Linear Equations
Ex 8B

Definition of a Linear Equation

- A linear equation in one variable x is an equation that can be written in the form
- $ax + b = 0$
- where a and b are real numbers and $a \neq 0$.

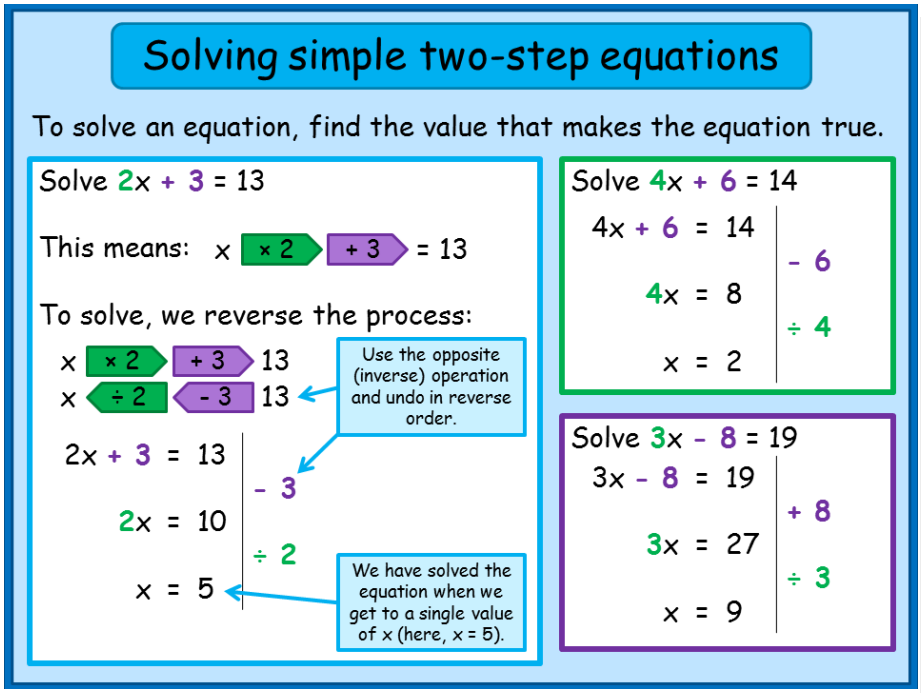
ONE STEP SUBTRACTION EXAMPLE

The Opposite of Subtraction is Addition

$$x - 120 = 80$$
$$+120 \quad +120$$
$$x = 200 \checkmark$$

The value which makes the equation true is 200.

x – 4 = 7	Original problem
x – 4 = 7	We want to remove the minus 4.
x – 4 +4 = 7 +4	The opposite of minus 4 is plus 4, so I added 4 to BOTH sides of the equation.
x = 11	-4+4 = 0, so x remains on the left and 7+4 = 11; therefore x = 11
Check: x – 4 = 7 11 – 4 = 7	 This is a correct statement, so my answer is x = 11 is correct!



Q1

Answer :

Let the numbers be 8x and 3x.

$$8x + 3x = 143$$
$$\Rightarrow 11x = 143$$
$$\Rightarrow x = \frac{143}{11}$$
$$\Rightarrow x = 13$$

$\therefore$ One number $= 8x = 8 \times 13 = 104$

Other number $= 3x = 3 \times 13 = 39$

Q2.

Answer :

Let the original number be x.

$\frac{2}{3}$ of the number is 20 less than the original number.

$$\therefore \frac{2}{3}x = x - 20$$
$$\Rightarrow \frac{2x}{3} = x - 20$$
$$\Rightarrow 2x = 3\left(x - 20\right) \quad \left(\text{by cross multiplication}\right)$$
$$\Rightarrow 2x = 3x - 60$$
$$\Rightarrow 2x - 3x = -60$$
$$\Rightarrow -x = -60$$
$$\Rightarrow x = 60$$

Therefore, the original number is 60.

Q3.

Answer :

Let the number be x.

Four fifths of the number is 10 more than two thirds of the number.

$$\therefore \frac{4}{5}x = 10 + \frac{2}{3}x$$
$$\Rightarrow \frac{4x}{5} = 10 + \frac{2x}{3}$$
$$\Rightarrow \frac{4x}{5} = \frac{30 + 2x}{3} \quad \left(L.C.M. \text{ of } 1 \text{ and } 3 \text{ is } 3\right)$$
$$\Rightarrow 3\left(4x\right) = 5\left(30 + 2x\right) \quad \left(\text{by cross multiplication}\right)$$
$$\Rightarrow 12x = 150 + 10x$$
$$\Rightarrow 12x - 10x = 150$$
$$\Rightarrow 2x = 150$$
$$\Rightarrow x = \frac{150}{2} = 75$$

Therefore, the number is 75.

Q4.

Answer :

Let one part be x.

7 times the first part $= 7x$

Let the other part be $\left(24-x\right)$.

5 times the second part $= 5\left(24-x\right)$

$\therefore 7x + 5\left(24 - x\right) = 146$

$\Rightarrow 7x + 120 - 5x = 146$

$\Rightarrow 7x - 5x = 146 - 120$

$\Rightarrow 2x = 26$

$\Rightarrow x = \frac{26}{2} = 13$

Therefore, one part is 13.

Other part $= \left(24 - x\right) = \left(24 - 13\right) = 11$

Q5.

Answer :

Let the number be x.

Fifth part increased by $5 = \frac{x}{5} + 5$

Fourt part diminished by $5 = \frac{x}{4} - 5$

$\therefore \frac{x}{5} + 5 = \frac{x}{4} - 5$

$\Rightarrow 5 + 5 = \frac{x}{4} - \frac{x}{5}$

$\Rightarrow 10 = \frac{5x - 4x}{20}$

$\Rightarrow 10 = \frac{x}{20}$

$\Rightarrow 200 = x$

$\Rightarrow x = 200$

Therefore, the number is 200.

Q6.

Answer :

Let the common multiple for the given three numbers be x.

Then, the three numbers would be 4x, 5x and 6x.

$\therefore 4x + 6x = 5x + 55$

$\Rightarrow 10x = 5x + 55$

$\Rightarrow 10x - 5x = 55$

$\Rightarrow 5x = 55$

$\Rightarrow x = \frac{55}{5} = 11$

$\therefore$ Smallest number $= 4x = 4\left(11\right) = 44$

Largest number is $= 6x = 6\left(11\right) = 66$

Third number $= 5x = 5\left(11\right) = 55$

Therefore, *the* three numbers are 44, 55 and 66.

Q7.

Answer :

Let the number be x.

$\therefore 10 + 4x = 5x - 5$

$\Rightarrow 10 + 5 = 5x - 4x$

$\Rightarrow 15 = x$

$\Rightarrow x = 15$ (by transposition)

Therefore, the number is 15.

Q8.

Answer :

Let us consider *x* as the common multiple of both the number.

Then, first number = 3*x*

Second number = 5*x*

$\therefore \frac{3x + 10}{5x + 10} = \frac{5}{7}$

$\Rightarrow 7\left(3x + 10\right) = 5\left(5x + 10\right)$ (by cross multiplication)

$\Rightarrow 21x + 70 = 25x + 50$

$\Rightarrow 21x - 25x = 50 - 70$

$\Rightarrow -4x = -20$

$\Rightarrow x = \frac{-20}{-4} = 5$

Therefore, the common multiple of both the numbers is 5.

First number = $3x = 3 \times 5 = 15$

Second number = $5x = 5 \times 5 = 25$

Q9.

Answer :

Let the first odd number be x.

Let the second odd number be $\left(x+2\right)$.

Let the third odd number be $\left(x+4\right)$.

$\therefore x + \left(x+2\right) + \left(x+4\right) = 147$

$\Rightarrow x + x + 2 + x + 4 = 147$

$\Rightarrow 3x + 6 = 147$

$\Rightarrow 3x = 147 - 6$

$\Rightarrow 3x = 141$

$\Rightarrow x = \frac{141}{3} = 47$

Therefore, the first odd number is 47.

$Second\ odd\ number = \left(x+2\right) = \left(47+2\right) = 49$

$Third\ odd\ number = \left(x+4\right) = \left(47+4\right) = 51$

Q10.

Answer :

Let the first even number *be* x.

Let the second even number *be* $x+2$.

Let the third even number be $x+4$.

$\therefore x + x+2+x+4 = 234$

$\Rightarrow x + x + 2 + x + 4 = 234$

$\Rightarrow 3x + 6 = 234$

$\Rightarrow 3x = 234 - 6$

$\Rightarrow 3x = 228$

$\Rightarrow x = \frac{228}{3} = 76$

$\therefore First\ even\ number = x = 76$

$Second\ even\ number = x+2 = 76+2 = 78$

$Third\ even\ number = x+4 = 80$

Q11. **Answer :**

Let the digit in *the* units place be x.

$Digit\ in\ the\ tens\ place = \left(12-x\right)$

$\therefore\ Original\ number = 10\left(12 - x\right) + x = 120 - 9x$

On reversing the digits, we have x at the tens place and $\left(12-x\right)$ at the units place.

$\therefore New\ number = 10x + 12 - x = 9x + 12$

$New\ number - Original\ number = 54$

$\Rightarrow 9x + 12 - \left(120 - 9x\right) = 54$

$\Rightarrow 9x + 12 - 120 + 9x = 54$

$\Rightarrow 18x - 108 = 54$

$\Rightarrow 18x = 54 + 108$

$\Rightarrow 18x = 162$

$\Rightarrow x = \frac{162}{18} = 9$

Therefore, the digit in *the* units place is 9.

$Digit\ in\ tens\ place = \left(12-x\right) = \left(12-9\right) = 3$

Therefore, the original number is 39.

Check :

The original number is 39.

$Sum\ of\ the\ digits\ in\ the\ original\ number = \left(3+9\right) = 12$

$New\ number\ obtained\ on\ reversing\ the\ digits = 93$

$New\ number - Original\ number = \left(93 - 39\right) = 54$

Thus, both the given conditions are satisfied by 39.

Hence, the original number *is* 39.

Q12.

Answer :

Let the digit in *the* units place be x.
*D*igit in *the* tens place $= 3x$
*O*riginal number $= 10\left(3x\right) + x = 30x + x$

On reversing the digits, we have x at the tens place and $\left(3x\right)$ at the units place.

$\therefore$ *N*ew number $= 10\left(x\right) + 3x = 10x + 3x$

New number $=$ Original number $- 36$
$\Rightarrow 10x + 3x = 30x + x - 36$
$\Rightarrow 13x = 31x - 36$
$\Rightarrow 36 = 31x - 13x$
$\Rightarrow 36 = 18x$
$\Rightarrow 18x = 36$
$\Rightarrow x = \frac{36}{18} = 2$
Therefore, the digit in *the* units place is 2.
*D*igit in *the* tens place $= \left(3x\right) = 3 \times 2 = 6$
Therefore, the original number is 62.

Check :
New number $+ 36 =$ Original Number
$26 + 36 = 62$
Hence, both the conditions are satisfied.
Therefore, the original number *is* 62.

Q13.

Answer :

Let the numerator be x.
The denominator is greater than the numerator by 7.
$\therefore \left(x+7\right)$
$\therefore \frac{x+17}{(x+7)-6} = 2$
$\Rightarrow \frac{x+17}{x+1} = 2$
$\Rightarrow x + 17 = 2\left(x + 1\right)$ $\left(\text{by cross multiplication}\right)$
$\Rightarrow x + 17 = 2x + 2$
$\Rightarrow x - 2x = 2 - 17$
$\Rightarrow -x = -15$
$\Rightarrow x = 15$
Therefore, the numerator is 15.
Denominator $= \left(x+7\right) = \left(15+7\right) = 22$
$\therefore$ Original number $= \frac{15}{22}$

Q14.

Denominator, d = x

It is given that twice the numerator is equal to two more than the denominator.

$\therefore$ Twice of numerator, $2n = x + 2$

$\therefore$ Numerator, $n = \frac{x+2}{2}$

$\therefore \frac{n+3}{d+3} = \frac{2}{3}$

$\Rightarrow 3(n + 3) = 2(d + 3)$ (*by cross multiplication*)

$\Rightarrow 3n + 9 = 2d + 6$

$\Rightarrow 3n - 2d = 6 - 9$

$\Rightarrow 3n - 2d = -3$

On replace d by x and n by $\frac{x+2}{2}$:

$\Rightarrow 3\left(\frac{x+2}{2}\right) - 2x = -3$

$\Rightarrow \frac{3x+6-4x}{2} = -3$ (taking the L.C.M. of 2 and 1 as 2)

$\Rightarrow 6 - x = -6$ (by cross multiplication)

$\Rightarrow -x = -6 - 6$

$\Rightarrow x = 12$

The denominator is 12.

$\therefore$ Numerator $= \frac{x+2}{2} = \frac{12+2}{2} = \frac{14}{2} = 7$

$\therefore$ Original fraction $= \frac{7}{12}$

Q15.

Answer :

Let the breadth of the original rectangle be x cm.

Then, its length will be (x + 7) cm.

The area of the rectangle will be (x)(x + 7) cm^2.

$\therefore (x + 3)(x + 7 - 4) = (x)(x + 7)$

$\Rightarrow (x + 3)(x + 3) = x^2 + 7x$

$\Rightarrow x^2 + 3x + 3x + 9 = x^2 + 7x$

$\Rightarrow x^2 + 6x + 9 = x^2 + 7x$

$\Rightarrow 9 = x^2 - x^2 + 7x - 6x$

$\Rightarrow 9 = x$

$\Rightarrow x = 9$ (by transposition)

Breadth of the original rectangle = 9 cm

Length of the original rectangle = (x + 7) = (9 + 7) = 16 cm

Q16.

Answer :

Let the width of the rectangle be x cm.

It is $\frac{2}{3}$ of the length of the rectangle.

This means that the length of the rectangle will be $\frac{3}{2}x$.

Perimeter of the rectangle $= 2(x) + 2\left(\frac{3}{2}\right)x = 180$ m

$\therefore 2x + \frac{6x}{2} = 180$

$\Rightarrow \frac{4x+6x}{2} = 180$ (taking the L.C.M. of 1 on the L.H.S. of the equation)

$\Rightarrow 10x = 2 \times 180$ (by cross multiplication)

$\Rightarrow 10x = 360$

$\Rightarrow x = \frac{360}{10} = 36$

Therefore, the width of the rectangle is 36 m.

Length of the rectangle will be $= \frac{3}{2}x = \frac{3}{2}(36) = 54$ m

Q17.

Let the length of the base of the triangle be x cm.

Then, its altitude will be $\frac{5}{3}x$ cm.

Area of the triangle $= \frac{1}{2}\left(x\right)\left(\frac{5}{3}x\right) = \frac{5}{6}x^2$

$\therefore \frac{1}{2}\left(x-2\right)\left(\frac{5}{3}x+4\right) = \frac{5}{6}x^2$

$\Rightarrow \left(\frac{x-2}{2}\right)\left(\frac{5x+12}{3}\right) = \frac{5x^2}{6}$

$\Rightarrow \frac{(x-2)(5x+12)}{6} = \frac{5x^2}{6}$

$\Rightarrow \frac{5x^2+12x-10x-24}{6} = \frac{5x^2}{6}$

$\Rightarrow 5x^2 + 2x - 24 = 5x^2$ (cancelling the denominators from both the sides since they are same)

$\Rightarrow 5x^2 - 5x^2 + 2x = 24$

$\Rightarrow 2x = 24$

$\Rightarrow x = \frac{24}{2} = 12\,m$

Therefore, the base of the triangle is 12 m.

Altitude of the triangle $= \frac{5}{3}x = \frac{5}{3}\left(12\right) = 20\,m$

Q18

Answer :

Let the common multiple of all the three angles be x.

Then, the first angle will be 4x.

And the second angle *will* be 5x.

In a triangle, sum of all the three angles will be equal to 180°.

$\therefore$ *T*hird angle $= 180 - \left(4x + 5x\right) = 180 - 9x$

$\therefore 4x + 5x = 180 - 9x$

$\Rightarrow 9x = 180 - 9x$

$\Rightarrow 9x + 9x = 180$

$\Rightarrow 18x = 180$

$\Rightarrow x = \frac{180}{18} = 10$

First angle $= 4x = 4 \times 10 = 40^\circ$

Second angle $= 5x = 5 \times 10 = 50^\circ$

Third angle $= 4x + 5x = 9x = 9 \times 10 = 90^\circ$

Q19

Answer :

Let the speed of the steamer in still water be x km/h.

Speed (downstream) $= (x + 1)$ km/h

Speed (upstream) $= (x - 1)$ km/h

Distance covered in 9 hours while going downstream $= 9(x+1)$ km

Distance covered in 10 hours while going upstream $= 10(x-1)$ km

But both of these distances will be same.

$9(x + 1) = 10(x - 1)$

$\Rightarrow 9x + 9 = 10x - 10$

$\Rightarrow 9 + 10 = 10x - 9x$

$\Rightarrow 19 = x$

$\Rightarrow x = 19$

Therefore, the speed of the steamer in still water is 19 km/h.

Distance between the ports $= 9(x+1) = 9(19+1) = 9 \times 20 = 180$ km

Q20

Answer :

Let the speed of one motorcyclist be x km/h.

So, the speed of the other motorcyclist will be $\left(x+7\right)$ km/h.

Distance travelled by the first motorcyclist in 2 hours $= 2x$ km

Distance travelled by the second motorcyclist in 2 hours $= 2\left(x+7\right)$ km

Therefore,

$300 - \left(2x + \left(2x + 14\right)\right) = 34$

$\Rightarrow 300 - \left(2x + 2x + 14\right) = 34$

$\Rightarrow 300 - 4x - 14 = 34$

$\Rightarrow 286 - 4x = 34$

$\Rightarrow 286 - 34 = 4x$

$\Rightarrow 252 = 4x$

$\Rightarrow x = \frac{252}{4} = 63$

Therefore, the speed of the first motorcyclist is 63 km/h.

The speed of the second motorcyclist is $\left(x+7\right) = \left(63+7\right) = 70$ km/h.

Check :

The distance covered by the first motorcyclist in 2 hours $= 63 \times 2 = 126$ km

The distance covered by the second motorcyclist in 2 hours $= 70 \times 2 = 140$ km

The distance between the motorcyclists after 2 hours $= 300 - \left(126 + 140\right) =$ 34 km (which is the same as given)

Therefore, the speeds of the motorcyclists are 63 km/h and 70 km/h, respectively.

Q21

Answer :

Let the first number be x.

Then, the second number will be $\frac{5}{6}x$.

Third number $= \frac{4}{5}\left(\frac{5}{6}x\right) = \frac{2}{3}x$

$\therefore x + \frac{5x}{6} + \frac{2x}{3} = 150$

$\Rightarrow \frac{6x+5x+4x}{6} = 150$ (multiplying the L.H.S. by 6, which is the L.C.M. of 1, 6 and 3)

$\Rightarrow 15x = 150 \times 6$ (by cross multiplication)

$\Rightarrow 15x = 900$

$\Rightarrow x = \frac{900}{15} = 60$

Therefore, the first number is 60.

Second number $= \frac{5}{6}x = \frac{5}{6}\left(60\right) = 50$

Third number $= \frac{2}{3}x = \frac{2}{3}\left(60\right) = 40$

Q22

Answer :

Let the first part be x.

Let the second part be $\left(4500 - x\right)$.

$\therefore 5\%\ of\ x = 10\%\ of\ \left(4500 - x\right)$

$\Rightarrow \left(\frac{5}{100}\right)x = \left(\frac{10}{100}\right)\left(4500 - x\right)$

$\Rightarrow \frac{5x}{100} = \frac{45000 - 10x}{100}$

$\Rightarrow 5x = 45000 - 10x$ (by cancellation of same denominators from both the sides) $\Rightarrow 5x + 10x = 45000 \Rightarrow 15x = 45000 \Rightarrow x = \frac{45000}{15} = 3000$ Therefore, the first part is 3000. Second part $= \left(4500 - x\right) = \left(4500 - 3000\right) = 1500$

Q23

Answer :

Let the present age of Rakhi be x.
Then, the present age of Rakhi's mother will be 4x.
After five years, Rakhi's age will be (x + 5).
After five years, her mother's age will be (4x + 5).

$4x + 5 = 3(x + 5)$
$\Rightarrow 4x + 5 = 3x + 15$
$\Rightarrow 4x - 3x = 15 - 5$
$\Rightarrow x = 10$

Present age of Rakhi $= 10$ years
Present age of Rakhi's mother $= 4(x) = 4 \times 10 = 40$ years

Q24

Answer :

Let the age of Monu's father be x years.
The age of Monu's grandfather will be $(x + 26)$.
Then, the age of Monu will be $(x - 29)$.

$\therefore x + (x + 26) + (x - 29) = 135$
$\Rightarrow x + x + 26 + x - 29 = 135$
$\Rightarrow 3x - 3 = 135$
$\Rightarrow 3x = 135 + 3$
$\Rightarrow 3x = 138$
$\Rightarrow x = \frac{138}{3} = 46$

$\therefore$ Age of Monu's father $= 46$ years
Age of Monu's grandfather $= (x + 26) = (46 + 26) = 72$ years
Age of Monu $= (x - 29) = 46 - 29 = 17$ years

Q25

Answer :

Let the age of the grandson be x years.
Then, his grandfather's age will be 10x.
Also, the grandfather is 54 years older than his grandson.
$\therefore$ Age of the grandson $= x + 54$

$10x = x + 54$
$\Rightarrow 10x - x = 54$
$\Rightarrow 9x = 54$
$\Rightarrow x = \frac{54}{9} = 6$

Therefore, the grandson's age is 6 years.
Grandfather's age $= 10(x) = 10 \times 6 = 60$ years

Q26

Answer :

Let the age of the younger cousin be x.
Then, the age of the elder cousin will be $(x + 10)$.
15 years ago :
Age of the younger cousin $= (x - 15)$
Age of elder cousin $= (x + 10 - 15)$
$= (x - 5)$

$\therefore (x - 5) = 2(x - 15)$
$\Rightarrow x - 5 = 2x - 30$
$\Rightarrow x - 2x = -30 + 5$
$\Rightarrow -x = -25$
$\Rightarrow x = 25$

Therefore, the present age of the younger cousin is 25 years.
Present age of elder cousin $= (x + 10) = (25 + 10) = 35$ years

Q27

Answer :

Let the number of deer in the herd be x.

The number of deer grazing in the field is $\left(\frac{1}{2}\right)$x.

Remaining deer $= x - \frac{x}{2} = \frac{x}{2}$

Number of deer playing nearby $= \frac{3}{4}\left(\frac{x}{2}\right) = \frac{3}{8}x$

The number of deer drinking water from the pond is 9.

$\therefore 9 + \frac{3}{8}x + \frac{1}{2}x = x$

$\Rightarrow \frac{72+3x+4x}{8} = x$ $\left(\text{multiplying the L.H.S. by } 8, \text{ which is the L.C.M. of } 1, 8 \text{ and } 2\right)$

$\Rightarrow 72 + 7x = 8x$ $\left(\text{by cross multiplication}\right) \Rightarrow 72 = 8x - 7x \Rightarrow 72 = x \Rightarrow x = 72$Total number of deer in the herd $= 72$