Linear Equations
Ex 8A

Definition of a Linear Equation

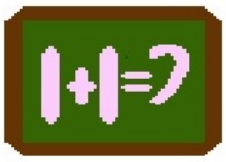
- A linear equation in one variable x is an equation that can be written in the form
- $ax + b = 0$
- where a and b are real numbers and $a \neq 0$.

Equation is a mathematical sentence indicating that two expressions are equal. The symbol "=" is used to indicate equality.

Ex.

$2x + 5 = 9$ is a conditional equation since its truth or falsity depends on the value of x

$2 + 9 = 11$ is identity equation since both of its sides are identical to the same number 11.



Solution Set of a Linear Equation

Example

$4x + 2 = 10$	this statement is either true of false
If $x = 1$, then $4x + 2 = 10$	is false because $4(1) + 2$ is $\neq 10$
If $x = 2$, then $4x + 2 = 10$	is true because $4(2) + 2 = 10$

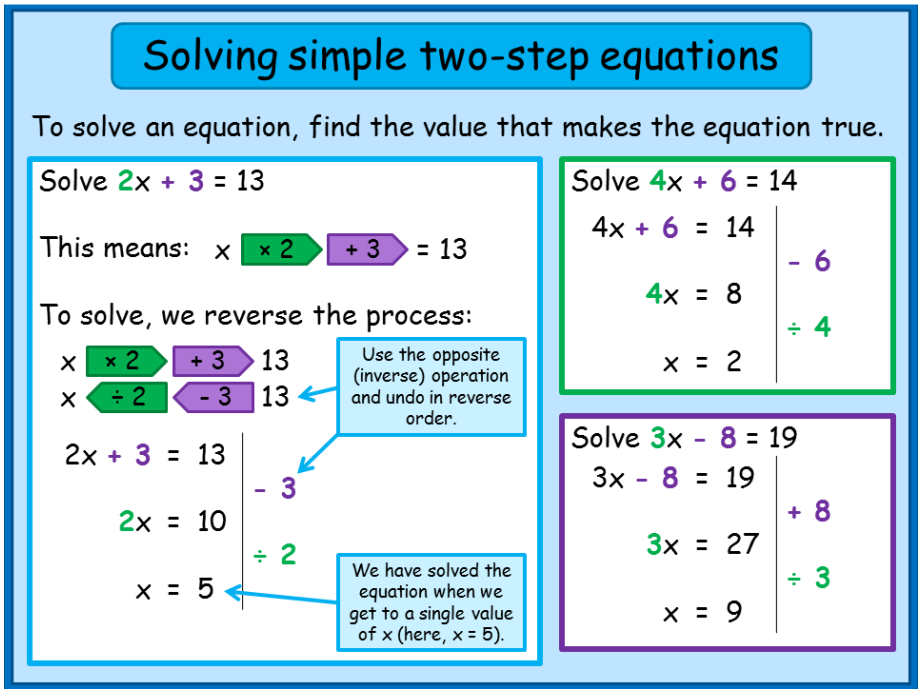
ONE STEP SUBTRACTION EXAMPLE

The Opposite of Subtraction is Addition

$$x - 120 = 80$$
$$+120 \quad +120$$
$$x = 200 \checkmark$$

The value which makes the equation true is 200.

$x - 4 = 7$	Original problem
$x - 4 = 7$	We want to remove the minus 4.
$x - 4 + 4 = 7 + 4$	The opposite of minus 4 is plus 4, so I added 4 to BOTH sides of the equation.
$x = 11$	-4+4 = 0, so x remains on the left and 7+4 = 11; therefore x = 11
Check: $x - 4 = 7$ $11 - 4 = 7$	This is a correct statement, so my answer is x = 11 is correct!



Q1

Answer :

$$
\begin{aligned}
&8x + 3 = 27 + 2x \\
&\Rightarrow 8x - 2x = 27 - 3 \\
&\Rightarrow 6x = 24 \\
&\Rightarrow x = \tfrac{24}{6} = 4 \\
&\therefore x = 4
\end{aligned}
$$

Q2

Answer :

$$
\begin{aligned}
&5x + 7 = 2x - 8 \\
&\Rightarrow 5x - 2x = -8 - 7 \\
&\Rightarrow 3x = -15 \\
&\Rightarrow x = \tfrac{-15}{3} = -5 \\
&\therefore x = -5
\end{aligned}
$$

Q3.

Answer :

$$
\begin{aligned}
&2z - 1 = 14 - z \\
&\Rightarrow 2z + z = 14 + 1 \\
&\Rightarrow 3z = 15 \\
&\Rightarrow z = \tfrac{15}{3} = 5 \\
&\therefore z = 5
\end{aligned}
$$

Q4.

Answer :

$$
\begin{aligned}
&9x + 5 = 4(x - 2) + 8 \\
&\Rightarrow 9x + 5 = 4x - 8 + 8 \\
&\Rightarrow 9x + 5 = 4x \\
&\Rightarrow 9x - 4x = -5 \\
&\Rightarrow 5x = -5 \\
&\Rightarrow x = \tfrac{-5}{5} = -1 \\
&\therefore x = -1
\end{aligned}
$$

Q5.

Answer :

$\frac{7y}{5} = y - 4$

By cross multiplication :

$\Rightarrow 7y = 5\left(y - 4\right)$

$\Rightarrow 7y = 5y - 20$

$\Rightarrow 7y - 5y = -20$

$\Rightarrow 2y = -20$

$\Rightarrow y = \frac{-20}{2} = -10$

$\therefore y = -10$

Q6.

Answer :

$3x + \frac{2}{3} = 2x + 1$

$\Rightarrow 3x - 2x = 1 - \frac{2}{3}$

$\Rightarrow x = \frac{1}{1} - \frac{2}{3}$ $\left(\text{L.C.M. of } 1 \text{ and } 3 \text{ is } 3\right)$ $\Rightarrow x = \frac{3-2}{3}$

$\Rightarrow x = \frac{1}{3}$

$\Rightarrow x = \frac{1}{3}$

$\therefore x = \frac{1}{3}$

Q7.

Answer :

$15\left(y - 4\right) - 2\left(y - 9\right) + 5\left(y + 6\right) = 0$

$\Rightarrow 15y - 60 - 2y + 18 + 5y + 30 = 0$

$\Rightarrow 15y - 2y + 5y - 60 + 18 + 30 = 0$

$\Rightarrow 18y - 12 = 0$

$\Rightarrow 18y = 12$

$\Rightarrow y = \frac{12}{18} = \frac{2}{3}$

$\therefore y = \frac{2}{3}$

Q8.

Answer :

$3\left(5x - 7\right) - 2\left(9x - 11\right) = 4\left(8x - 13\right) - 17$

$\Rightarrow 15x - 21 - 18x + 22 = 32x - 52 - 17$

$\Rightarrow 15x - 18x - 21 + 22 = 32x - 69$

$\Rightarrow -3x + 1 = 32x - 69$

$\Rightarrow 1 + 69 = 32x + 3x$

$\Rightarrow 70 = 35x$

$\Rightarrow 35x = 70$ $\left(by \text{ transposition}\right)$

$\Rightarrow x = \frac{70}{35} = 2$

$\therefore x = 2$

Q9.

Answer :

$\frac{x-5}{2} - \frac{x-3}{5} = \frac{1}{2}$

$\Rightarrow 10\left(\frac{x-5}{2}\right) - 10\left(\frac{x-3}{5}\right) = 10\left(\frac{1}{2}\right)$ $\left(\text{multiplying throughout by } 10, \text{ which is the L.C.M. of } 2, 2 \text{ and } 5\right)$

$\Rightarrow 5\left(x - 5\right) - 2\left(x - 3\right) = 5 \Rightarrow 5x - 25 - 2x + 6 = 5 \Rightarrow 5x - 2x - 25 + 6 = 5 \Rightarrow 3x - 19 = 5 \Rightarrow 3x = 5 + 19 \Rightarrow 3x = 24 \Rightarrow x = \frac{24}{3} = 8 \therefore x = 8$

Q10.

Answer :

$\frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2}{3} - t$

$\Rightarrow \frac{3t-2}{4} - \frac{2t+3}{3} = \frac{2-3t}{3}$ $\left(3 \text{ is the L.C.M. of } 1 \text{ and } 3\right)$

$\Rightarrow 12\left(\frac{3t-2}{4}\right) - 12\left(\frac{2t+3}{3}\right) = 12\left(\frac{2-3t}{3}\right)$ $\left(\text{multiplying throughout by } 12, \text{ which is the L.C.M. of } 4, 3 \text{ and } 3\right)$

$\Rightarrow 3\left(3t-2\right) - 4\left(2t+3\right) = 4\left(2-3t\right)$

$\Rightarrow 9t - 6 - 8t - 12 = 8 - 12t$

$\Rightarrow 9t - 8t - 6 - 12 = 8 - 12t$

$\Rightarrow t - 18 = 8 - 12t$

$\Rightarrow t + 12t = 18 + 8$

$\Rightarrow 13t = 26$

$\Rightarrow t = \frac{26}{13} = 2$

$\therefore t = 2$

Q11.

Answer :

$\frac{2x+7}{5} - \frac{3x+11}{2} = \frac{2x+8}{3} - 5$

$\Rightarrow \frac{2x+7}{5} - \frac{3x+11}{2} = \frac{2x+8-15}{3}$ $\left(\text{L.C.M. of } 3 \text{ and } 1 \text{ is } 3\right)$

$\Rightarrow 30\left(\frac{2x+7}{5}\right) - 30\left(\frac{3x+11}{2}\right) = 30\left(\frac{2x+8-15}{3}\right)$

$\left(\text{multiplying } g \text{ throughout by } 30, \text{ which is the L.C.M. of } 5, 2 \text{ and } 3\right)$

$\Rightarrow 6\left(2x+7\right) - 15\left(3x+11\right) = 10\left(2x+8-15\right) \Rightarrow 12x + 42 - 45x - 165 = 20x - 70 \Rightarrow 12x - 45x + 42 - 165 = 20x - 70 \Rightarrow -33x - 123 = 20x - 70 \Rightarrow -33x - 20x = 123 - 70 \Rightarrow -53x = 53 \Rightarrow x = \frac{53}{-53} \Rightarrow x = -1 \therefore x = -1$

Q12.

Answer :

$\frac{5x-4}{6} = 4x + 1 - \frac{3x+10}{2}$

$\Rightarrow \frac{5x-4}{6} = \frac{2\left(4x+1\right)-3x-10}{2}$ $\left(\text{L.C.M. of } 1 \text{ and } 2 \text{ is } 2\right)$

$\Rightarrow \frac{5x-4}{6} = \frac{8x+2-3x-10}{2}$

$\Rightarrow \frac{5x-4}{6} = \frac{8x-3x+2-10}{2}$

$\Rightarrow \frac{5x-4}{6} = \frac{5x-8}{2}$

$\Rightarrow 2\left(5x-4\right) = 6\left(5x-8\right)$

$\Rightarrow 10x - 8 = 30x - 48$

$\Rightarrow 10x - 30x = -48 + 8$

$\Rightarrow -20x = -40$

$\Rightarrow x = \frac{-40}{-20} = 2$

$\therefore x = 2$

Q13.

Answer :

$5x - \frac{1}{3}\left(x + 1\right) = 6\left(x + \frac{1}{30}\right)$

$\Rightarrow 5x - \frac{1(x+1)}{3} = 6\left(\frac{30x+1}{30}\right)$ $\left(\text{L.C.M. of } 1 \text{ and } 30 \text{ is } 30\right)$

$\Rightarrow 5x - \frac{(x+1)}{3} = \frac{30x+1}{5}$

$\Rightarrow \frac{15x-x-1}{3} = \frac{30x+1}{5}$ $\left(\text{L.C.M. of } 1 \text{ and } 3 \text{ is } 3\right)$

$\Rightarrow \frac{14x-1}{3} = \frac{30x+1}{5}$

$\Rightarrow 5\left(14x - 1\right) = 3\left(30x + 1\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 70x - 5 = 90x + 3$

$\Rightarrow 70x - 90x = 3 + 5$

$\Rightarrow -20x = 8$

$\Rightarrow x = \frac{8}{-20} = \frac{-2}{5}$

$\therefore x = -\frac{2}{5}$

Q14.

Answer :

$4 - \frac{2(z-4)}{3} = \frac{1}{2}\left(2z+5\right)$

$\Rightarrow \frac{12-2(z-4)}{3} = \frac{1(2z+5)}{2}$ $\left(\text{L.C.M. of } 1 \text{ and } 3 \text{ is } 3\right)$

$\Rightarrow \frac{12-2z+8}{3} = \frac{2z+5}{2}$

$\Rightarrow \frac{20-2z}{3} = \frac{2z+5}{2}$

$\Rightarrow 2\left(20-2z\right) = 3\left(2z+5\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 40 - 4z = 6z + 15$

$\Rightarrow 40 - 15 = 6z + 4z$

$\Rightarrow 25 = 10z$

$\Rightarrow 10z = 25$ $\left(\text{by transposition}\right)$

$\Rightarrow z = \frac{25}{10} = \frac{5}{2}$

$\therefore z = \frac{5}{2}$

Q15.

Answer :

$\frac{3(y-5)}{4} - 4y = 3 - \frac{(y-3)}{2}$

$\Rightarrow \frac{3y-15}{4} - 4y = 3 - \frac{y-3}{2}$

$\Rightarrow \frac{3y-15-16y}{4} = 3 - \frac{y-3}{2}$ $\left(\text{L.C.M. of } 4 \text{ and } 1 \text{ is } 4\right)$

$\Rightarrow \frac{-13y-15}{4} = \frac{6-y+3}{2}$

$\Rightarrow \frac{-13y-15}{4} = \frac{9-y}{2}$

$\Rightarrow 2\left(-13y-15\right) = 4\left(9 - y\right)$

$\Rightarrow -26y - 30 = 36 - 4y$

$\Rightarrow -26y + 4y = 36 + 30$

$\Rightarrow -22y = 66$

$\Rightarrow 22y = -66$ $\left(\text{multiplying both the sides with a } -\text{ve sign}\right)$

$\Rightarrow y = -\frac{66}{22} = -3$

$\therefore y = -3$

Q16.

Answer :

$\frac{8x-3}{3x} = 2$

$\Rightarrow 8x - 3 = 2\left(3x\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 8x - 3 = 6x$

$\Rightarrow 8x - 6x = 3$

$\Rightarrow 2x = 3$

$\Rightarrow x = \frac{3}{2}$

$\therefore x = \frac{3}{2}$

Q17.

Answer :

$\frac{9x}{7-6x} = 15$

$\Rightarrow \frac{9x}{7-6x} = \frac{15}{1}$

$\Rightarrow 1\left(9x\right) = 15\left(7-6x\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 9x = 105 - 90x$

$\Rightarrow 9x + 90x = 105$

$\Rightarrow 99x = 105$

$\Rightarrow x = \frac{105}{99} = \frac{35}{33}$

$\therefore x = \frac{35}{33}$

Q18.

Answer :

$\frac{3x}{5x+2} = -4$

$\Rightarrow \frac{3x}{5x+2} = \frac{-4}{1}$

$\Rightarrow 1\left(3x\right) = -4\left(5x+2\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 3x = -20x - 8$

$\Rightarrow 3x + 20x = -8$

$\Rightarrow 23x = -8$

$\Rightarrow x = \frac{-8}{23}$

$\therefore x = \frac{-8}{23}$

Q20.

Answer :

$\frac{2-9z}{17-4z} = \frac{4}{5}$

$\Rightarrow 5\left(2-9z\right) = 4\left(17-4z\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 10 - 45z = 68 - 16z$

$\Rightarrow 10 - 68 = 45z - 16z$

$\Rightarrow -58 = 29z$

$\Rightarrow 29z = -58$ $\left(\text{by transposition}\right)$

$\Rightarrow z = \frac{-58}{29} = -2$

$\therefore z = -2$

Q21.

Answer :

$\frac{4x+7}{9-3x} = \frac{1}{4}$

$\Rightarrow 4\left(4x+7\right) = 1\left(9-3x\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 16x + 28 = 9 - 3x$

$\Rightarrow 16x + 3x = 9 - 28$

$\Rightarrow 19x = -19$

$\Rightarrow x = \frac{-19}{19} = -1$

$\therefore x = -1$

Q22.

Answer :

$\frac{7y+4}{y+2} = \frac{-4}{3}$

$\Rightarrow 3\left(7y+4\right) = -4\left(y+2\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 21y + 12 = -4y - 8$

$\Rightarrow 21y + 4y = -8 - 12$

$\Rightarrow 25y = -20$

$\Rightarrow y = \frac{-20}{25} = \frac{-4}{5}$

$\therefore y = \frac{-4}{5}$

Q23.

Answer :

$\frac{15\left(2-y\right)-5\left(y+6\right)}{1-3y} = 10$

$\Rightarrow \frac{30-15y-5y-30}{1-3y} = 10$

$\Rightarrow \frac{-20y}{1-3y} = 10$

$\Rightarrow 1\left(-20y\right) = 10\left(1-3y\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow -20y = 10 - 30y$

$\Rightarrow -20y + 30y = 10$

$\Rightarrow 10y = 10$

$\Rightarrow y = \frac{10}{10} = 1$

$\therefore y = 1$

Q24.

Answer :

$\frac{2x-(7-5x)}{9x-(3+4x)} = \frac{7}{6}$

$\Rightarrow \frac{2x-7+5x}{9x-3-4x} = \frac{7}{6}$

$\Rightarrow \frac{7x-7}{5x-3} = \frac{7}{6}$

$\Rightarrow 6\left(7x-7\right) = 7\left(5x-3\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 42x - 42 = 35x - 21$

$\Rightarrow 42x - 35x = 42 - 21$

$\Rightarrow 7x = 21$

$\Rightarrow x = \frac{21}{7} = 3$

$\therefore x = 3$

Q25.

Answer :

$m - \frac{(m-1)}{2} = 1 - \frac{(m-2)}{3}$

$\Rightarrow \frac{2m-m+1}{2} = 1 - \frac{(m-2)}{3}$ $\left(L.C.M. \text{ of } 1 \text{ and } 2 \text{ is } 2\right)$

$\Rightarrow \frac{m+1}{2} = \frac{3-m+2}{3}$ $\left(L.C.M. \text{ of } 1 \text{ and } 3 \text{ is } 3\right)$

$\Rightarrow \frac{m+1}{2} = \frac{5-m}{3}$

$\Rightarrow 3\left(m+1\right) = 2\left(5-m\right)$ $\left(\text{by cross multiplication}\right)$

$\Rightarrow 3m + 3 = 10 - 2m$

$\Rightarrow 3m + 2m = 10 - 3$

$\Rightarrow 5m = 7$

$\Rightarrow m = \frac{7}{5}$

$\therefore m = \frac{7}{5}$

Q26.

Answer :

$\frac{3x+5}{4x+2} = \frac{3x+4}{4x+7}$

$\Rightarrow (4x+7)(3x+5) = (4x+2)(3x+4)$ (by cross multiplication)

$\Rightarrow 12x^2 + 20x + 21x + 35 = 12x^2 + 16x + 6x + 8$

$\Rightarrow 12x^2 + 41x + 35 = 12x^2 + 22x + 8$

$\Rightarrow 12x^2 - 12x^2 + 41x - 22x = 8 - 35$

$\Rightarrow 19x = -27$

$\Rightarrow x = \frac{-27}{19}$

$\therefore x = \frac{-27}{19}$

Q27.

Answer :

$\frac{9x-7}{3x+5} = \frac{3x-4}{x+6}$

$\Rightarrow (x+6)(9x-7) = (3x+5)(3x-4)$

(by cross multiplication)

$\Rightarrow 9x^2 - 7x + 54x - 42 = 9x^2 - 12x + 15x - 20$

$\Rightarrow 9x^2 + 47x - 42 = 9x^2 + 3x - 20$

$\Rightarrow 9x^2 - 9x^2 + 47x - 3x = -20 + 42$

$\Rightarrow 44x = 22$

$\Rightarrow x = \frac{22}{44} = \frac{1}{2}$

$\therefore x = \frac{1}{2}$

Q28.

Answer :

$\frac{2-7x}{1-5x} = \frac{3+7x}{4+5x}$

$\Rightarrow (4+5x)(2-7x) = (1-5x)(3+7x)$ (by cross multiplication)

$\Rightarrow 8 - 28x + 10x - 35x^2 = 3 + 7x - 15x - 35x^2$

$\Rightarrow -35x^2 - 18x + 8 = -35x^2 - 8x + 3$

$\Rightarrow -35x^2 + 35x^2 - 18x + 8x = -8 + 3$

$\Rightarrow -10x = -5$

$\Rightarrow x = \frac{-5}{-10} = \frac{1}{2}$

$\therefore x = \frac{1}{2}$