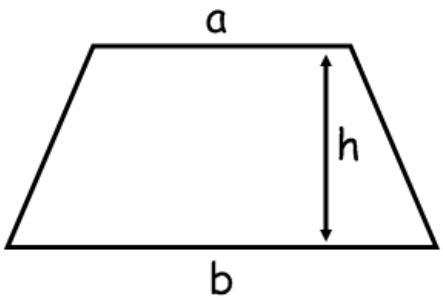
Area of Trapezium and Polygon
Ex 18B



Area of Trapezium $= \frac{1}{2}h(a+b)$

$\therefore$ Area of the trapezium = Area of the rectangle + Area of the triangle

$$= bh + \frac{1}{2}(a-b)h$$
$$= h\left[b + \frac{1}{2}(a-b)\right]$$
$$= h\left[\frac{2b}{2} + \frac{a-b}{2}\right]$$
$$= h\left[\frac{2b+a-b}{2}\right]$$
$$= h\left(\frac{a+b}{2}\right)$$
$$= \begin{pmatrix}\text{Half the sum of}\\ \text{parallel sides}\end{pmatrix} \times \begin{pmatrix}\text{Perpendicular distance}\\ \text{between the parallel sides}\end{pmatrix}$$

Q1.

Answer :

Area of quadrilateral ABCD = (Area of $\triangle$ ADC) + (Area of $\triangle$ ACB)

$$= \left(\tfrac{1}{2} \times AC \times DM\right) + \left(\tfrac{1}{2} \times AC \times BL\right)$$
$$= \left[\left(\tfrac{1}{2} \times 24 \times 7\right) + \left(\tfrac{1}{2} \times 24 \times 8\right)\right] \text{ cm}^2$$
$$= (84 + 96) \text{ cm}^2$$
$$= 180 \text{ cm}^2$$

Hence, the area of the quadrilateral is 180 cm^2.

Q2.

Answer :

Area of quadrilateral ABCD = (Area of $\triangle$ ABD) + (Area of $\triangle$ BCD)

$$= \left(\tfrac{1}{2} \times BD \times AL\right) + \left(\tfrac{1}{2} \times BD \times CM\right)$$
$$= \left[\left(\tfrac{1}{2} \times 36 \times 19\right) + \left(\tfrac{1}{2} \times 36 \times 11\right)\right] \text{ m}^2$$
$$= (342 + 198) \text{ m}^2$$
$$= 540 \text{ m}^2$$

Hence, the area of the field is 540 m^2.

Q3.

Answer :

Area of pentagon ABCDE = (Area of $\triangle$ AEN) + (Area of trapezium EDMN) + (Area of $\triangle$ DMC) + (Area of $\triangle$ ACB)

$$= \left(\tfrac{1}{2} \times AN \times EN\right) + \left(\tfrac{1}{2} \times (EN + DM) \times NM\right) + \left(\tfrac{1}{2} \times MC \times DM\right) + \left(\tfrac{1}{2} \times AC \times BL\right)$$
$$= \left(\tfrac{1}{2} \times AN \times EN\right) + \left(\tfrac{1}{2} \times (EN + DM) \times (AM - AN)\right) + \left(\tfrac{1}{2} \times (AC - AM) \times DM\right) + \left(\tfrac{1}{2} \times AC \times BL\right)$$
$$= \left[\left(\tfrac{1}{2} \times 6 \times 9\right) + \left(\tfrac{1}{2} \times (9 + 12) \times (14 - 6)\right) + \left(\tfrac{1}{2} \times (18 - 14) \times 12\right) + \left(\tfrac{1}{2} \times 18 \times 4\right)\right] \text{ cm}^2$$
$$= (27 + 84 + 24 + 36) \text{ cm}^2$$
$$= 171 \text{ cm}^2$$

Hence, the area of the given pentagon is 171 cm^2.

Q4.

Answer :

$$\text{Area of hexagon ABCDEF} = (\text{Area of}\triangle \text{AFP}) + (\text{Area of trapezium FENP}) + (\text{Area of}\triangle \text{ALB})$$
$$= \left(\tfrac{1}{2} \times \text{AP} \times \text{FP}\right) + \left(\tfrac{1}{2} \times (\text{FP} + \text{EN}) \times \text{PN}\right) + \left(\tfrac{1}{2} \times \text{ND} \times \text{EN}\right) + \left(\tfrac{1}{2} \times \text{MD} \times \text{CM}\right) + \left(\tfrac{1}{2} \times (\text{CM} + \text{BL}) \times \text{LM}\right) + \left(\tfrac{1}{2} \times \text{AL} \times \text{BL}\right)$$
$$= \left(\tfrac{1}{2} \times \text{AP} \times \text{FP}\right) + \left(\tfrac{1}{2} \times (\text{FP} + \text{EN}) \times (\text{PL} + \text{LN})\right) + \left(\tfrac{1}{2} \times (\text{NM} + \text{MD}) \times \text{CM}\right) + \left(\tfrac{1}{2} \times \text{MD} \times \text{CM}\right) + \left(\tfrac{1}{2} \times (\text{CM} + \text{BL}) \times (\text{LN} + \text{NM})\right) + \left(\tfrac{1}{2} \times (\text{AP} + \text{PL}) \times \text{BL}\right)$$
$$= \left[\left(\tfrac{1}{2} \times 6 \times 8\right) + \left(\tfrac{1}{2} \times (8 + 12) \times (2 + 8)\right) + \left(\tfrac{1}{2} \times (2 + 3) \times 12\right) + \left(\tfrac{1}{2} \times 3 \times 6\right) + \left(\tfrac{1}{2} \times (6 + 8) \times (8 + 2)\right) + \left(\tfrac{1}{2} \times (6 + 2) \times 8\right)\right] \text{ cm}^2$$
$$= (24 + 100 + 30 + 9 + 70 + 32) \text{ cm}^2$$
$$= 265 \text{ cm}^2$$

Hence, the area of the hexagon is 265 cm^2.

Q5.

Answer :

$$\text{Area of pentagon ABCDE} = (\text{Area of } \triangle \text{ABC}) + (\text{Area of } \triangle \text{ACD}) + (\text{Area of } \triangle \text{ADE})$$
$$= \left(\tfrac{1}{2} \times \text{AC} \times \text{BL}\right) + \left(\tfrac{1}{2} \times \text{AD} \times \text{CM}\right) + \left(\tfrac{1}{2} \times \text{AD} \times \text{EM}\right)$$
$$= \left[\left(\tfrac{1}{2} \times 10 \times 3\right) + \left(\tfrac{1}{2} \times 12 \times 7\right) + \left(\tfrac{1}{2} \times 12 \times 5\right)\right] \text{ cm}^2$$
$$= (15 + 42 + 30) \text{ cm}^2$$
$$= 87 \text{ cm}^2$$

Hence, the area of the pentagon is 87 cm^2.

Q6.

Answer :

$$\text{Area enclosed by the given figure} = (\text{Area of trapezium FEDC}) + (\text{Area of square ABCF})$$
$$= \left[\left\{\tfrac{1}{2} \times (6 + 20) \times 8\right\} + (20 \times 20)\right] \text{cm}^2$$
$$= (104 + 400) \text{cm}^2$$
$$= 504 \text{ cm}^2$$

Hence, the area enclosed by the figure is 504 cm^2.

Q7.

Answer :

We will find the length of AC.
From the right triangles ABC and HGF, we have :

$$\text{AC}^2 = \text{HF}^2 = \left\{(5)^2 - (4)^2\right\} \text{ cm}$$
$$= (25 - 16)cm$$
$$= 9\,cm$$
$$\text{AC} = \text{HF} = \sqrt{9}\,cm$$
$$= 3\,cm$$

$$\text{Area of the given figure ABCDEFGH} = (\text{Area of rectangle ADEH}) + (\text{Area of } \triangle \text{ABC}) + (\text{Area of } \triangle \text{HGF})$$
$$= (\text{Area of rectangle ADEH}) + 2(\text{Area of } \triangle \text{ABC})$$
$$= (\text{AD} \times \text{DE}) + 2(\text{Area of } \triangle \text{ABC})$$
$$= \{(\text{AC} + \text{CD}) \times \text{DE}\} + 2\left(\tfrac{1}{2} \times \text{BC} \times \text{AC}\right)$$
$$= \{(3 + 4) \times 8\} + 2\left(\tfrac{1}{2} \times 4 \times 3\right) \text{ cm}^2$$
$$= (56 + 12) \text{ cm}$$
$$= 68 \text{ cm}^2$$

Hence, the area of the given figure is 68 cm^2.

Q8.

Answer :

Let AL = DM = x cm
LM = BC = 13 cm
$\therefore x + 13 + x = 23$
$\Rightarrow 2x + 13 = 23$
$\Rightarrow 2x = (23 - 13)$
$\Rightarrow 2x = 10$
$\Rightarrow x = 5$
$\therefore$ AL = 5 cm
From the right $\triangle$ AFL, we have :
$FL^2 = AF^2 - AL^2$
$\Rightarrow FL^2 = \left\{\left(13^2\right) - (5)^2\right\}$
$\Rightarrow FL^2 = (169 - 25)$
$\Rightarrow FL^2 = 144$
$\Rightarrow FL = \sqrt{144}$
$\Rightarrow FL = 12\ cm$
$\therefore$ FL = BL = 12 cm
Area of a regular hexagon = (*A*rea of *the* trapezium ADEF)

Area of a regular hexagon = (*A*rea of *the* trapezium ADEF)
+ (*A*rea of *the* trapezium ABCD)

$$= 2(Area\ of\ trapezium\ ADEF)$$
$$= 2\left\{\frac{1}{2} \times (AD + EF) \times FL\right\}$$
$$= 2\left\{\frac{1}{2} \times (23 + 13) \times 12\right\} cm^2$$
$$= 2\left(\frac{1}{2} \times 36 \times 12\right) cm^2$$
$$= 432\ cm^2$$

Hence, the area of the given regular hexagon is 432 cm^2.