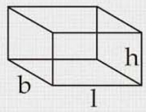
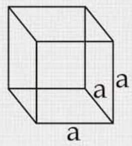
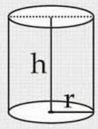
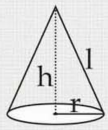
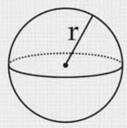
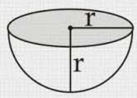
Volume and Surface Area of Solids
Ex 20.A

Name of the solid	Figure	Volume	Laterial/Curved Surface Area	Total Surface Area
Cuboid		lbh	2lh + 2bh or 2h(l+b)	2lh+2bh+2lb or 2(lh+bh+lb)
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi rh$	$2\pi rh + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3}\pi r^2 h$	πrl	$\pi rl + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3}\pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3}\pi r^3$	$2\pi r^2$	$2\pi r^2+\pi r^2$ or $3\pi r^2$

Q1.

Answer :

Volume of a cylinder = $\pi r^2 h$
Lateral surface$= 2\pi rh$
Total surface area $= 2\pi r(h+r)$

(i) Base radius = 7 cm; height = 50 cm
Now, we have the following:
Volume$= \frac{22}{7} \times 7 \times 7 \times 50 = 7700\ cm^3$
Lateral surface area$= 2\pi rh$$= 2 \times \frac{22}{7} \times 7 \times 50 = 2200\ cm^2$
Total surface area $= 2\pi r(h+r)$$= 2 \times \frac{22}{7} \times 7(50+7) = 2508\ cm^2$

(ii) Base radius = 5.6 m; height = 1.25 m
Now, we have the following:
Volume$= \frac{22}{7} \times 5.6 \times 5.6 \times 1.25 = 123.2\ m^3$
Lateral surface area$= 2\pi rh$$= 2 \times \frac{22}{7} \times 5.6 \times 1.25 = 44\ m^2$
Total surface area $= 2\pi r(h+r)$$= 2 \times \frac{22}{7} \times 5.6(1.25+5.6) = 241.12\ m^2$

(iii) Base radius = 14 dm = 1.4 m, height = 15 m
Now, we have the following:
Volume$= \frac{22}{7} \times 1.4 \times 1.4 \times 15 = 92.4\ m^3$
Lateral surface area$= 2\pi rh$$= 2 \times \frac{22}{7} \times 1.4 \times 15 = 132\ m^2$
Total surface area $= 2\pi r(h+r)$$= 2 \times \frac{22}{7} \times 1.4(15+1.4) = 144.32\ cm^2$

Q2.

Answer :

$r = 1.5\ \mathrm{m}$
$h = 10.5\ \mathrm{m}$
$\text{Capacity of the tank} = \text{volume of the tank} = \pi r^2 h = \frac{22}{7} \times 1.5 \times 1.5 \times 10.5 = 74$
$.25\ \mathrm{m}^3$
$\text{We know that } 1\ \mathrm{m}^3 = 1000\ \mathrm{L}$
$\therefore\ 74.25\ \mathrm{m}^3 = 74250\ \mathrm{L}$

Q3.

Answer :

Height = 7 m
Radius = 10 cm = 0.1 m
Volume$= \pi \mathrm{r}^2 \mathrm{h} = \frac{22}{7} \times 0.1 \times 0.1 \times 7 = 0.22\ \mathrm{m}^3$
Weight of wood = 225 kg/m^3
∴ Weight of the pole$= 0.22 \times 225 = 49.5\ kg$

Q4.

Answer :

Diameter = 2*r* = 140 cm
i.e., radius, *r* = 70 cm = 0.7 m

Now, volume $= \pi \mathrm{r}^2 \mathrm{h} = 1.54\ \mathrm{m}^3$

$\Rightarrow \frac{22}{7} \times 0.7 \times 0.7 \times h = 1.54$
$\therefore\ h = \frac{1.54 \times 7}{0.7 \times 0.7 \times 22} = \frac{154 \times 7}{154 \times 7} = 1\ m$

Q5.

Answer :

Volume$= \pi \mathrm{r}^2 \mathrm{h} = 3850\ \mathrm{cm}^3$
Height = 1 m =100 cm

Now, radius, $r = \sqrt{\frac{3850}{\pi \times \mathrm{h}}} = \sqrt{\frac{3850 \times 7}{22 \times 100}} = 1.75 \times 7 = 3.5\ cm$
∴ Diameter =2(radius) $= 2 \times 3.5 = 7\ cm$

Q6.

Answer :

Diameter = 14 m
Radius $= \frac{14}{2} = 7\ m$
Height = 5 m

∴ Area of the metal sheet required = total surface area

$= 2\pi \mathrm{r}\left(\mathrm{h} + \mathrm{r}\right)$
$= 2 \times \frac{22}{7} \times 7\left(5 + 7\right)\ m^2$
$= 44 \times 12\ m^2$
$= 528\ \mathrm{m}^2$

Q7.

Answer :

Circumference of the base = 88 cm
Height = 60 cm

Area of the curved surface $= circumference \times height = 88 \times 60 = 5280\ cm^2$
Circumference $= 2\pi \mathrm{r} = 88\ cm$
Then radius$= r = \frac{88}{2\pi} = \frac{88 \times 7}{2 \times 22} = 14\ cm$
∴ Volume$= \pi \mathrm{r}^2 \mathrm{h} = \frac{22}{7} \times 14 \times 14 \times 60 = 36960\ \mathrm{cm}^3$

Q8.

Answer :

Length = height = 14 m

Lateral surface area $= 2\pi \mathrm{rh} = 220\ \mathrm{m}^2$

Radius $= r = \frac{220}{2\pi \mathrm{h}} = \frac{220\times7}{2\times22\times14} = \frac{10}{4} = 2.5\ m$

$\therefore$ Volume $= \pi \mathrm{r}^2\mathrm{h} = \frac{22}{7} \times 2.5 \times 2.5 \times 14 = 275\ \mathrm{m}^3$

Q9.

Answer :

Height = 8 cm

Volume $= \pi \mathrm{r}^2\mathrm{h} = 1232\ \mathrm{cm}^3$

Now, radius $= r = \sqrt{\frac{1232}{\pi \mathrm{h}}} = \sqrt{\frac{1232\times7}{22\times8}} = \sqrt{49} = 7cm$

Also, curved surface area $= 2\pi \mathrm{rh} = 2 \times \frac{22}{7} \times 7 \times 8 = 352\ \mathrm{cm}^2$

$\therefore$ Total surface area

$= 2\pi \mathrm{r}\left(\mathrm{h}+\mathrm{r}\right) = \left(2 \times \frac{22}{7} \times 7 \times 8\right) + \left(2 \times \frac{22}{7} \times (7)^2\right) = 352 + 308 = 660\ \mathrm{cm}^2$

Q10.

Answer :

We have: $\frac{radius}{height} = \frac{7}{2}$

i.e., $r = \frac{7}{2}h$

Now, volume $= \pi \mathrm{r}^2\mathrm{h} = \pi\left(\frac{7}{2}\mathrm{h}\right)^2\mathrm{h} = 8316\ \mathrm{cm}^3$

$\Rightarrow \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times h^3 = 8316$

$\Rightarrow h^3 = \frac{8316\times2}{11\times7} = 108 \times 2 = 216$

$\Rightarrow h = \sqrt[3]{216} = 6\ cm$

Then $r = \frac{7}{2}h = \frac{7}{2} \times 6 = 21\ cm$

$\therefore$ Total surface area $= 2\pi \mathrm{r}\left(h+r\right) = 2 \times \frac{22}{7} \times 21 \times \left(6+21\right) = 3564\ \mathrm{cm}^2$

Q11.

Answer :

Curved surface area $= 2\pi rh = 4400\ cm^2$

Circumference $= 2\pi \mathrm{r} = 110\ \mathrm{cm}$

Now, height $= h = \frac{curved\ surface\ area}{circumference} = \frac{4400}{110} = 40\ cm$

Also, radius, $r = \frac{4400}{2\pi \mathrm{h}} = \frac{4400\times7}{2\times22\times40} = \frac{35}{2}$

$\therefore$ Volume $= \pi \mathrm{r}^2\mathrm{h} = \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \times 40 = 22 \times 5 \times 35 \times 10 = 38500\ \mathrm{cm}^3$

Q12.

Answer :

For the cubic pack:
Length of the side, a = 5 cm
Height = 14 cm
Volume= $a^2h = 5 \times 5 \times 14 = 350\ cm^3$

For the cylindrical pack:
Base radius = $r = 3.5\ cm$
Height = 12 cm
Volume= $\pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5 \times 12 = 462\ cm^3$

We can see that the pack with a circular base has a greater capacity than the pack with a square base.
Also, difference in volume= $462 - 350 = 112\ cm^3$

Q13.

Answer :

Diameter = 48 cm
Radius = 24 cm = 0.24 m
Height = 7 m

Now, we have:
Lateral surface area of one pillar= $\pi dh = \frac{22}{7} \times 0.48 \times 7 = 10.56\ m^2$
Surface area to be painted = total surface area of 15 pillars $= 10.56 \times 15 = 158.4\ m^2$
∴ Total cost= $\text{Rs}\ (158.4 \times 2.5) = \text{Rs}\ 396$

Q14.

Answer :

Volume of the rectangular vessel $= 22 \times 16 \times 14 = 4928\ cm^3$
Radius of the cylindrical vessel = 8 cm
Volume= $\pi r^2 h$

As the water is poured from the rectangular vessel to the cylindrical vessel, we have:
Volume of the rectangular vessel = volume of the cylindrical vessel

∴ Height of the water in the cylindrical vessel= $\frac{volume}{\pi r^2} = \frac{4928 \times 7}{22 \times 8 \times 8} = \frac{28 \times 7}{8} = \frac{49}{2} = 24.5\ cm$

Q15.

Answer :

Diameter of the given wire = 1 cm
Radius = 0.5 cm
Length = 11 cm
Now, volume= $\pi r^2 h = \frac{22}{7} \times 0.5 \times 0.5 \times 11 = 8.643\ cm^3$
The volumes of the two cylinders would be the same.
Now, diameter of the new wire = 1 mm = 0.1 cm
Radius = 0.05 cm
∴ New length $= \frac{\text{volume}}{\pi r^2} = \frac{8.643 \times 7}{22 \times 0.05 \times 0.05} = 1100.02\ cm \cong$ 11 m

Q16.

Answer :

Length of the edge, a = 2.2 cm
Volume of the cube = $a^3 = (2.2)^3 = 10.648\ cm^3$
Volume of the wire= $\pi r^2 h$
Radius = 1 mm = 0.1 cm
As volume of cube = volume of wire, we have:

$h = \frac{volume}{\pi r^2} = \frac{10.648 \times 7}{22 \times 0.1 \times 0.1} = 338.8\ cm$

Q17.

Answer :

Diameter = 7 m
Radius = 3.5 m
Depth = 20 m

Volume of the earth dug out $= \pi r^2 h = \frac{22}{7} \times 3.5 \times 3.5 \times 20 = 770\ \mathrm{m}^3$
Volume of the earth piled upon the given plot$= 28 \times 11 \times h = 770\ m^3$

$\therefore\ h = \frac{770}{28 \times 11} = \frac{70}{28} = 2.5\ m$

Q18.

Answer :

Inner diameter = 14 m
i.e., radius = 7 m
Depth = 12 m
Volume of the earth dug out$= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 12 = 1848\ \mathrm{m}^3$

Width of embankment = 7 m
Now, total radius $= 7 + 7 = 14\ m$

$$\begin{aligned}&\textbf{Volume of the embankment} = \textbf{total volume} - \textbf{inner volume}\\&= \pi r_o{}^2 h - \pi r_i{}^2 h = \pi h\left(r_o{}^2 - r_i{}^2\right)\\&= \frac{22}{7} h\left(14^2 - 7^2\right) = \frac{22}{7} h(196 - 49)\\&= \frac{22}{7} h \times 147 = 21 \times 22h\\&= 462 \times h\ m^3\end{aligned}$$

Since volume of embankment = volume of earth dug out, we have:
$1848 = 462\ h$
$\Rightarrow h = \frac{1848}{462} = 4\ m$
∴ Height of the embankment = 4 m

Q19.

Answer :

Diameter = 84 cm
i.e., radius = 42 cm
Length = 1 m = 100 cm
Now, lateral surface area $= 2\pi rh = 2 \times \frac{22}{7} \times 42 \times 100 = 26400\ \mathrm{cm}^2$
∴ Area of the road
$= \textbf{lateral surface area} \times \textbf{no. of rotations} = 26400 \times 750 = 19800000\ \mathrm{cm}^2 = 1980\ \mathrm{m}^2$

Q20.

Answer :

Thickness of the cylinder = 1.5 cm
External diameter = 12 cm
i.e., radius = 6 cm
also, internal radius = 4.5 cm
Height = 84 cm

Now, we have the following:
Total volume$= \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 84 = 9504\ \mathrm{cm}^3$
Inner volume $= \pi r^2 h = \frac{22}{7} \times 4.5 \times 4.5 \times 84 = 5346\ \mathrm{cm}^3$
Now, volume of the metal = total volume − inner volume $= 9504 - 5346 = 4158\ cm^3$
∴ Weight of iron $= 4158 \times 7.5 = 31185\ \mathrm{g} = 31.185\ \mathrm{kg}$ [Given: $1\ cm^3 = 7.5g$]

Q21.

Answer :

Length = 1 m = 100 cm
Inner diameter = 12 cm
Radius = 6 cm
Now, inner volume$= \pi r^2 h = \frac{22}{7} \times 6 \times 6 \times 100 = 11314.286 \text{ cm}^3$
Thickness = 1 cm
Total radius = 7 cm

Now, we have the following:
Total volume$= \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times 100 = 15400 \text{ cm}^3$
Volume of the tube $= \text{total volume} - \text{inner volume} = 15400 - 11314.286 = 4085.714\, cm^3$
Density of the tube = 7.7 g/cm^3
∴ Weight of the tube $= \text{volume} \times \text{density} = 4085.714 \times 7.7 = 31459.9978 \text{ g} = 31.459 \text{ kg}$