-x\left(\sec^2 x\right)}{\left(1+\tan x\right)^2}$$

$$= \frac{1 + \tan x - x \sec^2 x}{(1 + \tan x)^2}$$

Derivatives Ex 30.5 Q7

Using quotient rule, we have

$$\frac{d}{dx}\left(\frac{1}{ax^2 + bx + c}\right)$$

$$= \frac{(ax^2 + bx + c)\frac{d}{dx}(1) - 1 \times \frac{d}{dx}(ax^2 + bx + c)}{(ax^2 + bx + c)^2}$$

$$= \frac{-(2ax + b)}{(ax^2 + bx + c)^2}$$

$$\therefore \frac{d}{dx}\frac{1}{ax^2 + bx + c} = \frac{-(2ax + b)}{(ax^2 + bx + c)^2}$$

Derivatives Ex 30.5 Q8

We have,

$$\frac{d}{dx}\left(\frac{e^x}{1 + x^2}\right)$$

Using quotient rule,

$$= \frac{(1 + x^2)\frac{d}{dx}(e^x) - (e^x)\frac{d}{dx}(1 + x^2)}{(1 + x^2)^2}$$

$$= \frac{(1 + x^2)e^x - e^x \times 2x}{(1 + x^2)^2}$$

$$= \frac{e^x(1 + x^2 - 2x)}{(1 + x^2)^2}$$

$$= \frac{e^x (1-x)^2}{(1+x^2)^2}$$

Derivatives Ex 30.5 Q9

We have,

$$\frac{d}{dx}\left(\frac{e^x + \sin x}{1 + \log x}\right)$$

Using quotient rule, we get

$$= \frac{(1+\log x)\frac{d}{dx}(e^x + \sin x) - (e^x + \sin x)\frac{d}{dx}(1+\log x)}{(1+\log x)^2}$$

$$= \frac{(1+\log x)(e^x + \cos x) - (e^x + \sin x)\frac{1}{x}}{(1+\log x)^2}$$

$$= \frac{x(1+\log x)(e^x + \cos x) - (e^x + \sin x)}{x(1+\log x)^2}$$

Derivatives Ex 30.5 Q10

We have,

$$\frac{d}{dx}\left(\frac{x \tan x}{\sec x + \tan x}\right)$$

Using quotient rule, we get

$$= \frac{(\sec x + \tan x)\frac{d}{dx}(x \tan x) - (x \tan x)\frac{d}{dx}(\sec x + \tan x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)(x \sec^2 x + \tan x) - (x \tan x)(\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)^2} \quad \text{[Used product rule]}$$

$$= \frac{(\sec x + \tan x)(x \sec^2 x + \tan x) - x \sec x + \tan^2 x - x \tan x \sec^2 x}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)(x \sec^2 x + \tan x) - x \tan x (\sec x \tan x + \sec^2 x)}{(\sec x + \tan x)^2}$$

$$= \frac{(\sec x + \tan x)(x \sec^2 x + \tan x) - x \tan x \sec x (\sec x + \tan x)}{(\sec x + \tan x)^2}$$

$$= \frac{(x \sec^2 x + \tan x - x \tan x \sec x)(\sec x + \tan x)}{(\sec x + \tan x)^2}$$

$$= \frac{(x \sec^2 x + \tan x - x \tan x \sec x)}{(\sec x + \tan x)}$$

$$= \frac{x \sec x (\sec x - \tan x) + \tan x}{(\sec x + \tan x)}$$