Operations On Algebraic Expressions
Ex 6D

1. $(a + b)^2 = a^2 + 2ab + b^2 = (-a - b)^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(a - b)(a + b) = a^2 - b^2$
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
5. $(a + b - c)^2 = a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$
6. $(a - b + c)^2 = a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$
7. $(-a + b + c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
8. $(a - b - c)^2 = a^2 + b^2 + c^2 - 2ab + 2bc - 2ca$
9. $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
10. $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$
11. $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$
    $= (a + b)(a^2 - ab + b^2)$
12. $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$
    $= (a - b)(a^2 + ab + b^2)$
13. $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

if $a + b + c = 0$ then $a^3 + b^3 + c^3 = 3abc$

Q1

Answer :

(i) We have:

$$\left(x+6\right)\left(x+6\right)$$
$$=\left(x+6\right)^2$$
$$=x^2+6^2+2\times x\times 6 \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=x^2+36+12x$$

(ii) We have:

$$\left(4x+5y\right)\left(4x+5y\right)$$
$$=\left(4x+5y\right)^2$$
$$=(4x)^2+(5y)^2+2\times 4x\times 5y \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=16x^2+25y^2+40xy$$

(iii) We have:

$$\left(7a+9b\right)\left(7a+9b\right)$$
$$=\left(7a+9b\right)^2$$
$$=(7a)^2+(9b)^2+2\times 7a\times 9b \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=49a^2+81b^2+126ab$$

(iv) We have:

$$\left(\tfrac{2}{3}x+\tfrac{4}{5}y\right)\left(\tfrac{2}{3}x+\tfrac{4}{5}y\right)$$
$$=\left(\tfrac{2}{3}x+\tfrac{4}{5}y\right)^2$$
$$=\left(\tfrac{2}{3}x\right)^2+\left(\tfrac{4}{5}y\right)^2+2\times\tfrac{2}{3}x\times\tfrac{4}{5}y \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=\tfrac{4}{9}x^2+\tfrac{16}{25}y^2+\tfrac{16}{15}xy$$

(v) We have:

$$\left(x^2+7\right)\left(x^2+7\right)$$
$$=\left(x^2+7\right)^2$$
$$=\left(x^2\right)^2+7^2+2\times x^2\times 7 \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=x^4+49+14x^2$$

(vi) We have:

$$\left(\frac{5}{6}a^2+2\right)\left(\frac{5}{6}a^2+2\right)$$
$$=\left(\frac{5}{6}a^2+2\right)^2$$
$$=\left(\frac{5}{6}a^2\right)^2+(2)^2+2\times\frac{5}{6}a^2\times 2 \qquad \left[\text{using } \left(a+b\right)^2=a^2+b^2+2ab\right]$$
$$=\frac{25}{36}a^4+4+\frac{10}{3}a^2$$

Q2

Answer :

(i) We have:

$$\left(x-4\right)\left(x-4\right)$$
$$=\left(x-4\right)^2$$
$$=x^2-2\times x\times 4+4^2 \qquad \left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$$
$$=x^2-8x+16$$

(ii) We have:

$$\left(2x-3y\right)\left(2x-3y\right)$$
$$=\left(2x-3y\right)^2$$
$$=(2x)^2-2\times 2x\times 3y+(3y)^2 \qquad \left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$$
$$=4x^2-12xy+9y^2$$

(iii) We have:

$$\left(\frac{3}{4}x-\frac{5}{6}y\right)\left(\frac{3}{4}x-\frac{5}{6}y\right)$$
$$=\left(\frac{3}{4}x-\frac{5}{6}y\right)^2$$
$$=\left(\frac{3}{4}x\right)^2-2\times\frac{3}{4}x\times\frac{5}{6}y+\left(\frac{5}{6}y\right)^2 \qquad \left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$$
$$=\frac{9}{16}x^2-\frac{15}{12}xy+\frac{25}{36}y^2$$

(iv) We have:

$$\left(x-\frac{3}{x}\right)\left(x-\frac{3}{x}\right)$$
$$=\left(x-\frac{3}{x}\right)^2$$
$$=(x)^2-2\times x\times\frac{3}{x}+\left(\frac{3}{x}\right)^2 \qquad \left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$$
$$=x^2-6+\frac{9}{x^2}$$

(v) We have:

$$\left(\frac{1}{3}x^2-9\right)\left(\frac{1}{3}x^2-9\right)$$
$$=\left(\frac{1}{3}x^2-9\right)^2$$
$$=\left(\frac{1}{3}x^2\right)^2-2\times\frac{1}{3}x^2\times 9+(9)^2 \qquad \left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$$
$$=\frac{1}{9}x^4-6x^2+81$$

(vi) We have:

$$\left(\frac{1}{2}y^2-\frac{1}{3}y\right)\left(\frac{1}{2}y^2-\frac{1}{3}y\right)$$
$$=\left(\frac{1}{2}y^2-\frac{1}{3}y\right)^2$$
$$=\left(\frac{1}{2}y^2\right)^2-2\times\frac{1}{2}y^2\times\frac{1}{3}y+\left(\frac{1}{3}y\right)^2 \qquad \left[\text{using } \left(a-b\right)^2=a^2-2ab+b^2\right]$$
$$=\frac{1}{4}y^4-\frac{1}{3}y^3+\frac{1}{9}y^2$$

Q3

Answer :

We shall use the identities $(a+b)^2=a^2+b^2+2ab$ and $(a-b)^2=a^2+b^2-2ab$.

(i) We have:

$$\left(8a+3b\right)^2$$
$$=(8a)^2+2\times 8a\times 3b+(3b)^2$$
$$=64a^2+48ab+9b^2$$

(ii)We have:

$$\left(7x+2y\right)^2$$
$$=(7x)^2+2\times 7x\times 2y+(2y)^2$$
$$=49x^2+28xy+4y^2$$

(iii) We have :

$$\left(5x+11\right)^2$$
$$=(5x)^2+2\times 5x\times 11+(11)^2$$
$$=25x^2+110x+121$$

(iv) We have:

$$\left(\frac{a}{2}+\frac{2}{a}\right)^2$$
$$=\left(\frac{a}{2}\right)^2+2\times\frac{a}{2}\times\frac{2}{a}+\left(\frac{2}{a}\right)^2$$
$$=\frac{a^2}{4}+2+\frac{4}{a^2}$$

(v) We have:

$$\left(\frac{3x}{4}+\frac{2y}{9}\right)^2$$
$$=\left(\frac{3x}{4}\right)^2+2\times\frac{3x}{4}\times\frac{2y}{9}+\left(\frac{2y}{9}\right)^2$$
$$=\frac{9x^2}{16}+\frac{1}{3}xy+\frac{4y^2}{81}$$

(vi) We have:

$$\left(9x-10\right)^2$$
$$(9x)^2-2\times 9x\times 10+(10)^2$$
$$=81x^2-180x+100$$

(vii) We have:

$$\left(x^2y - yz^2\right)^2$$
$$\left(x^2y\right)^2 - 2 \times x^2y \times yz^2 + \left(yz^2\right)^2$$
$$= x^4y^2 - 2x^2y^2z^2 + y^2z^4$$

(viii) We have:

$$\left(\frac{x}{y} - \frac{y}{x}\right)^2$$
$$= \left(\frac{x}{y}\right)^2 - 2 \times \frac{x}{y} \times \frac{y}{x} + \left(\frac{y}{x}\right)^2$$
$$= \frac{x^2}{y^2} - 2 + \frac{y^2}{x^2}$$

(ix) We have:

$$\left(3m - \frac{4}{5}n\right)^2$$
$$= (3m)^2 - 2 \times 3m \times \frac{4}{5}n + \left(\frac{4}{5}n\right)^2$$
$$= 9m^2 - \frac{24mn}{5} + \frac{16}{25}n^2$$

Q4

Answer :

(i) We have:

$$\left(x+3\right)\left(x-3\right)$$
$$= x^2 - 9 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(ii) We have:

$$\left(2x+5\right)\left(2x-5\right)$$
$$= 4x^2 - 25 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(iii) We have:

$$\left(8+x\right)\left(8-x\right)$$
$$= 64 - x^2 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(iv) We have:

$$\left(7x+11y\right)\left(7x-11y\right)$$
$$= 49x^2 - 121y^2 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(v) We have:

$$\left(5x^2 + \frac{3}{4}y^2\right)\left(5x^2 - \frac{3}{4}y^2\right)$$
$$= 25x^4 - \frac{9}{16}y^4 \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right]$$

(vi) We have:

$$\left(\frac{4x}{5} - \frac{5y}{3}\right)\left(\frac{4x}{5} + \frac{5y}{3}\right)$$
$$= \frac{16x^2}{25} - \frac{25y^2}{9} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right) = a^2 - b^2\right)\right]$$

(vii) We have:

$$\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

$$=x^2-\frac{1}{x^2} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

(viii) We have:

$$\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x}-\frac{1}{y}\right)$$

$$=\frac{1}{x^2}-\frac{1}{y^2} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

(ix) We have:

$$\left(2a+\frac{3}{b}\right)\left(2a-\frac{3}{b}\right)$$

$$=4a^2-\frac{9}{b^2} \qquad \left[\text{using } \left(a+b\right)\left(a-b\right)=a^2-b^2\right]$$

Q5

Answer :

We shall use the identity $(a+b)^2=a^2+b^2+2ab$.

(i)

$$(54)^2$$
$$=\left(50+4\right)^2$$
$$=(50)^2+2\times50\times4+(4)^2$$
$$=2500+400+16$$
$$=2916$$

(ii)

$$(82)^2$$
$$=\left(80+2\right)^2$$
$$=(80)^2+2\times80\times2+(2)^2$$
$$=6400+320+4$$
$$=6724$$

(iii)

$$(103)^2$$
$$=\left(100+3\right)^2$$
$$=(100)^2+2\times100\times3+(3)^2$$
$$=10000+600+9$$
$$=10609$$

(iv)

$$(704)^2$$
$$=\left(700+4\right)^2$$
$$=(700)^2+2\times700\times4+(4)^2$$
$$=490000+5600+16$$
$$=495616$$

Q6

Answer :

We shall use the identity $(a-b)^2 = a^2 + b^2 - 2ab$.

(i)

$$(69)^2$$
$$= \left(70-1\right)^2$$
$$= (70)^2 - 2 \times 70 \times 1 + 1$$
$$= 4900 - 140 + 1$$
$$= 4761$$

(ii)

$$(78)^2$$
$$= \left(80-2\right)^2$$
$$= (80)^2 - 2 \times 80 \times 2 + 4$$
$$= 6400 - 320 + 4$$
$$= 6084$$

(iii)

$$(197)^2$$
$$= \left(200-3\right)^2$$
$$= (200)^2 - 2 \times 200 \times 3 + 9$$
$$= 40000 - 1200 + 9$$
$$= 38809$$

(iv)

$$(999)^2$$
$$= \left(1000-1\right)^2$$
$$= (1000)^2 - 2 \times 1000 \times 1 + 1$$
$$= 1000000 - 2000 + 1$$
$$= 998001$$

Q7

Answer :

We shall use the identity $(a-b)(a+b) = a^2 - b^2$.

(i)

$$(82)^2 - (18)^2$$
$$= (82-18)(82+18)$$
$$= (64)(100)$$
$$= 6400$$

(ii)

$$(128)^2 - (72)^2$$
$$= (128-72)(128+72)$$
$$= (56)(200)$$
$$= 11200$$

(iii)

$$197 \times 203$$
$$= \left(200-3\right)\left(200+3\right)$$
$$= (200)^2 - (3)^2$$
$$= 40000 - 9$$
$$= 39991$$

(iv)
$$\frac{198\times198-102\times102}{96}$$
$$=\frac{(198)^2-(102)^2}{96}$$
$$=\frac{(198-102)(198+102)}{96}$$
$$=\frac{(96)(300)}{96}$$
$$=300$$

(v)
$$(14.7\times15.3)$$
$$=(15-0.3)\times(15+0.3)$$
$$=(15)^2-(0.3)^2$$
$$=225-0.09$$
$$=224.91$$

(vi)
$$(8.63)^2-(1.37)^2$$
$$=(8.63-1.37)(8.63+1.37)$$
$$=(7.26)(10)$$
$$=72.6$$

Q8

Answer :

$$(9x^2+24x+16)$$
$$Given,\ x=12$$
$$\Rightarrow(3x)^2+2(3x)(4)+(4)^2$$
$$\Rightarrow(3x+4)^2$$
$$\Rightarrow(3(12)+4)^2$$
$$\Rightarrow(36+4)^2$$
$$\Rightarrow(40)^2=1600$$

Therefore, the value of the expression ($9x^2 + 24x + 16$), when $x = 12$, is 1600.

Q9

Answer :

$$(64x^2+81y^2+144xy)$$
$$Given:$$
$$x=11$$
$$y=\frac{4}{3}$$
$$\Rightarrow(8x)^2+(9y)^2+2(8x)(9y)$$
$$\Rightarrow(8x+9y)^2$$
$$\Rightarrow\left(8\left(11\right)+9\left(\frac{4}{3}\right)\right)^2$$
$$\Rightarrow(88+12)^2$$
$$\Rightarrow(100)^2$$
$$\Rightarrow10000$$

Therefore, the value of the expression ($64x^2 + 81y^2 + 144xy$), when $x = 11$ and $y = \frac{4}{3}$, is 10000.

Q10

Answer :

$(36x^2 + 25y^2 - 60xy)$
$\Rightarrow x = \frac{2}{3}, \ y = \frac{1}{5}$
$= (6x)^2 + (5y)^2 - 2(6x)(5y)$
$= (6x - 5y)^2$
$= \left(6\left(\frac{2}{3}\right) - 5\left(\frac{1}{5}\right)\right)^2$
$= (4 - 1)^2$
$= (3)^2$
$\Rightarrow 9$

Q11

Answer :

$(i) \ \left(x + \frac{1}{x}\right) = 4$
Squaring both the sides :
$\Rightarrow \left(x + \frac{1}{x}\right)^2 = (4)^2$
$\Rightarrow \left(x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right)\right) = 16$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) + 2 = 16$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 16 - 2$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 14$

Therefore, the value of $x^2 + \frac{1}{x^2}$ is 14.

$\left(x^2 + \frac{1}{x^2}\right) = 14$
Squaring both the sides :
$\Rightarrow \left(x^4 + \frac{1}{x^4} + 2(x^2)\left(\frac{1}{x^2}\right)\right) = (14)^2$
$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) + 2 = 196$
$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 196 - 2$
$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 194$

Therefore, the value of $x^4 + \frac{1}{x^4}$ is 194.

Q12

Answer :

$(i) \ \left(x - \frac{1}{x}\right) = 5$
$\Rightarrow$ Squaring both the sides :
$\Rightarrow \left(x - \frac{1}{x}\right)^2 = (5)^2$
$\Rightarrow \left(x^2 + \frac{1}{x^2} - 2(x)\left(\frac{1}{x}\right)\right) = 25$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 2 = 25$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 25 + 2$
$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) = 27$
Therefore, the value of $\left(x^2 + \frac{1}{x^2}\right)$ is 27.

$\left(x^2 + \frac{1}{x^2}\right) = 27$

$\Rightarrow$ Squaring both the sides :

$\Rightarrow \left(x^4 + \frac{1}{x^4} - 2(x^2)\left(\frac{1}{x^2}\right)\right) = (27)^2$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) - 2 = 729$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 729 + 2$

$\Rightarrow \left(x^4 + \frac{1}{x^4}\right) = 731$

Therefore, the value of $\left(x^4 + \frac{1}{x^4}\right)$ is 731.

Q13

Answer :

(i) $(x+1)(x-1)(x^2+1)$

$\Rightarrow (x^2 - x + x - 1)(x^2+1)$

$\Rightarrow (x^2-1)(x^2+1)$

$\Rightarrow (x^2)^2 - (1^2)^2$ $\quad$ $\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow x^4 - 1.$

Therefore, the product of $(x+1)(x-1)(x^2+1)$ is $x^4 - 1$.

(ii) $(x-3)(x+3)(x^2+9)$

$\Rightarrow \left((x)^2 - (3)^2\right)(x^2+9)$ $\quad$ $\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow (x^2-9)(x^2+9)$

$\Rightarrow (x^2)^2 - (9)^2$ $\quad$ $\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow x^4 - 81$

Therefore, the product of $(x-3)(x+3)(x^2+9)$ is $x^4 - 81$.

(iii) $(3x-2y)(3x+2y)(9x^2+4y^2)$

$\Rightarrow \left((3x)^2 - (2y)^2\right)(9x^2+4y^2)$

$\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow (9x^2-4y^2)(9x^2+4y^2)$

$\Rightarrow (9x^2)^2 - (4y^2)^2$ $\quad$ $\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow 81x^4 - 16y^4.$

Therefore, the product of $(3x-2y)(3x+2y)(9x^2+4y^2)$ is $81x^4 - 16y^4$.

(iv) $(2p+3)(2p-3)(4p^2+9)$

$\Rightarrow \left((2p)^2 - (3)^2\right)(4p^2+9)$ $\quad$ $\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow (4p^2-9)(4p^2+9)$

$\Rightarrow (4p^2)^2 - (9)^2$ $\quad$ $\left[\text{according to the formula } a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow 16p^4 - 81.$

Therefore, the product of $(2p+3)(2p-3)(4p^2+9)$ is $16p^4 - 81$.

Q14

Answer :

$x+y = 12$

On squaring both the sides :

$\Rightarrow (x+y)^2 = (12)^2$

$\Rightarrow x^2+y^2+2xy = 144$

$\Rightarrow x^2+y^2 = 144 - 2xy$

Given :

$xy = 14$

$\Rightarrow x^2+y^2 = 144 - 2(14)$

$\Rightarrow x^2+y^2 = 144 - 28$

$\Rightarrow x^2+y^2 = 116$

Therefore, the value of x^2+y^2 is 116.

Q15

Answer :

$x-y = 7$

$\Rightarrow$ On squaring both the sides :

$\Rightarrow (x-y)^2 = (7)^2$

$\Rightarrow x^2+y^2-2xy = 49$

$\Rightarrow x^2+y^2 = 49 + 2xy$

Given :

$xy = 9$

$\Rightarrow x^2+y^2 = 49 + 2(9)$

$\Rightarrow x^2+y^2 = 49 + 18$

$\Rightarrow x^2+y^2 = 67.$

Therefore, the value of $x^2+y^2\ is\ 67$.