Time and Work

Ex 13A

1. **Work from Days:**
   If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.
2. **Days from Work:**
   If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.
3. **Ratio:**
   If A is thrice as good a workman as B, then:
   Ratio of work done by A and B = 3 : 1.
   Ratio of times taken by A and B to finish a work = 1 : 3.

Q1.

Answer :

Work done by Rajan in 1 day $= \frac{1}{24}$

Work done by Amit in 1 day $= \frac{1}{30}$

Work done by Amit and Rajan together in 1 day $= \frac{1}{24} + \frac{1}{30} = \frac{54}{720} = \frac{3}{40}$

Q2. $\therefore$ They can complete the work in $\frac{40}{3}$ days, i.e., $13\frac{1}{3}$ days if they work together.

Answer :

Time taken by Ravi $= 15$ h

Time taken by Raman $= 12$ h

Work done per hour by Ravi $= \frac{1}{15}$

Work done per hour by Raman $= \frac{1}{12}$

Work done per hour by Ravi and Raman together $= \frac{1}{15} + \frac{1}{12} = \frac{9}{60} = \frac{3}{20}$

$\therefore$ Time taken by Ravi and Raman together to finish the work $= \frac{20}{3}$ h $= 6\frac{2}{3}$ h

Q3.

Answer :

Time taken by A and B to finish a piece of work $= 6$ days

Work done per day by A and B $= \frac{1}{6}$

Time taken by A alone $= 9$ days

Work done per day by A alone $= \frac{1}{9}$

Work done per day by B $= \left(\text{work done by A and B}\right) - \left(\text{work done by A}\right)$

$= \frac{1}{6} - \frac{1}{9} = \frac{3-2}{18} = \frac{1}{18}$

$\therefore$ B alone will take 18 days to complete the work.

Q4.

Answer :

Time taken by Raju $= 15$ h

Work done by Raju in 1 h $= \frac{1}{15}$

Time taken by Raju and Siraj working together $= 6$ h

Work done by Raju and Siraj in 1 h $= \frac{1}{6}$

Work done by Siraj in 1 h $= \left(\text{work done by Raju and Siraj}\right)$

$- \left(\text{work done by Raju}\right)$

$= \frac{1}{6} - \frac{1}{15} = \frac{5-2}{30} = \frac{3}{30} = \frac{1}{10}$

$\therefore$ Siraj will take 10 h to overhaul the scooter by himself.

Q5.

Answer :

Time taken by A to complete the work $= 10$ days

Time taken by B to complete the work $= 12$ days

Time taken by C to complete the work $= 15$ days

Work done per day by A $= \frac{1}{10}$

Work done per day by B $= \frac{1}{12}$

Work done per day by C $= \frac{1}{15}$

Total work done per day $= \frac{1}{10} + \frac{1}{12} + \frac{1}{15} = \frac{6+5+4}{60} = \frac{15}{60} = \frac{1}{4}$

A, B and C will take 4 days to complete the work if they work together.

Q6.

Answer :

Time taken by A to complete the piece of work $= 24$ h
Work done per hour by A $= \frac{1}{24}$
Time taken by B to complete the work $= 16$ h
Work done per hour by B $= \frac{1}{16}$
Total time taken when A, B and C work together $= 8$ h
Work done per hour by A, B and C $= \frac{1}{8}$
Work done per hour by A, B and C $= \left(\text{work done per hour by A}\right) +$
$\left(\text{work done per hour by B}\right) + \left(\text{work done per hour by C}\right)$
$\left(\text{Work done per hour by C}\right) = \left(\text{work done per hour by A, B and C}\right) -$
$\left(\text{work done per hour by A}\right) - \left(\text{work done per hour by B}\right)$
$= \frac{1}{8} - \frac{1}{24} - \frac{1}{16} = \frac{6-2-3}{48} = \frac{1}{48}$
Thus, C alone will take 48 h to complete the work.

Q7.

Answer :

A can complete the work in 20 h.
Work done per hour by A $= \frac{1}{20}$
B can complete the work in 24 h.
Work done per hour by B $= \frac{1}{24}$
It takes 8 h to complete the work if A, B and C work together.
Work done together per hour by A, B and C $= \frac{1}{8}$
$\left(\text{Work done per hour by A, B and C}\right) = \left(\text{work done per hour by A}\right)$
$+ \left(\text{work done per hour by B}\right) + \left(\text{work done per hour by C}\right)$
OR
$\left(\text{Work done per hour by C}\right) = \left(\text{work done per hour by A, B and C}\right)$
$- \left(\text{work done per hour by A}\right) - \left(\text{work done per hour by B}\right)$
$= \frac{1}{8} - \frac{1}{24} - \frac{1}{20} = \frac{1}{30}$
$\therefore$ C alone will take 30 h to complete the work.

Q8.

Answer :

Time taken by A to complete the work $= 16$ days
Work done per day by A $= \frac{1}{16}$
Time taken by B to complete the work $= 12$ days
Work done per day by B $= \frac{1}{12}$
Work done per day by A and B $= \frac{1}{12} + \frac{1}{16} = \frac{4+3}{48} = \frac{7}{48}$
Work done by A in two days $= \frac{2}{16} = \frac{1}{8}$
Work left $= 1 - \frac{1}{8} = \frac{7}{8}$
A and B together can complete $\frac{7}{48}$ of the work in 1 day.
Then, time taken to complete $\frac{7}{8}$ of the work $= \frac{7}{8} \div \frac{7}{48} = \frac{7}{8} \times \frac{48}{7} = 6$ days
$\therefore$ Total time taken $= 6 + 2 = 8$ days.

Q9.

Answer :

Time taken by A to complete the work $= 14$ days
Work done by A in one day $= \frac{1}{14}$
Time taken by B to complete the work $= 21$ days
Work done by B in one day $= \frac{1}{21}$
Work done jointly by A and B in one day $= \frac{1}{14} + \frac{1}{21} = \frac{3+2}{42} = \frac{5}{42}$
Work done by A and B in 6 days $= \frac{5}{42} \times 6 = \frac{5}{7}$
Work left $= 1 - \frac{5}{7} = \frac{2}{7}$
With B working alone, time required to complete the work $= \frac{2}{7} \div \frac{1}{21} = \frac{2}{7} \times 21 = 2 \times 3 = 6$ days
So, the total time taken to complete the work $= 6 + 6 = 12$ days

Q10.

Answer :

A can do $\frac{2}{3}$ work in 16 days
So, work done by A in one day $= \frac{2}{48} = \frac{1}{24}$
B can do $\frac{1}{4}$ work in 3 days
So, work done by B in one day $= \frac{1}{12}$
Work done jointly by A and B in one day $= \frac{1}{24} + \frac{1}{12} = \frac{1+2}{24} = \frac{3}{24} = \frac{1}{8}$
So, A and B together will take 8 days to complete the work.

Q11.

Answer :

Time taken by A $= 15$ days
Time taken by B $= 12$ days
Time taken by C $= 20$ days
Work d by A in one day $= \frac{1}{15}$
Work done by B in one day $= \frac{1}{12}$
Work done by C in one day $= \frac{1}{20}$
Work done in one day by A, B and C together $= \frac{1}{15} + \frac{1}{12} + \frac{1}{20} = \frac{4+5+3}{60} = \frac{12}{60} = \frac{1}{5}$
Work done by A, B and C together in 2 days $= \frac{2}{5}$
Work remaining $= 1 - \frac{2}{5} = \frac{3}{5}$
Work done by A and B in one day $= \frac{1}{15} + \frac{1}{12} = \frac{9}{60} = \frac{3}{20}$
Time required by A and B to complete the remaining work together $= \frac{3}{5} \div \frac{3}{20} = \frac{3}{5} \times \frac{20}{3} = 4$ days

Q12.

Answer :

Time needed by A and B to finish the work $= 18$ days
Time needed by B and C to finish the work $= 24$ days
Time needed by C and A to finish the work $= 36$ days
Work done by A and B in one day $= \frac{1}{18}$
Work done by B and C in one day $= \frac{1}{24}$
Work done by C and A in one day $= \frac{1}{36}$
$2 \times$ Work done by A, B and C in one day $= \frac{1}{18} + \frac{1}{24} + \frac{1}{36} = \frac{4+3+2}{72} = \frac{9}{72} = \frac{1}{8}$
$\therefore$ Work done by A, B and C in one day $= \frac{1}{16}$
So, A, B and C working together will take 16 days to complete the work.

Q13.

Answer :

$(A+B)$ can complete the work in 12 days.

$(B+C)$ can complete the work in 15 days.

$(C+A)$ can complete the work in 20 days.

$(A+B)$'s 1 day work $= \frac{1}{12}$

$(B+C)$'s 1 day work $= \frac{1}{15}$

$(C+A)$'s 1 day work $= \frac{1}{20}$

$2(A+B+C)$'s 1 day work $= \frac{1}{12} + \frac{1}{15} + \frac{1}{20} = \frac{5+4+3}{60} = \frac{12}{60} = \frac{1}{5}$

$(A+B+C)$'s 1 day work $= \frac{1}{10}$

A's 1 day work $= \{(A+B+C)$'s 1 day work$\} - \{(B+C)$'s 1 day work$\} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30} = \frac{1}{30}$

A will take 30 days to complete the work, if he works alone.

Q14.

Answer :

A can fill a tank in 10 hours.
B can fill a tank in 15 hours.
Pipe A fills $\frac{1}{10}$ of the tank in one hour.

Pipe B fills $\frac{1}{15}$ of the tank in one hour.

Part of tank filled by pipes A and B together $= \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30} = \frac{1}{6}$

Thus, pipes A and B require 6 hours to fill the tank.

Q15.

Answer :

Pipe A can fill a tank in 5 hours.
Pipe B can empty a full tank in 6 hours.

Pipe A fills $\frac{1}{5}$ of the tank in one hour.

Pipe B empties $\frac{1}{6}$ of the tank in one hour.

Part of the tank filled in one hour using both pipes A and B $= \frac{1}{5} - \frac{1}{6} = \frac{6-5}{30} = \frac{1}{30}$

It takes $\frac{30}{1}$ or 30 hours to fill the tank completely.

Q16.

Answer :

Time taken by tap A to fill the tank $= 6$ hours
Time taken by tap B to fill the tank $= 8$ hours
Time taken by tap C to fill the tank $= 12$ hours

A fills $\frac{1}{6}$ of the tank in one hour.

B fills $\frac{1}{8}$ of the tank in one hour.

C fills $\frac{1}{12}$ of the tank in one hour.

Part of the tank filled in one hour using all the three pipes $= \frac{1}{6} + \frac{1}{8} + \frac{1}{12} = \frac{4+3+2}{24} = \frac{9}{24}$

Time taken by A, B and C together to fill the tank $= \frac{24}{9} = \frac{8}{3} = 2\frac{2}{3}$ hours

Q17.

Answer :

Inlet A can fill the cistern in 12 minutes.
Inlet B can fill the cistern in 15 minutes.
Outlet C empties the filled cistern in 10 minutes.

Part of the cistern filled by inlet A in one minute $= \frac{1}{12}$

Part of the cistern filled by inlet B in one minute $= \frac{1}{15}$

Part of the cistern emptied by outlet C in one minute $= -\frac{1}{10}$

(water flows out from C and empties the cistern)

Part of the cistern filled in one minute with A, B and C working together $= \frac{1}{12} + \frac{1}{15} - \frac{1}{10}$

$= \frac{5+4-6}{60} = \frac{3}{60} = \frac{1}{20}$

The time required to fill the cistern with all inlets, A, B and C, open is 20 minutes.

Q18.

Answer :

A pipe can fill a cistern in 9 hours.

Part of the cistern filled by the pipe in one hour $= \frac{1}{9}$

Let the leak empty the cistern in x hours.

Part of the cistern emptied by the leak in one hour $= -\frac{1}{x}$

(The leak drains out the water)

Considering the leak, the tank is filled in 10 hours.

Part of the tank filled in one hour $= \frac{1}{10}$

Therefore,

$\frac{1}{9} - \frac{1}{x} = \frac{1}{10}$ or, $\frac{1}{x} = \frac{1}{9} - \frac{1}{10} = \frac{10-9}{90} = \frac{1}{90}$ x $= 90$

The leak will empty the filled cistern in 90 hours.

Q19.

Answer :

Pipe A can fill a cistern in 6 hours.
Pipe B can fill a cistern in 8 hours.

Part of the cistern filled by pipe A in one hour $= \frac{1}{6}$

Part of the cistern filled by pipe B in one hour $= \frac{1}{8}$

Part of the cistern filled by pipes A and B in one hour $= \frac{1}{6} + \frac{1}{8} = \frac{4+3}{24} = \frac{7}{24}$

Part of the cistern filled by pipes A and B in 2 hours $= \frac{7}{24} \times 2 = \frac{7}{12}$

Part of the tank empty after 2 hours $= 1 - \frac{7}{12} = \frac{5}{12}$

Time taken by pipe B to fill the remaining tank $= \frac{5}{12} \div \frac{1}{8} = \frac{5}{12} \times 8 = \frac{10}{3} = 3\frac{1}{3}$ hours