RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.1

Limits Ex 29.1 Q1

$$\lim_{x\to 0} \frac{x}{|x|}$$

We know that $|x| = \begin{cases} x, \text{if } x \ge 0 \\ -x, \text{if } x < 0 \end{cases}$

$\therefore \quad \lim_{x\to 0^+} \frac{x}{|x|} = \lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$

Also, $\lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0^-} \frac{x}{-x} = \lim_{x\to 0^-} -1 = -1$

$\Rightarrow \quad$ LHL of $f(x) \ne$ RHL of $f(x)$

$\Rightarrow \quad \lim_{x\to 0} \frac{x}{|x|}$ does not exist

Limits Ex 29.1 Q2

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (2x+3)$$
$$= 2(2)+3$$
$$= 7$$

$\therefore \quad \lim_{x\to 2^-} f(x) = 7$

Also,

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (x+k)$$
$$= (2+k)$$

Since, $\lim_{x\to 2} f(x)$ exists (given)

$\therefore \quad \lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$\Rightarrow \quad 7 = 2+k$

$\Rightarrow \quad k = 5$

Limits Ex 29.1 Q3

Let $f(x) = \frac{1}{x}$, this function is defined for every value of x except at $x = 0$

As $x \to 0^+, \frac{1}{x} \to \infty$.

As $x \to 0^-, \frac{1}{x} \to -\infty$

$\therefore \quad \lim_{x\to 0} \frac{1}{x}$ does not exist.

Limits Ex 29.1 Q4

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{3x}{-x+2x} = \lim_{x\to 0^-} \frac{3x}{x} = 3 \qquad \left[\because \text{ as } x \to 0^-, |x| = -x\right]$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{3x}{x+2x} = 1 \qquad \left[\because \text{ as } x \to 0^+, |x| = x\right]$$

thus, $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

$\therefore$ $\lim_{x\to 0} f(x)$ does not exist.

Limits Ex 29.1 Q5

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x+1$$
$$= \lim_{h\to 0} (0+h)+1 = 1$$

Also, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x-1$

$\lim_{h\to 0} (0-h)-1 = -1$

$\Rightarrow$ $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$

Hence, limit does not exist.

Limits Ex 29.1 Q6

$$\begin{aligned} LHL &= \lim_{x\to 0^-} f(x) \\ &= \lim_{h\to 0} f(0-h) \\ &= \lim_{h\to 0} -h-4 \\ &= 0-4 \\ &= -4 \end{aligned}$$

$$\begin{aligned} RHL &= \lim_{x\to 0^+} f(x) \\ &= \lim_{h\to 0} f(0+h) \\ &= \lim_{h\to 0} h+5 \\ &= 0+5 \\ &= 5 \end{aligned}$$

$\therefore \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$

Hence $\lim_{x\to 0} f(x)$ does not exist.

Limits Ex 29.1 Q7

$$\lim_{x\to 3^+} f(x) = 4$$

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x+1) = \lim_{h\to 0} (3-h+1) = 3+1 = 4$$

Since, $\lim_{x\to 3^+} f(x) = 4 = \lim_{x\to 3^-} f(x)$

$\therefore$ $\lim_{x\to 3} f(x)$ is 4

Limits Ex 29.1 Q8

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 3(x+1) = 3$$

and

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} 2(x) + 3 = 3$$

$$\therefore \quad \lim_{x\to 0} f(x) = 3$$

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} 3(x+1) = 3+3 = 6$$

$$\therefore \quad \lim_{x\to 1} f(x) = 6$$

Limits Ex 29.1 Q9

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} x^2 - 1 = \lim_{h\to 0} (-1-h)^2 - 1 = 1 - 1 = 0$$

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} -x^2 - 1 = \lim_{h\to 0} -(1+h)^2 - 1 = -2$$

Since, $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

$\therefore \lim_{x\to 1} f(x)$ does not exist.

Limits Ex 29.1 Q10

$$\text{LHL} = \lim_{x\to 0^-} f(x)$$

$$\Rightarrow \quad \lim_{x\to 0^-} \frac{|x|}{x}$$

$$\Rightarrow \quad \lim_{h\to 0} \frac{|0-h|}{0-h}$$

$$\Rightarrow \quad \lim_{h\to 0} \frac{+h}{-h} = -1 \qquad \text{---(i)}$$

and,

$$\text{RHL} = \lim_{x\to 0^+} f(x)$$

$$\Rightarrow \quad \lim_{x\to 0^+} \frac{|x|}{x}$$

$$\Rightarrow \quad \lim_{h\to 0} \frac{|0+h|}{0+h} = \lim_{h\to 0} \frac{h}{h} = 1 \qquad \text{---(ii)}$$

So, LHL $\neq$ RHL

$\therefore \lim_{x\to 0} f(x)$ does not exist.

Limits Ex 29.1 Q11

$$\lim_{x \to a_1} f(x)$$

$\Rightarrow \lim_{x \to a_1} (x - a_1)(x - a_2)\ldots(x - a_n)$ [Putting limit $x \to a_1$]

$\Rightarrow (a_1 - a_1)(a_1 - a_2)\ldots(a_1 - a_n)$

$\Rightarrow 0$

And, $\lim_{x \to a} f(x)$

$\Rightarrow \lim_{x \to a} (x - a_1)(x - a_2)\ldots(x - a_n)$ [Putting limit $x \to a$]

$\Rightarrow (a - a_1)(a - a_2)\ldots(a - a_n).$

Limits Ex 29.1 Q12

$$\lim_{x \to 1^+} \frac{1}{(x-1)} = \lim_{h \to 0} \frac{1}{(1+h-1)} = \lim_{h \to 0} \frac{1}{h} = \infty$$

Limits Ex 29.1 Q13(i)

$$\lim_{x \to 2^+} \frac{x-3}{x^2-4}$$

$$= \lim_{h \to 0} \frac{(2+h)-3}{(2+h)^2 - 2^2} \qquad \left[\because \lim_{x \to 2^+} f(x) = \lim_{h \to 0} f(2+h)\right]$$

$$= \lim_{h \to 0} \frac{(2-3+h)}{(2+h-2)(2+h+2)}$$

$$= \lim_{h \to 0} \frac{(h-1)}{(h)(4+h)}$$

$$= \lim_{h \to 0} \frac{1 - \frac{1}{h}}{4+h}$$

$$= \frac{1 - \frac{1}{0}}{4} = -\infty$$

Limits Ex 29.1 Q13(ii)

$$\lim_{x \to 2^-} \frac{x-3}{(x^2-4)} = \lim_{h \to 0} \frac{(2-h)-3}{(2-h)^2 - 4}$$

$$= \lim_{h \to 0} \frac{(2-h-3)}{(2-h+2)(2-h-2)} \qquad \left[\because \lim_{x \to 2^-} f(x) = \lim_{h \to 0} f(2-h)\right]$$

$$= \lim_{h \to 0} \frac{-1-h}{(4-h)(-h)}$$

$$= \lim_{h \to 0} \frac{\frac{1}{h}+1}{(4-h)}$$

$$= \frac{\frac{1}{0}+1}{4} = \infty$$

Limits Ex 29.1 Q13(iii)

$$\lim_{x\to 0^+}\frac{1}{3x}$$

$$=\lim_{h\to 0}\frac{1}{3(0+h)}$$

$$=\lim_{h\to 0}\frac{1}{0+3h}$$

$$=\frac{1}{0}=\infty$$

Limits Ex 29.1 Q13(iv)

$$\lim_{x\to -8^+}\frac{2x}{x+8}$$

$$=\lim_{h\to 0}\frac{2(-8+h)}{(-8+h)+8}$$

$$=\lim_{h\to 0}\frac{-16+2h}{h}$$

$$=\lim_{h\to 0}\frac{-16}{h}+2$$

$$\Rightarrow \quad \frac{-16}{0}+2=\infty$$

Limits Ex 29.1 Q13(v)

$$\lim_{x\to 0^+}\frac{2}{x^{\frac{1}{5}}}$$

$$=\lim_{h\to 0}\frac{2}{(0+h)^{\frac{1}{5}}}$$

$$\Rightarrow \frac{2}{0}=\infty$$

Limits Ex 29.1 Q13(vi)

$$\lim_{x\to \frac{\pi}{2}^-}\tan x$$

$$=\lim_{h\to 0}\tan\left(\frac{\pi}{2}-h\right)$$

$$=\tan\left(\frac{\pi}{2}-0\right)$$

$$\Rightarrow \quad \tan\frac{\pi}{2}=\infty$$

Limits Ex 29.1 Q13(vii)

$$\lim_{x \to -\frac{\pi}{2}^+} \sec x$$

$$= \lim_{h \to 0} \sec\left(-\frac{\pi}{2} + h\right)$$

$$= \sec\left(-\frac{\pi}{2} + 0\right)$$

$$= \sec\left(-\frac{\pi}{2}\right)$$

$$= \frac{1}{\cos\left(-\frac{\pi}{2}\right)}$$

$$= \frac{-1}{\left(\cos\frac{\pi}{2}\right)}$$

$$= \frac{-1}{0} = -\infty$$

Limits Ex 29.1 Q13(viii)

$$\lim_{x \to 0^-} \frac{x^2 - 3x + 2}{x^3 - 2x^2}$$

$$= \lim_{x \to 0^-} \frac{x^2 - x - 2x + 2}{x^2(x - 2)}$$

$$= \lim_{x \to 0^-} \frac{x(x - 1) - 2(x - 1)}{x^2(x - 2)}$$

$$= \lim_{x \to 0^-} \frac{(x - 1)(x - 2)}{x^2(x - 2)}$$

$$= \lim_{x \to 0^-} \frac{(x - 1)}{x^2}$$

$$= \lim_{h \to 0} \frac{(0 - h - 1)}{(0 - h)^2}$$

$$= \frac{-h}{h^2} = \frac{-1}{h} = \frac{-1}{0} = -\infty$$

Limits Ex 29.1 Q13(ix)

$$\lim_{x \to -2^+} \frac{x^2 - 1}{2x + 4}$$

$$= \lim_{x \to 2^+} \frac{(x - 1)(x + 1)}{2(x + 2)}$$

$$= \lim_{h \to 0} \frac{(-2 + h - 1)(-2 + h + 1)}{2(-2 + h + 2)}$$

$$= \lim_{h \to 0} \frac{(-3 + h)(h - 1)}{2h}$$

$$\Rightarrow \quad \frac{-3 \times -1}{2 \times 0} = \frac{1}{0} = \infty$$

Limits Ex 29.1 Q13(x)

$$\lim_{x\to 0^-} 2-\cot x$$
$$= \lim_{h\to 0} 2-\cot(0-h)$$
$$= \lim_{h\to 0} 2-(-1)\coth$$
$$= \lim_{h\to 0} 2+\coth$$
$$= \lim_{h\to 0} 2+\frac{1}{\tanh}$$
$$\Rightarrow \quad 2+\frac{1}{0} \Leftarrow \infty$$

Limits Ex 29.1 Q13(xi)

$$\lim_{x\to 0^-} 1+\operatorname{cosec} x$$
$$= \lim_{x\to 0^-} 1+\operatorname{cosec}(0-h)$$
$$= \lim_{h\to 0} 1-\operatorname{cosec} h$$
$$= \lim_{h\to 0} 1-\frac{1}{\sinh}$$
$$\Rightarrow \quad 1-\frac{1}{0} = -\infty$$

Limits Ex 29.1 Q14

$$\lim_{x\to 0} e^{\frac{-1}{x}}$$
$$\lim_{x\to 0^+} e^{\frac{-1}{x}} = \lim_{x\to 0^+} \frac{1}{e^{\frac{1}{x}}} = \lim_{h\to 0} \frac{1}{e^{\frac{1}{0+h}}} = \frac{1}{e^{\frac{1}{0}}} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$
$$And, \lim_{x\to 0^-} e^{\frac{-1}{x}} = \lim_{x\to 0^-} \frac{1}{e^{\frac{1}{x}}} = \lim_{h\to 0} \frac{1}{e^{\frac{1}{0-h}}} = \lim_{h\to 0} \frac{1}{e^{-\frac{1}{h}}} = \frac{1}{e^{-\frac{1}{0}}} = \frac{1}{e^{-\infty}} = e^{\infty} = \infty$$
$$\Rightarrow \quad \lim_{x\to 0^+} e^{\frac{-1}{x}} \neq \lim_{x\to 0^-} e^{\frac{-1}{x}}$$
$$\therefore \quad \lim_{x\to 0} e^{\frac{-1}{x}} \text{ does not exist .}$$

Limits Ex 29.1 Q15

(i) $\lim_{x \to 2}[x]$

$$\lim_{x \to 2^-}[x] = 1$$
$$\lim_{x \to 2^+}[x] = 2$$

Thus, $\lim_{x \to 2}[x]$ does not exist.

(ii) $\lim_{x \to \frac{5}{2}}[x]$

$$\lim_{x \to \frac{5}{2}^+}[x] = 2$$
$$\lim_{x \to \frac{5}{2}^-}[x] = 2$$

$\Rightarrow \quad \lim_{x \to \frac{5}{2}}[x] = 2$

(iii) $\lim_{x \to 1}[x]$

$$\lim_{x \to 1^-}[x] = 0$$
$$\lim_{x \to 1^+}[x] = 1$$

$\Rightarrow \quad \lim_{x \to 1^-}[x] \neq \lim_{x \to 1^+}[x]$

Thus, $\lim_{x \to 1}[x]$ does not exist

Limits Ex 29.1 Q16

$\lim_{x \to a^+}[x]$

$\Rightarrow \quad \lim_{h \to 0^+}[a+h] = [a]$

$\Rightarrow \quad \lim_{h \to 0^+}[x] = [a] \forall a \in R$

Also, $\lim_{x \to 1^-}[x]$

$= \lim_{h \to 0}[1-h]$

$= 0$

$\Rightarrow \quad \lim_{x \to 1^-}[x] = 0$

Limits Ex 29.1 Q17

$\lim_{x \to 2^-} \frac{x}{[x]} = \lim_{x \to 2^-} \frac{x}{1} = \frac{2}{1} = 2 \qquad \left[\because \lim_{x \to k^-}[x] = k-1\right]$

Also, $\lim_{x \to 2^+} \frac{x}{[x]} = \lim_{x \to 2^+} \frac{x}{3} = \frac{2}{3} \qquad \left[\lim_{x \to k^+}[x] = k+1\right]$

$\Rightarrow \quad \lim_{x \to 2^-} \frac{x}{[x]} \neq \lim_{x \to 2^+} \frac{x}{[x]}$

$$\lim_{x\to 3^+} \frac{x}{[x]} = \lim_{x\to 3^+} \frac{x}{3} = \frac{3}{3} = 1$$

$$\lim_{x\to 3^-} \frac{x}{[x]} = \lim_{x\to 3^-} \frac{x}{2} = \frac{3}{2} = 1.5$$

Therefore, $\lim_{x\to 3^+} \frac{x}{[x]} \neq \lim_{x\to 3^-} \frac{x}{[x]}$

Limits Ex 29.1 Q19

$$\lim_{x\to \frac{5}{2}} [x]$$

$$\lim_{x\to \frac{5}{2}} [x] = \left[\frac{5}{2}\right],$$

$$= [2.5] = 2 \qquad \text{[By definition of greatest integer function]}$$

$$\Rightarrow \quad \lim_{x\to \frac{5}{2}} [x] = 2$$

Limits Ex 29.1 Q20

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x - [x])$$

$$= \lim_{x\to 2^-} x - \lim_{x\to 2^-} [x]$$

$$= 2 - 1 = 1 \qquad \left[\because \lim_{x\to k^-} [x] = k - 1\right]$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3x - 5) \qquad [\because x > 2]$$

$$= 3(2) - 5$$

$$= 6 - 5$$

$$= 1$$

Thus, $\lim_{x\to 2^-} f(x) = 1 = \lim_{x\to 2^+} f(x)$

$$\Rightarrow \quad \lim_{x\to 2} f(x) = 1$$

Limits Ex 29.1 Q21

$$\lim_{x\to 0^-} \sin\frac{1}{x} = \lim_{h\to 0} \sin\frac{1}{0-h} = -\lim_{h\to 0} \sin\frac{1}{h}$$

$= -$(An oscillating number which oscillates between -1 and 1).

So, $\lim_{x\to 0^-} \sin\frac{1}{x}$ does not exist.

Similarly, $\lim_{x\to 0^+} \sin\frac{1}{x}$ does not exist.

$\lim_{x\to 0} \sin\frac{1}{x}$ does not exist.

Limits Ex 29.1 Q22

Let $f(x)=\begin{cases} \dfrac{k\cos x}{\pi-2x}, & \text{where } x \neq \dfrac{\pi}{2} \\ 3, & \text{where } x = \dfrac{\pi}{2} \end{cases}$ and if $\lim\limits_{x\to\pi/2} f(x) = f\left(\dfrac{\pi}{2}\right)$, find the value of k.

$$\text{LHL} = \lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$= \lim_{h\to\frac{\pi}{2}} f\left(\frac{\pi}{2}-h\right)$$

$$= \lim_{h\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{\pi-2\left(\frac{\pi}{2}-h\right)}$$

$$= \lim_{h\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}-h\right)}{2h}$$

$$= \frac{k\cos\left(\frac{\pi}{2}-\frac{\pi}{2}\right)}{\pi}$$

$$= \frac{k}{\pi}$$

$$\text{RHL} = \lim_{x\to\frac{\pi}{2}^+} f(x)$$

$$= \lim_{h\to\frac{\pi}{2}} f\left(\frac{\pi}{2}+h\right)$$

$$= \lim_{h\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}+h\right)}{\pi-2\left(\frac{\pi}{2}+h\right)}$$

$$= \lim_{h\to\frac{\pi}{2}} \frac{k\cos\left(\frac{\pi}{2}+h\right)}{-2h}$$

$$= \frac{k\cos\left(\frac{\pi}{2}+\frac{\pi}{2}\right)}{-\pi}$$

$$= \frac{k}{\pi}$$

$$\therefore \lim_{x\to\frac{\pi}{2}^-} f(x) = \lim_{x\to\frac{\pi}{2}^+} f(x)$$

Hence $\lim\limits_{x\to\frac{\pi}{2}} f(x)$ exists.

$$\lim_{x\to\frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right)$$

$$\frac{k}{\pi} = 3$$

$$k = 3\pi$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.2

Limits Ex 29.2 Q1

$$\lim_{x\to 1}\frac{x^2+1}{x+1}=\frac{(1)^2+1}{1+1}=\frac{1+1}{2}=\frac{2}{2}=1$$

Limits Ex 29.2 Q2

$$\lim_{x\to 0}\frac{2x^2+3x+4}{x^2+3x+2}=\lim_{x\to 0}\frac{2x^2+3x+4}{(x+2)(x+1)}=\frac{2(0)+3(0)+4}{(0+2)(0+1)}=\frac{4}{2}=2$$

Limits Ex 29.2 Q3

$$\lim_{x\to 3}\frac{\sqrt{2x+3}}{x+3}=\frac{\sqrt{9}}{6}=\frac{1}{2}$$

Limits Ex 29.2 Q4

$$\lim_{x\to 1}\frac{\sqrt{x+8}}{\sqrt{x}}=\frac{\sqrt{(1+8)}}{\sqrt{1}}=\sqrt{9}=3$$

Limits Ex 29.2 Q5

$$\lim_{x\to a}\frac{\sqrt{x}+\sqrt{a}}{x+a}=\frac{\sqrt{a}+\sqrt{a}}{a+a}=\frac{2\sqrt{a}}{2a}=\frac{1}{\sqrt{a}}$$

Limits Ex 29.2 Q6

$$\lim_{x\to 1}\frac{1+(x-1)^2}{1+x^2}=\frac{1+0^2}{1+1}=\frac{1}{2}$$

Limits Ex 29.2 Q7

$$\lim_{x\to 0}\frac{x^{\frac{2}{3}}-9}{x-27}=\frac{-9}{-27}=\frac{1}{3}$$

Limits Ex 29.2 Q8

$$\lim_{x \to 0} 9 = 9$$

Limits Ex 29.2 Q9

$$\lim_{x \to 2} (3 - x) = (3 - 2) = 1$$

Limits Ex 29.2 Q10

$$\lim_{x \to -1} \left(4x^2 + 2\right) = \left(4(-1)^2 + 2\right) = 6$$

Limits Ex 29.2 Q11

$$\lim_{x \to -1} \frac{x^3 - 3x + 1}{x - 1} = \frac{(-1)^3 - 3(-1) + 1}{(-1 - 1)} = \frac{-1 + 3 + 1}{-2} = \frac{3}{-2} = \frac{-3}{2}$$

Limits Ex 29.2 Q12

$$\lim_{x \to 0} \frac{3x + 1}{x + 3} = \frac{3(0) + 1}{(0 + 3)} = \frac{1}{3}$$

Limits Ex 29.2 Q13

$$\lim_{x \to 3} \frac{x^2 - 9}{x + 2} = \frac{3^2 - 9}{3 + 2} = 0$$

Limits Ex 29.2 Q14

$$\lim_{x \to 0} \frac{ax + b}{(x + d)} = \frac{a \times 0 + b}{(0 + d)} = \frac{b}{d}$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.3

Limits Ex 29.3 Q1

$$\lim_{x\to-5}\frac{2x^2+9x-5}{x+5}=\lim_{x\to-5}\frac{(x+5)(2x-1)}{(x+5)}\qquad=\lim_{x\to-5}(2x-1)=2(-5)-1=-10-1=-11$$

Limits Ex 29.3 Q2

$$\lim_{x\to3}\frac{x^2-4x+3}{x^2-2x-3}$$

$$=\lim_{x\to3}\frac{x^2-3x-x+3}{x^2+x-3x-3}$$

$$=\lim_{x\to3}\frac{x(x-1)-3(x-1)}{x(x+1)-3(x+1)}$$

$$=\lim_{x\to3}\frac{(x-1)(x-3)}{(x+1)(x-3)}$$

$$=\lim_{x\to3}\frac{x-1}{x+1}$$

$$=\frac{3-1}{3+1}$$

$$=\frac{2}{4}=\frac{1}{2}$$

Limits Ex 29.3 Q3

$$\lim_{x\to3}\frac{x^4-81}{x^2-9}=\lim_{x\to3}\frac{(x^2-9)(x^2+9)}{(x^2-9)}=\lim_{x\to3}x^2+9\qquad=(3)^2+9=9+9=18$$

Limits Ex 29.3 Q4

$$\lim_{x\to2}\frac{x^3-8}{x^2-4}=\lim_{x\to2}\frac{(x-2)(x^2+4+2x)}{(x-2)(x+2)}=\frac{(2)^2+4+2(2)}{2+2}=\frac{4+4+4}{4}=\frac{12}{4}=3$$

Limits Ex 29.3 Q5

$$\lim_{x \to -\frac{1}{2}} \frac{8x^3+1}{2x+1} = \lim_{x \to -\frac{1}{2}} \frac{8\left(x^3+\frac{1}{8}\right)}{2\left(x+\frac{1}{2}\right)}$$

$$= \frac{8}{2} \lim_{x \to -\frac{1}{2}} \frac{\left(x^3+\left(\frac{1}{2}\right)^3\right)}{x+\frac{1}{2}}$$

$$= 4 \lim_{x \to -\frac{1}{2}} \frac{\left(x+\frac{1}{2}\right)\left(x^2+\frac{1}{4}-\frac{1}{2}x\right)}{\left(x+\frac{1}{2}\right)}$$

$$= 4\left(\left(\frac{-1}{2}\right)^2+\frac{1}{4}-\frac{1}{2}\left(\frac{-1}{2}\right)\right)$$

$$= 4\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right)$$

$$= 3$$

Limits Ex 29.3 Q6

$$\lim_{x \to 4} \frac{x^2-7x+12}{x^2-3x-4}$$

$$= \lim_{x \to 4} \frac{x^2-3x-4x+12}{x^2+x-4x-4}$$

$$= \lim_{x \to 4} \frac{x(x-3)-4(x-3)}{x(x+1)-1(x+1)}$$

$$= \lim_{x \to 4} \frac{(x-3)(x-4)}{(x-4)(x+1)}$$

$$= \lim_{x \to 4} \frac{x-3}{x+1}$$

$$= \frac{4-3}{4+1}$$

$$= \frac{1}{5}$$

Limits Ex 29.3 Q7

$$\lim_{x\to 2} \frac{x^4 - 16}{x - 2}$$

$$= \lim_{x\to 2} \frac{(x^2 - 4)(x^2 + 4)}{(x - 2)}$$

$$= \lim_{x\to 2} \frac{(x - 2)(x + 2)(x^2 + 4)}{(x - 2)}$$

$$= \lim_{x\to 2} (x + 2)(x^2 + 4)$$

$$= (2 + 2)(4 + 4)$$

$$= 4(8)$$

$$= 32$$

Limits Ex 29.3 Q8

$$\lim_{x\to 5} \frac{x^2 - 9x + 20}{x^2 - 6x + 5}$$

$$= \lim_{x\to 5} \frac{x^2 - 4x - 5x + 20}{x^2 - x - 5x + 5}$$

$$= \lim_{x\to 5} \frac{x(x - 4) - 5(x - 4)}{x(x - 1) - 5(x - 1)}$$

$$= \lim_{x\to 5} \frac{(x - 5)(x - 4)}{(x - 5)(x - 1)}$$

$$= \lim_{x\to 5} \frac{x - 4}{x - 1}$$

$$= \frac{5 - 4}{5 - 1}$$

$$= \frac{1}{4}$$

Limits Ex 29.3 Q9

$$\lim_{x \to -1} \frac{x^3 + 1}{x + 1}$$

$$= \lim_{x \to -1} \frac{(x+1)\left(x^2 - x + 1\right)}{(x+1)} \qquad \left[a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right)\right]$$

$$= \lim_{x \to -1} \left(x^2 - x + 1\right)$$

$$= (-1)^2 - (-1) + 1$$

$$= 1 + 1 + 1$$

$$= 3$$

Limits Ex 29.3 Q10

$$\lim_{x \to 5} \frac{x^3 - 125}{x^2 - 7x + 10} = \lim_{x \to 5} \frac{(x-5)\left(x^2 + 25 + 5x\right)}{(x-2)(x-5)} = \frac{(5)^2 + 25 + 5(5)}{(5-2)} = \frac{25 + 25 + 25}{3} = \frac{75}{3} = 25$$

Limits Ex 29.3 Q11

$$\lim_{x\to\sqrt{2}} \frac{x^2-2}{x^2+\sqrt{2}x-4}$$

$$= \lim_{x\to\sqrt{2}} \frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x^2+2\sqrt{2}x-\sqrt{2}x-4}$$

$$= \lim_{x\to\sqrt{2}} \frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x\left(x+2\sqrt{2}\right)-\sqrt{2}\left(x+2\sqrt{2}\right)}$$

$$= \lim_{x\to\sqrt{2}} \frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{\left(x+2\sqrt{2}\right)\left(x-\sqrt{2}\right)}$$

$$= \frac{\sqrt{2}+\sqrt{2}}{\sqrt{2}+2\sqrt{2}} = \frac{2\sqrt{2}}{3\sqrt{2}}$$

$$= \frac{2}{3}$$

Limits Ex 29.3 Q12

$$\lim_{x\to\sqrt{3}} \frac{x^2-3}{x^2+3\sqrt{3}x-12}$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)}{x^2+4\sqrt{3}x-\sqrt{3}x-12}$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)}{x\left(x+4\sqrt{3}\right)-\sqrt{3}\left(x+4\sqrt{3}\right)}$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)}{\left(x-\sqrt{3}\right)\left(x+4\sqrt{3}\right)}$$

$$= \frac{\sqrt{3}+\sqrt{3}}{\sqrt{3}+4\sqrt{3}} = \frac{2\sqrt{3}}{5\sqrt{3}}$$

$$= \frac{2}{5}$$

Limits Ex 29.3 Q13

$$\lim_{x\to\sqrt{3}} \frac{x^4-9}{x^2+4\sqrt{3}x-15} = \lim_{x\to\sqrt{3}} \frac{\left(x-\sqrt{3}\right)\left(x+\sqrt{3}\right)\left(x^2+3\right)}{\left(x-\sqrt{3}\right)\left(x+5\sqrt{3}\right)} = \lim_{x\to\sqrt{3}} \frac{\left(x+\sqrt{3}\right)\left(x^2+3\right)}{\left(x+5\sqrt{3}\right)}$$

$$= \frac{\left(\sqrt{3}+\sqrt{3}\right)(3+3)}{\left(\sqrt{3}+5\sqrt{3}\right)} = \frac{\left(2\sqrt{3}\right)(6)}{6\sqrt{3}} = 2$$

Limits Ex 29.3 Q14

$$\lim_{x\to 2}\left(\frac{x}{x-2} - \frac{4}{x^2-2x}\right)$$

$$= \lim_{x\to 2}\left(\frac{x}{x-2} - \frac{4}{x(x-2)}\right)$$

$$= \lim_{x\to 2}\left(\frac{x(x)-4}{x(x-2)}\right)$$

$$= \lim_{x\to 2}\left(\frac{x^2-4}{x(x-2)}\right)$$

$$= \lim_{x\to 2}\frac{(x-2)(x+2)}{x(x-2)}$$

$$= \lim_{x\to 2}\frac{(x+2)}{x}$$

$$= \frac{2+2}{2}$$

$$= \frac{4}{2}$$

$$= 2$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.4

Limits Ex 29.4 Q1

$$\lim_{x\to 0}\frac{\sqrt{1+x+x^2}-1}{x}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x+x^2}-1\right)}{x}\frac{\left(\sqrt{1+x+x^2}+1\right)}{\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\lim_{x\to 0}\frac{\left(\left(1+x+x^2\right)-1\right)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\lim_{x\to 0}\frac{x\left(1+x\right)}{x\left(\sqrt{1+x+x^2}+1\right)}$$

$$=\lim_{x\to 0}\frac{1+x}{\sqrt{1+x+x^2}+1}$$

$$=\frac{1+0}{\sqrt{1+0+0}+1}$$

$$=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

Limits Ex 29.4 Q2

$$\lim_{x\to 0}\frac{2x}{\sqrt{a+x}-\sqrt{a-x}}$$

$$=\lim_{x\to 0}\frac{2x}{\left(\sqrt{a+x}-\sqrt{a-x}\right)}\times\frac{\sqrt{a+x}+\sqrt{a-x}}{\left(\sqrt{a+x}+\sqrt{a-x}\right)}$$

$$=\lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{\left((a+x)-(a-x)\right)}$$

$$=\lim_{x\to 0}\frac{2x\left(\sqrt{a+x}+\sqrt{a-x}\right)}{2x}$$

$$=\lim_{x\to 0}\left(\sqrt{a+x}+\sqrt{a-x}\right)$$

$$=\sqrt{a}+\sqrt{a}$$

$$=2\sqrt{a}$$

Limits Ex 29.4 Q3

$$\lim_{x\to 0}\frac{\sqrt{a^2+x^2}-a}{x^2}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{a^2+x^2}-a\right)}{x^2}\times\frac{\left(\sqrt{a^2+x^2}+a\right)}{\left(\sqrt{a^2+x^2}+a\right)}$$

$$=\lim_{x\to 0}\frac{\left(a^2+x^2-a^2\right)}{x^2\sqrt{a^2+x^2}+a}$$

$$=\lim_{x\to 0}\frac{x^2}{x^2\left(\sqrt{a^2+x^2}+a\right)}$$

$$=\lim_{x\to 0}\frac{1}{\sqrt{a^2+x^2}+a}$$

$$=\frac{1}{a+a}$$

$$=\frac{1}{2a}$$

Limits Ex 29.4 Q4

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1-x}}{2x}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-\sqrt{1-x}\right)}{2x}\times\frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{(1+x)-(1-x)}{2x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{2x}{2x\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\frac{1}{\sqrt{1}+\sqrt{1}}$$

$$=\frac{1}{2}$$

Limits Ex 29.4 Q5

$$\lim_{x\to 2}\frac{\sqrt{3-x}-1}{2-x}$$

$$=\lim_{x\to 2}\frac{\left(\sqrt{3-x}-1\right)}{2-x}\times\frac{\left(\sqrt{3-x}+1\right)}{\left(\sqrt{3-x}+1\right)}$$

$$=\lim_{x\to 2}\frac{(3-x)-1}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$=\lim_{x\to 2}\frac{(2-x)}{(2-x)\left(\sqrt{3-x}+1\right)}$$

$$=\lim_{x\to 2}\frac{1}{\left(\sqrt{3-x}+1\right)}$$

$$=\frac{1}{\sqrt{3-2}+1}=\frac{1}{1+1}$$

$$=\frac{1}{2}$$

Limits Ex 29.4 Q6

$$\lim_{x\to 3}\frac{x-3}{\sqrt{x-2}-\sqrt{4-x}}$$

$$=\lim_{x\to 3}\frac{x-3}{\left(\sqrt{x-2}-\sqrt{4-x}\right)}\times\frac{\left(\sqrt{x-2}+\sqrt{4-x}\right)}{\left(\sqrt{x-2}+\sqrt{4-x}\right)}$$

$$=\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{(x-2)-(4-x)}$$

$$=\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{x-2-4+x}$$

$$=\lim_{x\to 3}\frac{(x-3)\left(\sqrt{x-2}+\sqrt{4-x}\right)}{2(x-3)}$$

$$=\frac{1}{2}\lim_{x\to 3}\left(\sqrt{x-2}+\sqrt{4-x}\right)$$

$$=\frac{1}{2}\left(\sqrt{3-2}+\sqrt{4-3}\right)$$

$$=\frac{1}{2}\left(\sqrt{1}+\sqrt{1}\right)$$

$$=\frac{1}{2}(1+1)=\frac{2}{2}$$

$$=1$$

Limits Ex 29.4 Q7

$$\lim_{x\to 0}\frac{x}{\left(\sqrt{1+x}-\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{x}{\left(\sqrt{1+x}-\sqrt{1-x}\right)}\times\frac{\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}+\sqrt{1-x}\right)}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{\left(\sqrt{1+x}\right)^2-\left(\sqrt{1-x}\right)^2}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{1+x-1+x}$$

$$=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{2x}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\frac{\sqrt{1+x}+\sqrt{1-x}}{x}\right)x$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\sqrt{1+x}+\sqrt{1-x}\right)$$

$$=\frac{1}{2}\left(\sqrt{1}+\sqrt{1}\right)$$

$$=\frac{2}{2}=1$$

Limits Ex 29.4 Q8

$$\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{x-1}$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{x-1}\times\frac{\left(\sqrt{5x-4}+\sqrt{x}\right)}{\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{\left((5x-4)-x\right)}{(x-1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=4\lim_{x\to 1}\frac{(x-1)}{(x-1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=4\lim_{x\to 1}\frac{1}{\sqrt{5x-4}+\sqrt{x}}$$

$$=4\times\frac{1}{\sqrt{5-4}+\sqrt{1}}$$

$$=4\times\frac{1}{\sqrt{1}+\sqrt{1}}$$

$$=\frac{4}{2}=2$$

Limits Ex 29.4 Q9

$$\lim_{x\to 1}\frac{(x-1)}{\left(\sqrt{x^2+3}-2\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\times\left(\sqrt{x^2+3}+2\right)}{\left(\sqrt{x^2+3}-2\right)\left(\sqrt{x^2+3}+2\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2+3-4\right)}$$

$$=\lim_{x\to 1}\frac{(x-1)\left(\sqrt{x^2+3}+2\right)}{\left(x^2-1\right)}$$

$$=\lim_{x\to 1}\frac{\sqrt{x^2+3}+2}{x+1}$$

Putting the value $x = 1$

$$\Rightarrow \quad \frac{\sqrt{1+3}+2}{1+1}$$

$$=\frac{2+2}{2}$$

$$=\frac{4}{2}=2$$

Limits Ex 29.4 Q10

$$\lim_{x\to 3}\frac{\left(\sqrt{x+3}-\sqrt{6}\right)}{\left(x^2-9\right)}$$

$$=\lim_{x\to 3}\frac{\left(\sqrt{x+3}-\sqrt{6}\right)}{(x-3)(x+3)}\frac{\left(\sqrt{x+3}-\sqrt{6}\right)}{\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\lim_{x\to 3}\frac{\left((x+3)-6\right)}{(x-3)(x+3)\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\lim_{x\to 3}\frac{(x-3)}{(x-3)(x+3)\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\lim_{x\to 3}\frac{1}{(x+3)\left(\sqrt{x+3}+\sqrt{6}\right)}$$

$$=\frac{1}{(3+3)\sqrt{3+3}+\sqrt{6}}$$

$$=\frac{1}{6\left(\sqrt{6}+\sqrt{6}\right)}=\frac{1}{6\times 2\sqrt{6}}$$

$$=\frac{1}{12\sqrt{6}}$$

Limits Ex 29.4 Q11

$$\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{\left(x^2-1\right)}$$

$$=\lim_{x\to 1}\frac{\left(\sqrt{5x-4}-\sqrt{x}\right)}{(x-1)(x+1)}\times\frac{\left(\sqrt{5x-4}+\sqrt{x}\right)}{\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{\left((5x-4)-x\right)}{(x-1)(x+1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{4(x-1)}{(x-1)(x+1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\lim_{x\to 1}\frac{4}{(x+1)\left(\sqrt{5x-4}+\sqrt{x}\right)}$$

$$=\frac{4}{(1+1)\left(\sqrt{5-4}+\sqrt{1}\right)}$$

$$=\frac{4}{2(1+1)}$$

$$=\frac{4}{4}=1$$

Limits Ex 29.4 Q12

$$\lim_{x\to 0}\frac{\left(\sqrt{1+x}-1\right)}{x}$$

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+x}-1\right)}{x}\times\frac{\left(\sqrt{1+x}+1\right)}{\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{(1+x-1)}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{x}{x\left(\sqrt{1+x}+1\right)}$$

$$=\lim_{x\to 0}\frac{1}{\left(\sqrt{1+x}+1\right)}$$

$$=\frac{1}{\sqrt{1}+1}=\frac{1}{2}$$

Limits Ex 29.4 Q13

$$\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{(x-2)}$$

$$=\lim_{x\to 2}\frac{\sqrt{x^2+1}-\sqrt{5}}{(x-2)}\times\frac{\left(\sqrt{x^2+1}+\sqrt{5}\right)}{\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\lim_{x\to 2}\frac{\left(x^2+1-5\right)}{(x-2)\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\lim_{x\to 2}\frac{(x+2)(x-2)}{(x-2)\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\lim_{x\to 2}\frac{(x+2)}{\left(\sqrt{x^2+1}+\sqrt{5}\right)}$$

$$=\frac{(2+2)}{\sqrt{4+1}+\sqrt{5}}$$

$$=\frac{4}{2\sqrt{5}}=\frac{2}{\sqrt{5}}$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.5

Limits Ex 29.5 Q1

$$\lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{x-a}$$

$$= \lim_{x \to a} \frac{(x+2)^{\frac{5}{2}} - (a+2)^{\frac{5}{2}}}{(x+2)-(a+2)}$$

$$= \lim_{y \to b} \frac{y^{\frac{5}{2}} - b^{\frac{5}{2}}}{y-b}, \text{ where } x+2=y \text{ and } a+2=b$$

$$= \frac{5}{2} b^{\frac{5}{2}-1} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{5}{2}(a+2)^{\frac{5}{2}-1}$$

$$= \frac{5}{2}(a+2)^{\frac{3}{2}}$$

Limits Ex 29.5 Q2

$$\lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{x-a}$$

$$= \lim_{x \to a} \frac{(x+2)^{\frac{3}{2}} - (a+2)^{\frac{3}{2}}}{(x+2)-(a+2)}$$

Let $x+2=y$, $a+2=b$

$$\Rightarrow \lim_{(x+2) \to (a+2)} \frac{(y)^{\frac{3}{2}} - (b)^{\frac{3}{2}}}{(y)-(b)} \qquad \left[\text{Using formula } \lim_{x \to a} \frac{x^n - a^n}{x-a} = na^{n-1}\right]$$

$$= \frac{3}{2}(b)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(a+2)^{\frac{3}{2}-1}$$

$$= \frac{3}{2}(a+2)^{\frac{1}{2}}$$

Limits Ex 29.5 Q3

$$\lim_{x\to 0}\frac{(1+x)^6-1}{(1+x)^2-1}$$

$$=\lim_{x\to 0}\frac{\frac{(1+x)^6-1^6}{1+x-1}}{\frac{(1+x)^2-1^2}{1+x-1}}$$

$\Rightarrow$ Let $1+x=y$, as $x\to 0,\ y\to 1$

$$=\frac{\lim_{y\to 1}\frac{y^6-1^6}{y-1}}{\lim_{y\to 1}\frac{y^2-1}{y-1}}$$

$$=\frac{6(1)^{6-1}}{2(1)^{2-1}}\qquad\left[\text{Using formula }\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=\frac{6}{2}$$

$$=3$$

Limits Ex 29.5 Q4

$$\lim_{x\to a}\frac{x^{\frac{2}{7}}-a^{\frac{2}{7}}}{x-a}$$

Applying formula $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$, here, $n=\frac{2}{7}$

$$\Rightarrow \lim_{x\to a}\frac{x^{\frac{2}{7}}-a^{\frac{2}{7}}}{x-a}=\frac{2}{7}(a)^{\frac{2}{7}-1}$$

$$= \frac{2}{7} a^{\frac{-5}{7}}$$

Limits Ex 29.5 Q5

$$\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x^{\frac{2}{7}} - a^{\frac{2}{7}}}$$

$$= \lim_{x \to a} \frac{\frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$$ [Dividing numerator and denominator by $x - a$]

$$= \frac{\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = \frac{5}{7}$ is numerator and applying $\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator, where $m = \frac{2}{7}$

$$\Rightarrow \quad \frac{\lim_{x \to a} \frac{x^{\frac{5}{7}} - a^{\frac{5}{7}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{2}{7}} - a^{\frac{2}{7}}}{x - a}} = \frac{\frac{5}{7}(a)^{\frac{5}{7}-1}}{\frac{2}{7}(a)^{\frac{2}{7}-1}}$$

$$= \frac{\frac{5}{7} a^{\frac{-2}{7}}}{\frac{2}{7} a^{\frac{-5}{7}}}$$

$$= \frac{5}{2} a^{\frac{-2}{7}+\frac{5}{7}}$$

$$= \frac{5}{2} a^{\frac{3}{7}}$$

Limits Ex 29.5 Q6

$$\lim_{x \to -\frac{1}{2}} \frac{8x^3 + 1}{2x + 1}$$

$$= \frac{8}{2} \lim_{x \to -\frac{1}{2}} \frac{x^3 + \left(\frac{1}{2}\right)^3}{x + \frac{1}{2}}$$

$$= 4 \lim_{x \to -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)}$$

Applying formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

Here, $n = 3$, $a = \frac{-1}{2}$

$$\Rightarrow \quad 4 \lim_{x \to -\frac{1}{2}} \frac{x^3 - \left(-\frac{1}{2}\right)^3}{x - \left(-\frac{1}{2}\right)} = 4 \times 3 \left(-\frac{1}{2}\right)^{3-1}$$

$$= 4 \times 3 \times \frac{1}{4}$$

$$= 3$$

Limits Ex 29.5 Q7

$$\lim_{x\to 27}\frac{\left(x^{\frac{1}{3}}+3\right)\left(x^{\frac{1}{3}}-3\right)}{x-27}$$

$$=\lim_{x\to 27}\frac{\left(x^{\frac{2}{3}}-9\right)}{x-27}$$

$$=\lim_{x\to 27}\frac{x^{\frac{2}{3}}-27^{\frac{2}{3}}}{x-27}$$

Applying formula $\lim_{x\to 27}\frac{x^n-a^n}{x-a}=na^{n-1}$

$$=\frac{2}{3}(27)^{\frac{2}{3}-1}$$

$$=\frac{2}{3}(27)^{\frac{-1}{3}}$$

$$=\frac{2}{3}\times\frac{1}{(27)^{\frac{1}{3}}}$$

$$=\frac{2}{3}\times\frac{1}{3}$$

$$=\frac{2}{9}$$

Limits Ex 29.5 Q8

$$\lim_{x\to 4}\frac{x^3-64}{x^2-16}$$

$$=\lim_{x\to 4}\frac{x^3-4^3}{x^2-4^2}$$

$$=\lim_{x\to 4}\frac{\frac{x^3-4^3}{x-4}}{\frac{x^2-4^2}{x-4}}$$ [Dividing numerator and denominator by $x-4$]

$$=\frac{\lim_{x\to 4}\frac{x^3-4^3}{x-4}}{\lim_{x\to 4}\frac{x^2-4}{x-4}}$$

Applying $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ in numerator and $\lim_{x\to a}\frac{x^m-a^m}{x-a}=ma^{m-1}$ in denominator

$\Rightarrow \quad n=3,\ m=2$

$$\Rightarrow \quad \frac{\lim_{x\to 4}\frac{x^3-4^3}{x-4}}{\lim_{x\to 4}\frac{x^2-4}{x-4}}=\frac{3(4)^{3-1}}{2(4)^{2-1}}=\frac{3(4)^2}{2(4)}$$

$=6$

Limits Ex 29.5 Q9

$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$$

$$=\lim_{x\to 1}\frac{\frac{x^{15}-1^{15}}{x-1}}{\frac{x^{10}-1^{10}}{x-1}}$$ [Dividing numerator and denomintoar by $(x-1)$]

$$=\frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}}$$

Applying formula $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$ in numerator and $\lim_{x\to a}\frac{x^m-a^m}{x-a}=ma^{m-1}$ in denominator

Here, $n=15$, $m=10$

$$\Rightarrow \frac{\lim_{x\to 1}\frac{x^{15}-1^{15}}{x-1}}{\lim_{x\to 1}\frac{x^{10}-1^{10}}{x-1}}=\frac{15(1)^{15-1}}{10(1)^{10-1}}$$

$$=\frac{15}{10}$$

$$=\frac{3}{2}$$

Limits Ex 29.5 Q10

$$\lim_{x\to -1}\frac{x^3+1}{x+1}$$

$$=\lim_{x\to -1}\frac{x^3-(-1)^3}{x-(-1)}$$

Applying formula $\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$

Here, $n=3$, $a=-1$

$$\Rightarrow \lim_{x\to -1}\frac{x^3-(-1)^3}{x-(-1)}=na^{n-1}$$

$$=3(-1)^{3-1}$$

$$=3(-1)^2$$

$$=3$$

Limits Ex 29.5 Q11

$$\lim_{x\to a}\frac{x^{\frac{2}{3}}-a^{\frac{2}{3}}}{x^{\frac{3}{4}}-a^{\frac{3}{4}}}$$

$$=\lim_{x\to a}\frac{\frac{x^{\frac{2}{3}}-a^{\frac{2}{3}}}{x-a}}{\frac{x^{\frac{3}{4}}-a^{\frac{3}{4}}}{x-a}}$$ [Dividing numerator and denominator by $x-a$]

$$=\frac{\lim_{x\to a}\frac{x^{\frac{2}{3}}-a^{\frac{2}{3}}}{x-a}}{\lim_{x\to a}\frac{x^{\frac{3}{4}}-a^{\frac{3}{4}}}{x-a}}$$

Applying the formula $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$ in numerator and $\lim_{x \to a} \frac{x^m - a^m}{x - a} = ma^{m-1}$ in denominator respectively

Here, $n = \frac{2}{3}$, $m = \frac{3}{4}$

$$\Rightarrow \quad \frac{\lim_{x \to a} \frac{x^{\frac{2}{3}} - a^{\frac{2}{3}}}{x - a}}{\lim_{x \to a} \frac{x^{\frac{3}{4}} - a^{\frac{3}{4}}}{x - a}} = \frac{\frac{2}{3}(a)^{\frac{2}{3}-1}}{\frac{3}{4}(a)^{\frac{3}{4}-1}}$$

$$= \frac{8}{9} a^{\frac{-1}{3}+\frac{1}{4}}$$

$$= \frac{8}{9} a^{\frac{-1}{12}}$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.6

Limits Ex 29.6 Q1

$$\lim_{x\to\infty} \frac{(3x-1)(4x-2)}{(x+8)(x-1)} \qquad \left[\text{Expression is } \frac{\infty}{\infty}\right]$$

$$= \lim_{x\to\infty} \frac{(12x^2 - 10x + 2)}{(x^2 + 9x - 8)}$$

$$= \lim_{x\to\infty} \left(\frac{12 - \frac{10}{x} + \frac{2}{x^2}}{1 + \frac{9}{x} - \frac{8}{x^2}}\right)$$

$$= \frac{12 - 0 + 0}{1 + 0 - 1}$$

$$= 12$$

Limits Ex 29.6 Q2

$$\lim_{x\to\infty} \frac{3x^3 - 4x^2 + 6x - 1}{2x^3 + x^2 - 5x + 7}$$

$$= \lim_{x\to\infty} \frac{3 - \frac{4}{x} + \frac{6}{x^2} - \frac{1}{x^3}}{2 + \frac{1}{x} - \frac{5}{x^2} + \frac{7}{x^3}}$$

$$= \frac{3 - 0 + 0 - 0}{2 + 0 - 0 + 0}$$

$$= \frac{3}{2}$$

Limits Ex 29.6 Q3

$$\lim_{x\to\infty} \frac{5x^3 - 6}{\sqrt{9 + 4x^6}}$$

$$= \lim_{x\to\infty} \frac{5 - \frac{6}{x^3}}{\sqrt{\frac{9}{x^6} + \frac{4x^6}{x^6}}}$$

$$= \lim_{x\to\infty} \frac{\left(5 - \frac{6}{x^3}\right)}{\sqrt{\frac{9}{x^6} + 4}}$$

$$= \frac{5}{\sqrt{4}} = \frac{5}{2}$$

Limits Ex 29.6 Q4

$$\lim_{x\to\infty} \sqrt{x^2 + cx} - x$$

$$= \lim_{x\to\infty} \left(\left(\sqrt{x^2 + cx} - x\right)\frac{\left(\sqrt{x^2 + cx} + x\right)}{\sqrt{x^2 + cx} + x}\right)$$

$$= \lim_{x\to\infty} \frac{\left(x^2 + cx - x^2\right)}{\sqrt{x^2 + cx} + x}$$

$$= \lim_{x\to\infty} \frac{cx}{\sqrt{x^2 + cx} + x} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{x\to\infty} \frac{c}{\sqrt{1 + \frac{c}{x}} + 1}$$

$$= \frac{c}{1 + 1} = \frac{c}{2}$$

Limits Ex 29.6 Q5

$$\lim_{x\to\infty} \sqrt{x + 1} - \sqrt{x}$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x + 1} - \sqrt{x}\right)\left(\sqrt{x + 1} + \sqrt{x}\right)}{\left(\sqrt{x + 1} + \sqrt{x}\right)}$$

$$= \lim_{x\to\infty} \frac{(x + 1 - x)}{\sqrt{x + 1} + \sqrt{x}}$$

$$= \lim_{x\to\infty} \left(\frac{1}{\sqrt{x + 1} + \sqrt{x}}\right)$$

$$= \frac{1}{\infty}$$
$$= 0$$

Limits Ex 29.6 Q6

$$\lim_{x \to \infty} \sqrt{x^2 + 7x} - x$$

$$= \lim_{x \to \infty} \left(\frac{\left(\sqrt{x^2 + 7x} - x\right)\left(\sqrt{x^2 + 7x} + x\right)}{\sqrt{x^2 + 7x} + x} \right)$$

$$= \lim_{x \to \infty} \left(\frac{\left(x^2 + 7x\right) - x^2}{\sqrt{x^2 + 7x} + x} \right)$$

$$= \lim_{x \to \infty} \frac{7x}{\sqrt{x^2 + 7x} + x}$$

$$= \lim_{x \to \infty} \frac{7}{\sqrt{\frac{x^2}{x^2} + \frac{7x}{x^2}} + 1}$$

$$= \lim_{x \to \infty} \frac{7}{\sqrt{1 + \frac{7}{x}} + 1}$$

$$= \frac{7}{2}$$

Limits Ex 29.6 Q7

$$\lim_{x \to \infty} \frac{x}{\sqrt{4x^2 + 1} - 1}$$

$$= \lim_{x \to \infty} \frac{1}{\sqrt{4 + \frac{1}{x^2}} - \frac{1}{x}}$$

$$= \frac{1}{\sqrt{4} - 0}$$

$$= \frac{1}{2}$$

Limits Ex 29.6 Q8

$$\lim_{n\to\infty} \frac{n^2}{1+2+3+\ldots+n}$$

$$= \lim_{n\to\infty} \frac{n^2}{\frac{1}{2}n(n+1)} \qquad \left[\because 1+2+3+\ldots+n = \frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \frac{2n^2}{n^2+n}$$

$$= 2\lim_{n\to\infty} \frac{n^2}{n^2+n}$$

$$= 2\lim_{n\to\infty} \frac{n^2}{n^2\left(1+\frac{1}{n}\right)}$$

$$= 2\times\frac{1}{1+0}$$

$$= 2$$

Limits Ex 29.6 Q9

$$\lim_{n\to\infty} \frac{\frac{3}{x}+\frac{4}{x^2}}{\frac{5}{x}+\frac{6}{x^2}} \qquad \left[\frac{0}{0} \text{ form}\right]$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}\left(3+\frac{4}{x}\right)}{\frac{1}{x}\left(5+\frac{6}{x}\right)}$$

$$= \lim_{x\to\infty} \frac{(3+0)}{(5+0)} = \frac{3}{5}$$

Limits Ex 29.6 Q10

$$\lim_{x\to\infty}\frac{\sqrt{x^2+a^2}-\sqrt{x^2+b^2}}{\sqrt{x^2+c^2}-\sqrt{x^2+d^2}} \qquad \left[\frac{\infty}{\infty}\text{ form}\right]$$

$$\lim_{x\to\infty}\frac{\left(\sqrt{x^2+a^2}-\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)}\times\frac{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}{\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(\left(x^2+a^2\right)-\left(x^2-b^2\right)\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(\sqrt{x^2+c^2}-\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(x^2+c^2-x^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{x^2+c^2}+\sqrt{x^2+d^2}\right)}{\left(c^2-d^2\right)\left(\sqrt{x^2+a^2}+\sqrt{x^2+b^2}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(x\sqrt{1+\frac{c^2}{x^2}}+x\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(x\sqrt{1+\frac{a^2}{x^2}}+x\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$=\lim_{x\to\infty}\frac{\left(a^2-b^2\right)\left(\sqrt{1+\frac{c^2}{x^2}}+\sqrt{1+\frac{d^2}{x^2}}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+\frac{a^2}{x^2}}+\sqrt{1+\frac{b^2}{x^2}}\right)}$$

$$=\frac{\left(a^2-b^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}{\left(c^2-d^2\right)\left(\sqrt{1+0}+\sqrt{1+0}\right)}$$

$$=\frac{\left(a^2-b^2\right)(1+1)}{\left(c^2-d^2\right)(1+1)}$$

$$=\frac{\left(a^2-b^2\right)(2)}{\left(c^2-d^2\right)(2)}=\frac{a^2-b^2}{c^2-d^2}$$

Limits Ex 29.6 Q11

$$\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$$

We know that $(n+2) = (n+2)(n+1)!$

$$\Rightarrow \quad \lim_{n\to\infty} \frac{(n+2)(n+1)!+(n+1)!}{(n+2)(n+1)!-(n+1)!}$$

$$= \lim_{n\to\infty} \frac{(n+1)![(n+2)+1]}{(n+1)[(n+2)-1]}$$

$$= \lim_{n\to\infty} \frac{n+3}{n+1} \qquad \left[\frac{\infty}{\infty} \text{ form}\right]$$

$$= \lim_{n\to\infty} \frac{1+\frac{3}{n}}{1+\frac{1}{n}}$$

$$= \frac{1+0}{1+0} = 1$$

$$= 1$$

Limits Ex 29.6 Q12

$$\lim_{x\to\infty} x\left[\sqrt{x^2+1} - \sqrt{x^2-1}\right]$$

$$= \lim_{x\to\infty} x\left[\sqrt{x^2+1} - \sqrt{x^2-1}\right] \times \frac{\left(\sqrt{x^2+1} + \sqrt{x^2-1}\right)}{\left(\sqrt{x^2+1} + \sqrt{x^2+1}\right)}$$

$$= \lim_{x\to\infty} x \frac{x\left(x^2+1-x^2+1\right)}{\sqrt{x^2+1} + \sqrt{x^2-1}}$$

$$= \lim_{x\to\infty} \frac{x(2)}{x\left(\sqrt{1+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x^2}}\right)}$$

$$= \lim_{x\to\infty} \frac{2}{\sqrt{1+\frac{1}{x^2}} + \sqrt{1-\frac{1}{x^2}}}$$

$$= \frac{2}{2} = 1$$

Limits Ex 29.6 Q13

$$\lim_{x\to\infty}\left[\sqrt{x+1} - \sqrt{x}\right]\sqrt{x+2}$$

$$= \lim_{x\to\infty}\left[\sqrt{x+1} - \sqrt{x}\right]\frac{\left[\sqrt{x+1} + \sqrt{x}\right]}{\left[\sqrt{x+1} + \sqrt{x}\right]} \times \frac{\sqrt{x+2} \times \sqrt{x+2}}{\sqrt{x+2}}$$

$$= \lim_{x\to\infty}\frac{(x+1-x)}{\sqrt{x+1}+\sqrt{x}} \times \frac{(x+2)}{\sqrt{x+2}}$$

$$= \lim_{x\to\infty}\frac{1(x+2)}{\left(\sqrt{x+1}+\sqrt{x}\right)\left(\sqrt{x+2}\right)}$$

$$= \lim_{x\to\infty}\frac{x\left(1+\frac{2}{x}\right)}{\sqrt{x}\left(\sqrt{1+\frac{1}{x}}+\sqrt{1}\right)\left(\sqrt{1+\frac{2}{x}}\right)\sqrt{x}}$$

$$= \lim_{x\to\infty}\frac{\left(1+\frac{2}{x}\right)}{\left(\sqrt{1+\frac{1}{x}}+\sqrt{1}\right)\left(\sqrt{1+\frac{2}{x}}\right)}$$

$$= \lim_{x\to\infty}\frac{(1+0)}{(1+1)\times 1} = \frac{1}{2}$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.7

Limits Ex 29.7 Q1

$$\lim_{x\to 0}\frac{\sin 3x}{5x}$$

$$=\frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{x}$$

$$=\frac{1}{5}\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$

$$=\frac{3}{5}\lim_{3x\to 0}\frac{\sin 3x}{3x}$$

$$=\frac{3}{5}\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{3}{5}$$

Limits Ex 29.7 Q2

$$\lim_{x\to 0}\frac{\sin x^\circ}{x}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x\times\pi}{180}}{x} \qquad \left[\because 1^\circ=\frac{\pi}{180}\text{radians}\right]$$

$$=\lim_{x\to 0}\frac{\sin\frac{\pi x}{180}}{x\times\frac{\pi}{180}}\times\frac{\pi}{180}$$

$$=\frac{\pi}{180}\lim_{x\to 0}\frac{\frac{\sin\pi x}{180}}{\frac{\pi x}{180}}$$

$$=\frac{\pi}{180}\lim_{x\to 0}\frac{\frac{\sin\pi x}{180}}{\frac{\pi x}{180}}$$

$$=\frac{\pi}{180}\times 1=\frac{\pi}{180} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{\pi}{180}$$

Limits Ex 29.7 Q3

$$\lim_{x\to 0} \frac{x^2}{\sin x^2}$$

$$= \lim_{x\to 0} \frac{1}{\frac{\sin x^2}{x^2}}$$

$$= \frac{1}{\lim_{x\to 0} \frac{\sin x^2}{x^2}}$$

$$= \frac{1}{1} \qquad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1\right]$$

$$= 1$$

Limits Ex 29.7 Q4

$$\lim_{x\to 0} \frac{\sin x \cos x}{3x}$$

$$= \frac{1}{3} \lim_{x\to 0} \frac{\sin x \cos x}{x}$$

$$= \frac{1}{3} \lim_{x\to 0} \frac{\sin x}{x} \times \lim_{x\to 0} \cos x$$

$$= \frac{1}{3} \times 1 \times 1 \qquad \left[\because \lim_{x\to 0} \frac{\sin x}{x} = 1, \ \lim_{x\to 0} \cos x = \cos 0° = 1\right]$$

$$= \frac{1}{3}$$

Limits Ex 29.7 Q5

$$\lim_{x\to 0}\frac{3\sin x-4\sin^3 x}{x}$$

$$=\lim_{x\to 0}\frac{\sin 3x}{x} \qquad \left[\because \sin 3x = 3\sin x - 4\sin^3 x\right]$$

$$=\lim_{x\to 0}\frac{\sin 3x}{3x}\times 3$$

$$=3\times\lim_{x\to 0}\frac{\sin 3x}{3x}$$

$$=3\times\lim_{3x\to 0}\frac{\sin 3x}{3x} \qquad \left[\because x\to 0,\ 3x\to 0\right]$$

$$=3\times 1 \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=3$$

Limits Ex 29.7 Q6

$$\lim_{x\to 0}\frac{\tan 8x}{\sin 2x}$$

$$=\frac{\lim_{x\to 0}\tan 8x}{\lim_{x\to 0}\sin 2x}$$

$$=\frac{\lim_{x\to 0}\frac{\tan 8x}{8x}\times 8x}{\lim_{x\to 0}\frac{\sin 2x}{2x}\times 2x}$$

$$=\frac{\lim_{8x\to 0}\frac{\tan 8x}{8x}\times\frac{8x}{2x}}{\lim_{2x\to 0}\frac{\sin 2x}{2x}} \qquad \begin{bmatrix}\because x\to 0\\ 8x\to 0\\ 2x\to 0\end{bmatrix}$$

$$= \frac{1 \times \frac{8}{2}}{1}$$ $\left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1 \text{ and } \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$

$$= \frac{8}{2}$$

$$= 4$$

Limits Ex 29.7 Q7

$$\lim_{x \to 0} \frac{\tan mx}{\tan nx}$$

$$= \frac{\lim_{x \to 0} \tan mx}{\lim_{x \to 0} \tan nx}$$

$$= \frac{\lim_{mx \to 0} \frac{\tan mx}{mx} \times mx}{\lim_{nx \to 0} \frac{\tan nx}{nx} \times nx}$$ $\left[\because \text{If } x \to 0 \text{ then } mx \to 0 \text{ also } nx \to 0\right]$

$$= \frac{1 \times m}{1 \times n}$$ $\left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$

$$= \frac{m}{n}$$

Limits Ex 29.7 Q8

$$\lim_{x\to 0}\frac{\sin 5x}{\tan 3x}$$

$$=\frac{\lim\limits_{x\to 0}\sin 5x}{\lim\limits_{3x\to 0}\tan 3x}$$

$$=\frac{\lim\limits_{5x\to 0}\frac{\sin 5x}{5x}\times 5}{\lim\limits_{3x\to 0}\frac{\tan 3x}{3x}\times 3}\qquad\left[\because \text{If } x\to 0 \text{ then } 3x\to 0, 5x\to 0\right]$$

$$=\frac{5}{3}\times 1\qquad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1 \text{ also } \lim_{x\to 0}\frac{\tan x}{x}=1\right]$$

$$=\frac{5}{3}$$

Limits Ex 29.7 Q9

$$\lim_{x\to 0}\frac{\sin x^\circ}{x^\circ}$$

$$=\lim_{x\to 0}\frac{\sin\frac{x\times\pi}{180}}{x\times\frac{\pi}{180}}\qquad\left[\because 1^\circ=\frac{\pi}{180}\text{ radians}\right]$$

$$=\lim_{\frac{\pi x}{180}\to 0}\frac{\sin\frac{\pi x}{180}}{\frac{\pi x}{180}}\qquad\left[\because \text{If } x\to 0 \text{ then } \frac{\pi x}{180}\to 0\right]$$

$$=1\qquad\left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

Limits Ex 29.7 Q10

$$\lim_{x\to 0} \frac{7x\cos x - 3\sin x}{4x + \tan x}$$

$$= \lim_{x\to 0} \frac{7\cos x - \frac{3\sin x}{x}}{4 + \frac{\tan x}{x}}$$

$$= \frac{\lim_{x\to 0} 7\cos x - \lim_{x\to 0} \frac{3\sin x}{x}}{\lim_{x\to 0} 4 + \lim_{x\to 0} \frac{\tan x}{x}}$$

$$= \frac{7 \times \lim_{x\to 0} \cos x - 3\lim_{x\to 0} \frac{\sin x}{x}}{4 + \lim_{x\to 0} \frac{\tan x}{x}}$$

$$= \frac{7\times 1 - 3\times 1}{4+1} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x} = 1 \quad \lim_{x\to 0}\frac{\tan x}{x} = 1\right]$$

$$= \frac{4}{5}$$

Limits Ex 29.7 Q11

$$\lim_{x\to 0} \frac{\cos ax - \cos bx}{\cos cx - \cos dx} = \lim_{x\to 0} \frac{-2\sin\left(\frac{a+b}{2}\right)x \sin\left(\frac{a-b}{2}\right)x}{-2\sin\left(\frac{c+d}{2}\right)x \sin\left(\frac{c-d}{2}\right)x}$$

$$= \frac{\lim_{x\to 0} \sin\left(\frac{a+b}{2}\right)x \lim_{x\to 0} \sin\left(\frac{a+b}{2}\right)x}{\lim_{x\to 0} \sin\left(\frac{c+d}{2}\right)x \lim_{x\to 0} \sin\left(\frac{c-d}{2}\right)x}$$

$$= \frac{\left(\lim_{x\to 0} \frac{\sin\left(\frac{a+b}{2}\right)x}{\left(\frac{a+b}{2}\right)x} \times \left(\frac{a+b}{2}\right)x\right)\left(\lim_{x\to 0} \frac{\sin\left(\frac{a-b}{2}\right)x}{\left(\frac{a-b}{2}\right)x} \times \left(\frac{a-b}{2}\right)x\right)}{\left(\lim_{x\to 0} \frac{\sin\left(\frac{c+d}{2}\right)x}{\left(\frac{c+d}{2}\right)x} \times \left(\frac{c+d}{2}\right)x\right)\left(\lim_{x\to 0} \frac{\sin\left(\frac{c-d}{2}\right)x}{\left(\frac{c-d}{2}\right)x} \times \left(\frac{c-d}{2}\right)x\right)}$$

$$= \frac{(a+b)(a-b)}{(c+d)(c-d)} \qquad \left[\because \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1\right]$$

$$= \frac{a^2 - b^2}{c^2 - d^2}.$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.8

Limits Ex 29.8 Q1

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$Let\ y = \frac{\pi}{2} - x$$

$$as\ x \to \pi/2, \quad y \to 0$$

$$\lim_{x \to \pi/2}\left(\frac{\pi}{2} - x\right)\tan x$$

$$= \lim_{y \to 0} y \tan\left(\frac{\pi}{2} - y\right)$$

$$= \lim_{y \to 0} y \frac{\sin\left(\frac{\pi}{2} - y\right)}{\cos\left(\frac{\pi}{2} - y\right)}$$

$$= \lim_{y \to 0} y \frac{\cos y}{\sin y}$$

$$= \lim_{y \to 0} \cos y = \lim_{y \to 0} \frac{y}{\sin y}$$

$$= 1$$

Limits Ex 29.8 Q2

$$\lim_{x \to \frac{\pi}{2}} \frac{\sin 2x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 \sin x \cos x}{\cos x}$$

$$= 2 \lim_{x \to \frac{\pi}{2}} \sin x$$

$$= 2 \times \sin \frac{\pi}{2}$$

$$= 2 \times 1$$

$$= 2$$

Limits Ex 29.8 Q3

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin^2 x}{1 - \sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(1 - \sin x)(1 + \sin x)}{(1 - \sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} (1 + \sin x)$$

$$= 1 + \sin \frac{\pi}{2}$$

$$= 1 + 1$$

$$= 2$$

Limits Ex 29.8 Q4

$$\lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{1 - \sin^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{1 + \sin x}$$

$$= \frac{1}{1 + \sin \frac{\pi}{2}}$$

$$= \frac{1}{1 + 1}$$

$$= \frac{1}{2}$$

Limits Ex 29.8 Q5

$$\lim_{x\to a}\frac{\cos x-\cos a}{x-a}$$

$$=\lim_{x\to a}\frac{\left(-2\sin\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)\right)}{x-a}$$

$$=-2\lim_{x\to a}\sin\left(\frac{x+a}{2}\right)\lim_{x\to a}\frac{\sin\left(\frac{x-a}{2}\right)}{x-a}$$

$$=-2\times\sin\left(\frac{a+a}{2}\right)\times\left(\lim_{x-a\to 0}\frac{\sin\left(\frac{x-a}{2}\right)}{\frac{x-a}{2}}\right)\times\frac{1}{2}$$

$$=-2\sin a\times 1\times\frac{1}{2}\qquad\left[\because\lim_{x\to a}\frac{\sin x}{x}=1\right]$$

$$=-\sin a$$

Limits Ex 29.8 Q6

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\tan x}{x-\frac{\pi}{4}}$$

If $x\to\frac{\pi}{4}$, then $x-\frac{\pi}{4}\to 0$

Let $x-\frac{\pi}{4}=y\Rightarrow y\to 0$

$$= \lim_{y\to 0} \frac{1-\tan\left(y+\frac{\pi}{4}\right)}{y}$$

$$= \lim_{y\to 0} \frac{1-\left(\frac{\tan y+\tan\frac{\pi}{4}}{1-\tan y\tan\frac{\pi}{4}}\right)}{y}$$

$$= \lim_{y\to 0} \frac{(1-\tan y-\tan y-1)}{y(1-\tan y)} \qquad \left[\because \tan\frac{\pi}{4}=1\right]$$

$$= \lim_{y\to 0} \frac{(-2\tan y)}{y(1-\tan y)}$$

$$= -2\lim_{y\to 0}\frac{\tan y}{y}\times\frac{1}{\lim_{y\to 0}(1-\tan y)}$$

$$= -2\times 1\frac{1}{(1-0)} \qquad \left[\because \lim_{\theta\to 0}=\frac{\tan\theta}{\theta}=1\right]$$

$$= -2$$

Limits Ex 29.8 Q7

$$\lim_{x\to\frac{\pi}{2}} \frac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2}$$

If $x\to\frac{\pi}{2}, \frac{\pi}{2}-x\to 0$

Let $\frac{\pi}{2}-x=y$ they $y\to 0$

$$= \lim_{y\to 0} \frac{1-\sin\left(\frac{\pi}{2}-y\right)}{y^2}$$

$$= \lim_{y\to 0} \frac{1-\cos y}{y^2}$$

$$= \lim_{y\to 0} \frac{2\sin^2\frac{y}{2}}{y^2}$$

$$= 2\left(\lim_{y\to 0}\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2\times\frac{1}{4} \qquad \left[\because \lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Limits Ex 29.8 Q8

$$\lim_{x \to \frac{\pi}{3}} \frac{\sqrt{3} - \tan x}{\pi - 3x}$$

If $x \to \frac{\pi}{3}, \frac{\pi}{3} - x \to 0, \pi - 3x \to 0$

Let $\frac{\pi}{3} - x = y$ then $y \to 0$

$$= \lim_{y \to 0} \frac{\sqrt{3} - \tan\left(\frac{\pi}{3} - y\right)}{3\left(\frac{\pi}{3} - x\right)}$$

$$= \lim_{y \to 0} \left(\frac{\left(\sqrt{3} - \frac{\tan\frac{\pi}{3} - \tan y}{1 + \tan\frac{\pi}{3} . \tan y} \right)}{3y} \right)$$

$$= \lim_{y \to 0} \left(\frac{\left(\sqrt{3} - \frac{\sqrt{3} - \tan y}{1 + \sqrt{3} \tan y} \right)}{3y} \right)$$

$$= \lim_{y \to 0} \frac{\left(\sqrt{3} + 3\tan y - \sqrt{3} + \tan y\right)}{3\left(1 + \sqrt{3} \tan y\right) y}$$

$$= \lim_{y \to 0} \frac{4 \tan y}{3\left(1 + \sqrt{3} \tan y\right) y}$$

$$= \frac{4}{3} \times \lim_{y \to 0} \frac{\tan y}{y} \times \frac{1}{\lim_{y \to 0}\left(1 + \sqrt{3} \frac{\tan y}{y} \times y\right)}$$

$$= \frac{4 \times 1}{3} \times \frac{1}{1 + 0}$$

$$= \frac{4}{3}$$

Limits Ex 29.8 Q9

$$\lim_{x \to a} \frac{a \sin x - x \sin a}{ax^2 - xa^2} = \lim_{x \to a} \frac{(a \sin x - x \sin a)}{ax(x-a)}$$

Let $t = x - a$

Then, as $x \to a$, $t \to 0$

$$\therefore \lim_{x \to a} \frac{(a \sin x - x \sin a)}{ax(x-a)} = \lim_{t \to 0} \frac{(a \sin(t+a) - (t+a)\sin a)}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a + a \sin a \cos t - t \sin a - a \sin a}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a + a \sin a(\cos t - 1) - t \sin a}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a + a \sin a(2\sin^2(t/2)) - t \sin a}{a(t+a)t}$$

$$= \lim_{t \to 0} \frac{a \sin t \cos a}{a(t+a)t} + \lim_{t \to 0} \frac{a \sin a(2\sin^2(t/2))}{a(t+a)t} - \lim_{t \to 0} \frac{t \sin a}{a(t+a)t}$$

$$= \frac{a \cos a}{a^2} + 0 - \frac{\sin a}{a^2}$$

$$= \frac{a \cos a - \sin a}{a^2}$$

Limits Ex 29.8 Q10

$$\lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \cdot \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2 - 1 - \sin x}{\cos^2 x\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1 - \sin x}{(1 - \sin^2 x)\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1}{(1 + \sin x)\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

$$= \frac{1}{(1+1)\left(\sqrt{2} + \sqrt{2}\right)}$$

$$= \frac{1}{(4\sqrt{2})}$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.9

Limits Ex 29.9 Q1

$$= \lim_{x \to \pi} \frac{1+\cos x}{\tan^2 x}$$

As $x \to \pi, x - \pi \to 0$, let $x - \pi = y$

$$= \lim_{y \to 0} \frac{1+\cos(\pi + y)}{\tan^2(\pi + y)}$$

$$= \lim_{y \to 0} \frac{1-\cos y}{\tan^2 y}$$

$$= \lim_{y \to 0} \frac{2\sin^2 \frac{y}{2}}{\tan^2 y}$$

$$= \frac{\lim_{y \to 0} 2\sin^2 \frac{y}{2}}{\lim_{y \to 0} \tan^2 y}$$

$$= \frac{2\left(\lim_{y \to 0} \frac{\sin \frac{y}{2}}{\frac{y}{2}}\right)^2 \times \frac{y^2}{4}}{\left(\lim_{y \to 0} \frac{\tan y}{y}\right) \times y^2}$$

$$= \frac{2 \times 1 \times \frac{y^2}{4}}{1 \times y^2} \qquad \left[\because \lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1,\ \lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1\right]$$

$$= 2 \times 1 \times \frac{1}{4}$$

$$= \frac{1}{2}$$

Limits Ex 29.9 Q2

$$\lim_{x\to\frac{\pi}{4}} \frac{\operatorname{cosec}^2 x - 2}{\cot x - 1}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cot^2 x + 1 - 2}{\cot x - 1}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{\cot^2 x - 1}{\cot x - 1}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{(\cot x - 1)(\cot x + 1)}{(\cot x - 1)}$$

$$= \lim_{x\to\frac{\pi}{4}} (\cot x + 1)$$

$$= \cot\frac{\pi}{4} + 1$$

$$= 1 + 1$$

$$= 2$$

Limits Ex 29.9 Q3

$$\lim_{x\to\frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

$$= \lim_{x\to\frac{\pi}{6}} \frac{(\operatorname{cosec} x - 2)(\operatorname{cosec} x + 2)}{(\operatorname{cosec} x - 2)}$$

$$= \lim_{x\to\frac{\pi}{6}} (\operatorname{cosec} x + 2)$$

$$= \operatorname{cosec}\frac{\pi}{6} + 2$$

$$= 2 + 2$$

$$= 4$$

Limits Ex 29.9 Q4

$$\lim_{x\to\frac{\pi}{4}} \frac{2-\operatorname{cosec}^2 x}{1-\cot x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{2-\left(1+\cot^2 x\right)}{1-\cot x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{2-1-\cot^2 x}{1-\cot x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{1-\cot^2 x}{1-\cot x}$$

$$= \lim_{x\to\frac{\pi}{4}} \frac{(1-\cot x)(1+\cot x)}{(1-\cot x)}$$

$$= \lim_{x\to\frac{\pi}{4}} (1+\cot x)$$

$$= 1+\cot\frac{\pi}{4}$$

$$= 1+1$$

$$= 2$$

Limits Ex 29.9 Q5

$$\lim_{x\to\pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2}$$

$$= \lim_{x\to\pi} \frac{\sqrt{2+\cos x}-1}{(\pi-x)^2} \times \frac{\sqrt{2+\cos x}+1}{\sqrt{2+\cos x}+1}$$

$$= \lim_{x\to\pi} \frac{(2+\cos x)-1}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

$$= \lim_{x\to\pi} \frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)}$$

Let $\pi - x = y, x \to \pi, y \to 0$

$$\Rightarrow \lim_{x\to\pi} \frac{1+\cos x}{(\pi-x)^2\left(\sqrt{2+\cos x}+1\right)} = \lim_{y\to 0} \frac{1+\cos(\pi-y)}{y^2\left(\sqrt{2+\cos(\pi-y)}+1\right)}$$

$$= \lim_{y\to 0}\frac{1-\cos y}{y^2\sqrt{2-\cos y}+1}$$

$$= \lim_{y\to 0}\frac{2\sin^2\frac{y}{2}}{y^2\sqrt{2-\cos y}+1}$$

$$= 2\lim_{y\to 0}\left(\frac{\sin\frac{y}{2}}{\frac{y}{2}}\right)^2\times\frac{1}{4}\frac{1}{\sqrt{2-\cos y}+1}$$

$$= 2\times\left(\lim_{y\to 0}\frac{\sin y}{2}\right)^2\times\frac{1}{4}\frac{1}{\lim_{y\to 0}\sqrt{2-\cos y}+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{2-\cos 0}+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{\sqrt{2-1}+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{1+1}$$

$$= 2\times 1\times\frac{1}{4}\times\frac{1}{2}$$

$$= \frac{1}{4}$$

Limits Ex 29.9 Q6

$$\lim_{x\to\frac{3\pi}{2}}\frac{1+\operatorname{cosec}^3 x}{\cot^2 x}$$

$$= \lim_{x\to\frac{3\pi}{2}}\frac{(1+\operatorname{cosec}x)\left(1+\operatorname{cosec}^2x-\operatorname{cosec}x\right)}{\left(\operatorname{cosec}^2x-1\right)}$$

$$= \lim_{x\to\frac{3\pi}{2}}\frac{(\operatorname{cosec}x+1)\left(1+\operatorname{cosec}^2x-\operatorname{cosec}x\right)}{(\operatorname{cosec}x-1)(\operatorname{cosec}x+1)}$$

$$= \lim_{x\to\frac{3\pi}{2}}\frac{\left(1+\operatorname{cosec}^2x-\operatorname{cosec}x\right)}{\operatorname{cosec}x-1}$$

$$= \frac{1+\operatorname{cosec}^2\frac{3\pi}{2}-\operatorname{cosec}\frac{3\pi}{2}}{\operatorname{cosec}\frac{3\pi}{2}-1}$$

$$= \frac{1+(-1)^2-(-1)}{(-1)-1} \qquad \left[\because \operatorname{cosec}\frac{3\pi}{2}=-1\right]$$

$$= \frac{1+1+1}{-2}$$

$$= \frac{-3}{2}$$

RD Sharma Solutions Class 11 Maths Chapter 29 Ex 29.11

Limits Ex 29.11 Q1

$$\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^{\lim_{n\to\infty}\left(\frac{x}{n}\right)n}$$

$$= e^x$$

Limits Ex 29.11 Q2

$$\lim_{x\to 0^+}\left\{1+\tan^2\sqrt{x}\right\}^{1/2x}$$

$$= e^{\lim_{x\to 0^+}\left\{\frac{\tan^2\sqrt{x}}{2x}\right\}}$$

$$= e^{\lim_{x\to 0^+}\left\{\frac{\tan^2\sqrt{x}}{2x}\right\}}$$

$$= e^{\lim_{x\to 0^+}\left\{\frac{\sin^2\sqrt{x}}{2x\cos^2\sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}\lim_{x\to 0^+}\left\{\left(\frac{\sin\sqrt{x}}{\sqrt{x}}\right)^2\right\}\lim_{x\to 0^+}\left\{\frac{1}{\cos^2\sqrt{x}}\right\}}$$

$$= e^{\frac{1}{2}}$$

$$= \sqrt{e}$$

Limits Ex 29.11 Q3

$$\lim_{x\to 0}(\cos x)^{1/\sin x} = \lim_{x\to 0}(1+\cos x-1)^{1/\sin x}$$

$$= \lim_{x\to 0}(1-(1-\cos x))^{1/\sin x}$$

$$= \lim_{x\to 0}\left(1-2\sin^2\left(\frac{x}{2}\right)\right)^{1/\sin x}$$

$$= e^{\lim_{x\to 0}\left(-2\sin^2\left(\frac{x}{2}\right)\right)\times(1/\sin x)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{\sin x}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{-2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}\right)}$$

$$= e^{\lim_{x\to 0} -\tan x}$$

$$= e^{0}$$

$$= 1$$

Limits Ex 29.11 Q4

$$\lim_{x\to 0}\left(\cos x+\sin x\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1+\left(\cos x+\sin x-1\right)\right)^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{(\cos x+\sin x-1)}{x}}$$

$$= e^{\lim_{x\to 0}\frac{(\sin x-(1-\cos x))}{x}}$$

$$= e^{\lim_{x\to 0}\frac{\left(\sin x-2\sin^2(x/2)\right)}{x}}$$

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}-\lim_{x\to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$= e^{\lim_{x\to 0}\frac{\sin x}{x}-\lim_{x\to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$= e^{1-0}$$

$$= e$$

Limits Ex 29.11 Q5

$$\lim_{x\to 0}\left(\cos x + a\sin bx\right)^{1/x}$$

$$= \lim_{x\to 0}\left(1 + (\cos x + a\sin bx - 1)\right)^{1/x}$$

$$= e^{\lim_{x\to 0}\frac{(\cos x + a\sin bx - 1)}{x}}$$

$$= e^{\lim_{x\to 0}\frac{(a\sin bx - (1-\cos x))}{x}}$$

$$= e^{\lim_{x\to 0}\frac{(a\sin bx - 2\sin^2(x/2))}{x}}$$

$$= e^{\lim_{x\to 0}\frac{ab\sin bx}{bx} - \lim_{x\to 0}\frac{2\sin(x/2)\sin(x/2)}{2\left(\frac{x}{2}\right)}}$$

$$= e^{\lim_{x\to 0}\frac{ab\sin bx}{bx} - \lim_{x\to 0}\frac{\sin(x/2)\sin(x/2)}{\left(\frac{x}{2}\right)}}$$

$$= e^{ab-0}$$

$$= e^{ab}$$

Limits Ex 29.11 Q6

$$\lim_{x\to\infty}\left\{\frac{x^2+2x+3}{2x^2+x+5}\right\}^{\frac{3x-2}{3x+2}}$$

$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3x-2}{3x+2}\right)\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right)\right\}}$$

$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}}\right)\left(\ln\left(\frac{x^2+2x+3}{2x^2+x+5}\right)\right)\right\}}$$

$$= e^{\lim_{x\to\infty}\left\{\left(\frac{3-\frac{2}{x}}{3+\frac{2}{x}}\right)\left(\ln\left(\frac{1+\frac{2}{x}+\frac{3}{x^2}}{2+\frac{1}{x}+\frac{5}{x^2}}\right)\right)\right\}}$$

$$= e^{1.\ln\left(\frac{1}{2}\right)}$$

$$= \frac{1}{2}$$

Limits Ex 29.11 Q7

$$\lim_{x\to 1}\left\{\frac{x^3+2x^2+x+1}{x^2+2x+3}\right\}^{\frac{1-\cos(x-1)}{(x-1)^2}}$$

$$= e^{\lim_{x\to 1}\left\{\frac{1-\cos(x-1)}{(x-1)^2}\ln\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)\right\}}$$

$$= e^{\lim_{x\to 1}\left\{\frac{2\sin^2(x-1)}{4\left(\frac{x-1}{2}\right)^2}\ln\left(\frac{x^3+2x^2+x+1}{x^2+2x+3}\right)\right\}}$$

$$= e^{\frac{2}{4}\ln\left(\frac{5}{6}\right)}$$

$$= e^{\ln\left(\frac{5}{6}\right)^{\frac{1}{2}}}$$

$$= \left(\frac{5}{6}\right)^{\frac{1}{2}} = \sqrt{\frac{5}{6}}$$

Limits Ex 29.11 Q8

$$\lim_{x\to 0}\left\{\frac{e^x+e^{-x}-2}{x^2}\right\}^{1/x^2}$$

$$= e^{\lim_{x\to 0}\frac{\left\{\frac{e^x+e^{-x}-2}{x^2}\right\}}{x^2}}$$

Applying L'Hospital's Rule

$$= e^{\lim_{x\to 0}\left\{\frac{2+e^x(-2+x)+x}{2(-1+e^x)x^2}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{1+e^x(-1+x)}{x(-2+e^x(2+x))}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{e^x x}{-2+e^x(2+4x+x^2))}\right\}}$$

Applying L'Hospital's Rule again

$$= e^{\frac{1}{2}\lim_{x\to 0}\left\{\frac{1+x}{6+6x+x^2}\right\}}$$

$$= e^{\frac{1}{2}\left\{\frac{\lim_{x\to 0}(1+x)}{\lim_{x\to 0}(6+6x+x^2)}\right\}}$$

$$= e^{\frac{1}{12}}$$

$$= \sqrt[12]{e}$$

Limits Ex 29.11 Q9

$$\lim_{x \to a}\left\{\frac{\sin x}{\sin a}\right\}^{\frac{1}{x-a}}$$

$$= \lim_{x \to a}\left\{1+\left(\frac{\sin x}{\sin a}-1\right)\right\}^{\frac{1}{x-a}}$$

$$= e^{\lim_{x \to a}\left\{\frac{\left(\frac{\sin x}{\sin a}-1\right)}{x-a}\right\}}$$

$$= e^{\lim_{x \to a}\left\{\frac{\left(\frac{\sin x-\sin a}{\sin a}\right)}{x-a}\right\}}$$

$$= e^{\lim_{x \to a}\left\{\left(\frac{\sin x-\sin a}{\sin a(x-a)}\right)\right\}}$$

$$= e^{\lim_{x \to a}\left\{\left(\frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{\sin a(x-a)}\right)\right\}}$$

$$= e^{\lim_{x \to a}\left\{\left(\frac{2\cos\left(\frac{x+a}{2}\right)}{\sin a}\right)\right\}\lim_{x \to a}\left\{\left(\frac{\sin\left(\frac{x-a}{2}\right)}{2\left(\frac{x-a}{2}\right)}\right)\right\}}$$

$$= e^{\frac{2\cos a}{2\sin a}}$$

$$= e^{\cot a}$$

Limits Ex 29.11 Q10

$$\lim_{x \to \infty}\left\{\frac{3x^2+1}{4x^2-1}\right\}^{\frac{x^3}{1+x}}$$

$$= \lim_{x \to \infty}\left\{1+\frac{-x^2+2}{4x^2-1}\right\}^{\frac{x^3}{1+x}}$$

$$= e^{\lim_{x \to \infty}\left\{\left(\frac{-x^2+2}{4x^2-1}\right)\left(\frac{x^3}{1+x}\right)\right\}}$$

$$= e^{\lim_{x \to \infty}\left\{\left(\frac{-x^5+2x^3}{4x^2-1+4x^3-x}\right)\right\}}$$

$$= e^{-\infty}$$

$$= 0$$