Rational Numbers Ex.A

RATIONAL NUMBERS

A <u>rational number</u> is a number that can be expressed as a fraction (ratio) in the form $^a/_b$, where a and b are integers and b is not zero.

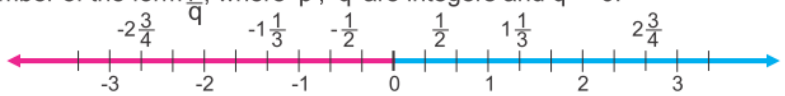
Examples: ½, 8, $^5/_3$, √4, $7^1/_9$, -12, 6.25, $0.31\overline{1}$

When a rational number fraction is divided to form a decimal value, it becomes a terminating or repeating decimal.

$^2/_5$ can be represented as $2\overline{)5.0}$ = 0.4 which is a terminating decimal.

$^1/_3$ can be represented as $1\overline{)3.0}$ = $0.\overline{3}$ which is a repeating decimal.

A number of the form $\frac{p}{q}$, where 'p', 'q' are integers and q ≠ 0.



Property	Operations on Rational Numbers			
Name	Addition	Subtraction	Multiplication	Division*
Closure	a + b ∈ Q	a - b ∈ Q	a × b ∈ Q	a ÷ b ∈ Q
Commutative	a + b = b + a	a - b ≠ b - a	a × b = b × a	a ÷ b ≠ b ÷ a
Associative	(a + b) + c = a + (b + c)	(a - b) - c ≠ a - (b - c)	(a × b) × c = a × (b × c)	(a ÷ b) ÷ c ≠ a ÷ (b ÷ c)
Distributive	a × (b + c) = ab + ac	a × (b - c) = ab - ac	Not applicable	Not applicable
Where a, b, c ∈ Q (set of rational numbers), *b is a non-zero rational number				

Q1.

Answer :

If $\frac{a}{b}$ is a fraction and m is a non-zero integer, then $\frac{a}{b} = \frac{a\times m}{b\times m}$.

Now,

(i) $\frac{-3}{5} = \frac{-3\times4}{5\times4} = \frac{-12}{20}$

(ii) $\frac{-3}{5} = \frac{-3\times-6}{5\times-6} = \frac{18}{-30}$

(iii) $\frac{-3}{5} = \frac{-3\times7}{5\times7} = \frac{-21}{35}$

(iv) $\frac{-3}{5} = \frac{-3\times-8}{5\times-8} = \frac{24}{-40}$

Q2.

Answer :

If $\frac{a}{b}$ is a rational number and m is a common divisor of a and b, then $\frac{a}{b} = \frac{a\div m}{b\div m}$.

$\therefore \frac{-42}{98} = \frac{-42\div14}{98\div14} = \frac{-3}{7}$

Q3.

Answer :

If $\frac{a}{b}$ is a rational integer and m is a common divisor of a and b, then $\frac{a}{b} = \frac{a\div m}{b\div m}$.

$\therefore \frac{-48}{60} = \frac{-48\div12}{60\div12} = \frac{-4}{5}$

Q4.

Answer :

A rational number $\frac{a}{b}$ is said to be in the standard form if a and b have no common divisor other than unity and $b > 0$.
Thus,

(i) The greatest common divisor of 12 and 30 is 6.

$\therefore \frac{-12}{30} = \frac{-12 \div 6}{30 \div 6} = \frac{-2}{5}$ (In the standard form)

(ii)The greatest common divisor of 14 and 49 is 7.

$\therefore \frac{-14}{49} = \frac{-14 \div 7}{49 \div 7} = \frac{-2}{7}$ (In the standard form)

(iii) $\frac{24}{-64} = \frac{24 \times (-1)}{-64 \times -1} = \frac{-24}{64}$

The greatest common divisor of 24 and 64 is 8.

$\therefore \frac{-24}{64} = \frac{-24 \div 8}{64 \div 8} = \frac{-3}{8}$ (In the standard form)

(iv) $\frac{-36}{-63} = \frac{-36 \times (-1)}{-63 \times -1} = \frac{36}{63}$

The greatest common divisor of 36 and 63 is 9.

$\therefore \frac{36}{63} = \frac{36 \div 9}{63 \div 9} = \frac{4}{7}$ (In the standard form)

Q5.
Answer :

We know:
(i) Every positive rational number is greater than 0.
(ii) Every negative rational number is less than 0.

Thus, we have:

(i)$\frac{3}{8}$ is a positive rational number.
$\therefore \frac{3}{8} > 0$

(ii)$\frac{-2}{9}$ is a negative rational number.
$\therefore \frac{-2}{9} < 0$

(iii) $\frac{-3}{4}$ is a negative rational number.
$\therefore \frac{-3}{4} < 0$
Also,
$\frac{1}{4}$ is a positive rational number.
$\therefore \frac{1}{4} > 0$
Combining the two inequalities, we get:
$\frac{-3}{4} < \frac{1}{4}$

(iv)Both $\frac{-5}{7}$ and $\frac{-4}{7}$ have the same denominator, that is, 7.

So, we can directly compare the numerators.

$\therefore \frac{-5}{7} < \frac{-4}{7}$

(v)The two rational numbers are $\frac{2}{3}$ and $\frac{3}{4}$.

The LCM of the denominators 3 and 4 is 12.

Now,

$\frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12}$

Also,

$\frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$

Further

$\frac{8}{12} < \frac{9}{12}$

$\therefore \frac{2}{3} < \frac{3}{4}$

(vi)The two rational numbers are $\frac{-1}{2}$ and -1.

We can write $-1 = \frac{-1}{1}$.

The LCM of the denominators 2 and 1 is 2.

Now,

$\frac{-1}{2} = \frac{-1\times1}{2\times1} = \frac{-1}{2}$

Also,

$\frac{-1}{1} = \frac{-1\times2}{1\times2} = \frac{-2}{2}$

$\because \frac{-2}{1} < \frac{-1}{1}$

$\therefore -1 < \frac{-1}{2}$

Q6.

Answer :

1. The two rational numbers are $\frac{-4}{3}$ and $\frac{-8}{7}$.

The LCM of the denominators 3 and 7 is 21.

Now,

$\frac{-4}{3} = \frac{-4\times7}{3\times7} = \frac{-28}{21}$

Also,

$\frac{-8}{7} = \frac{-8\times3}{7\times3} = \frac{-24}{21}$

Further,

$\frac{-28}{21} < \frac{-24}{21}$

$\therefore \frac{-4}{3} < \frac{-8}{7}$

2. The two rational numbers are $\frac{7}{-9}$ and $\frac{-5}{8}$.

The first fraction can be expressed as $\frac{7}{-9} = \frac{7\times -1}{-9\times -1} = \frac{-7}{9}$.

The LCM of the denominators 9 and 8 is 72.

Now,

$\frac{-7}{9} = \frac{-7\times 8}{9\times 8} = \frac{-56}{72}$

Also,

$\frac{-5}{8} = \frac{-5\times 9}{8\times 9} = \frac{-45}{72}$

Further,

$\frac{-56}{72} < \frac{-45}{72}$

$\therefore \frac{7}{-9} < \frac{-5}{8}$

3. The two rational numbers are $\frac{-1}{3}$ and $\frac{4}{-5}$.

$\frac{4}{-5} = \frac{4\times -1}{-5\times -1} = \frac{-4}{5}$

The LCM of the denominators 3 and 5 is 15.

Now,

$\frac{-1}{3} = \frac{-1\times 5}{3\times 5} = \frac{-5}{15}$

Also,

$\frac{-4}{5} = \frac{-4\times 3}{5\times 3} = \frac{-12}{15}$

Further,

$\frac{-12}{15} < \frac{-5}{15}$

$\therefore \frac{4}{-5} < \frac{-1}{3}$

4. The two rational numbers are $\frac{9}{-13}$ and $\frac{7}{-12}$.

Now, $\frac{9}{-13} = \frac{9\times -1}{-13\times -1} = \frac{-9}{13}$ and $\frac{7}{-12} = \frac{7\times -1}{-12\times -1} = \frac{-7}{12}$

The LCM of the denominators 13 and 12 is 156.

Now,

$\frac{-9}{13} = \frac{-9\times 12}{13\times 12} = \frac{-108}{156}$

Also,

$\frac{-7}{12} = \frac{-7\times 13}{12\times 13} = \frac{-91}{156}$

Further,

$\frac{-108}{156} < \frac{-91}{156}$

$\therefore \frac{9}{-13} < \frac{7}{-12}$

5. The two rational numbers are $\frac{4}{-5}$ **and** $\frac{-7}{10}$.

$\therefore \frac{4}{-5} = \frac{4\times-1}{-5\times-1} = \frac{-4}{5}$

The LCM of the denominators 5 and 10 is 10.

Now,

$\frac{-4}{5} = \frac{-4\times2}{5\times2} = \frac{-8}{10}$

Also,

$\frac{-7}{10} = \frac{-7\times1}{10\times1} = \frac{-7}{10}$

Further,

$\frac{-8}{10} < \frac{-7}{10}$

$\therefore \frac{-4}{5} < \frac{-7}{10}$, *or*, $\frac{4}{-5} < \frac{-7}{10}$

6. The two rational numbers are $\frac{-12}{5}$ **and** -3.
-3 **can be written as** $\frac{-3}{1}$.

The LCM of the denominators is 5.

Now,

$\frac{-3}{1} = \frac{-3\times5}{1\times5} = \frac{-15}{5}$

Because $\frac{-15}{5} < \frac{-12}{5}$, we can conclude that $-3 < \frac{-12}{5}$.

Q7.
Thus,

$\frac{-3}{7} > \frac{6}{-13}$

(ii) We will write each of the given numbers with positive denominators.

One number = $\frac{5}{-13} = \frac{5\times(-1)}{-13\times(-1)} = \frac{-5}{13}$

Other number = $\frac{-35}{91}$

LCM of 13 and 91 = 91

$\therefore \frac{-5}{13} = \frac{-5\times7}{13\times7} = \frac{-35}{91}$ and $\frac{-35}{91}$

Clearly,

$-35 = -35$

$\therefore \frac{-35}{91} = \frac{-35}{91}$

Thus,

$\frac{-5}{13} = \frac{-35}{91}$

(iii) We will write each of the given numbers with positive denominators.

One number = -2

We can write -2 as $\frac{-2}{1}$.
Other number = $\frac{-13}{5}$

LCM of 1 and 5 = 5

$\therefore \frac{-2}{1} = \frac{-2\times5}{1\times5} = \frac{-10}{5}$ and $\frac{-13}{5} = \frac{-13\times1}{5\times1} = \frac{-13}{5}$

Clearly,

$-10 > -13$

$\therefore \frac{-10}{5} > \frac{-13}{5}$

Thus,

$\frac{-2}{1} > \frac{-13}{5}$

$-2 > \frac{-13}{5}$

(iv) We will write each of the given numbers with positive denominators.

One number = $\frac{-2}{3}$
Other number = $\frac{5}{-8} = \frac{5\times(-1)}{-8\times(-1)} = \frac{-5}{8}$

LCM of 3 and 8 = 24

$\therefore \frac{-2}{3} = \frac{-2\times8}{3\times8} = \frac{-16}{24}$ and $\frac{-5}{8} = \frac{-5\times3}{8\times3} = \frac{-15}{24}$

Clearly,

$-16 < -15$

$\therefore \frac{-16}{24} < \frac{-15}{24}$

Thus,

$\frac{-2}{3} < \frac{-5}{8}$
$\frac{-2}{3} < \frac{5}{-8}$

(v) $\frac{-3}{-5} = \frac{-3\times-1}{-5\times-1} = \frac{3}{5}$

$\frac{3}{5}$ is a positive number.

Because every positive rational number is greater than 0, $\frac{3}{5} > 0 \Rightarrow 0 < \frac{3}{5}$.

(vi) We will write each of the given numbers with positive denominators.

One number = $\frac{-8}{9}$

Other number = $\frac{-9}{10}$

LCM of 9 and 10 = 90

$\therefore \frac{-8}{9} = \frac{-8\times10}{9\times10} = \frac{-80}{90}$ and $\frac{-9}{10} = \frac{-9\times9}{10\times9} = \frac{-81}{90}$

Clearly,

$-81 < -80$

$\therefore \frac{-81}{90} < \frac{-80}{90}$

Thus,

$\frac{-9}{10} < \frac{-8}{9}$

Q8.

Answer :

(i) We will write each of the given numbers with positive denominators.

We have:

$\frac{4}{-9} = \frac{4\times(-1)}{-9\times(-1)} = \frac{-4}{9}$ and $\frac{7}{-18} = \frac{7\times(-1)}{-18\times(-1)} = \frac{-7}{18}$

Thus, the given numbers are $\frac{-4}{9}, \frac{-5}{12}, \frac{-7}{18}$ and $\frac{-2}{3}$.

LCM of 9, 12, 18 and 3 is 36.

Now,

$\frac{-4}{9} = \frac{-4\times4}{9\times4} = \frac{-16}{36}$

$\frac{-5}{12} = \frac{-5\times3}{12\times3} = \frac{-15}{36}$

$\frac{-7}{18} = \frac{-7\times2}{18\times2} = \frac{-14}{36}$

$\frac{-2}{3} = \frac{-2\times12}{3\times12} = \frac{-24}{36}$

Clearly,

$\frac{-24}{36} < \frac{-16}{36} < \frac{-15}{36} < \frac{-14}{36}$

$\therefore \frac{-2}{3} < \frac{-4}{9} < \frac{-5}{12} < \frac{-7}{18}$

That is

$\frac{-2}{3} < \frac{4}{-9} < \frac{-5}{12} < \frac{7}{-18}$

(ii) We will write each of the given numbers with positive denominators.

We have:

$\frac{5}{-12} = \frac{5\times(-1)}{-12\times(-1)} = \frac{-5}{12}$ and $\frac{9}{-24} = \frac{9\times(-1)}{-24\times(-1)} = \frac{-9}{24}$

Thus, the given numbers are $\frac{-3}{4}, \frac{-5}{12}, \frac{-7}{16}$ and $\frac{-9}{24}$.

LCM of 4, 12, 16 and 24 is 48.

Now,

$\frac{-3}{4}=\frac{-3\times12}{4\times12}=\frac{-36}{48}$

$\frac{-5}{12}=\frac{-5\times4}{12\times4}=\frac{-20}{48}$

$\frac{-7}{16}=\frac{-7\times3}{16\times3}=\frac{-21}{48}$

$\frac{-9}{24}=\frac{-9\times2}{24\times2}=\frac{-18}{48}$

Clearly,

$\frac{-36}{48}<\frac{-21}{48}<\frac{-20}{48}<\frac{-18}{48}$

$\therefore \frac{-3}{4}<\frac{-7}{16}<\frac{-5}{12}<\frac{-9}{24}$

That is

$\frac{-3}{4}<\frac{-7}{16}<\frac{5}{-12}<\frac{9}{-24}$

(iii) We will write each of the given numbers with positive denominators.

We have:

$\frac{3}{-5}=\frac{3\times(-1)}{-5\times(-1)}=\frac{-3}{5}$

Thus, the given numbers are $\frac{-3}{5}$, $\frac{-7}{10}$, $\frac{-11}{15}$ and $\frac{-13}{20}$.

LCM of 5, 10, 15 and 20 is 60.

Now,

$\frac{-3}{5}=\frac{-3\times12}{5\times12}=\frac{-36}{60}$

$\frac{-7}{10}=\frac{-7\times6}{10\times6}=\frac{-42}{60}$

$\frac{-11}{15}=\frac{-11\times4}{15\times4}=\frac{-44}{60}$

$\frac{-13}{20}=\frac{-13\times3}{20\times3}=\frac{-39}{60}$

Clearly,

$\frac{-44}{60}<\frac{-42}{60}<\frac{-39}{60}<\frac{-36}{60}$

$\therefore \frac{-11}{15}<\frac{-7}{10}<\frac{-13}{20}<\frac{-3}{5}$.

That is

$\frac{-11}{15}<\frac{-7}{10}<\frac{-13}{20}<\frac{3}{-5}$

(iv) We will write each of the given numbers with positive denominators.

We have:

$\frac{13}{-28}=\frac{13\times(-1)}{-28\times(-1)}=\frac{-13}{28}$

Thus, the given numbers are $\frac{-4}{7}$, $\frac{-9}{14}$, $\frac{-13}{28}$ and $\frac{-23}{42}$.

LCM of 7, 14, 28 and 42 is 84.

Now,

$\frac{-4}{7} = \frac{-4\times12}{7\times12} = \frac{-48}{84}$

$\frac{-9}{14} = \frac{-9\times6}{14\times6} = \frac{-54}{84}$

$\frac{-13}{28} = \frac{-13\times3}{28\times3} = \frac{-39}{84}$

$\frac{-23}{42} = \frac{-23\times2}{42\times2} = \frac{-46}{84}$

Clearly,

$\frac{-54}{84} < \frac{-48}{84} < \frac{-46}{84} < \frac{-39}{84}$

$\therefore \frac{-9}{14} < \frac{-4}{7} < \frac{-23}{42} < \frac{-13}{28}$.

That is

$\frac{-9}{14} < \frac{-4}{7} < \frac{-23}{42} < \frac{13}{-28}$

Q9.

Answer :

(i) We will first write each of the given numbers with positive denominators. We have:

$\frac{8}{-3} = \frac{8\times(-1)}{-3\times(-1)} = \frac{-8}{3}$

Thus, the given numbers are $-2,\ \frac{-13}{6},\ \frac{-8}{3}$ **and** $\frac{1}{3}$

LCM of 1, 6, 3 and 3 is 6

Now,

$\frac{-2}{1} = \frac{-2\times6}{1\times6} = \frac{-12}{6}$

$\frac{-13}{6} = \frac{-13\times1}{6\times1} = \frac{-13}{6}$

$\frac{-8}{3} = \frac{-8\times2}{3\times2} = \frac{-16}{6}$

and

$\frac{1}{3} = \frac{1\times2}{3\times2} = \frac{2}{6}$

Clearly,Thus,

$\frac{2}{6} > \frac{-12}{6} > \frac{-13}{6} > \frac{-16}{6}$

$\therefore \frac{1}{3} > -2 > \frac{-13}{6} > \frac{-8}{3}$. i.e $\frac{1}{3} > -2 > \frac{-13}{6} > \frac{8}{-3}$

(ii) We will first write each of the given numbers with positive denominators. We have:

$\frac{7}{-15} = \frac{7\times(-1)}{-15\times(-1)} = \frac{-7}{15}$ and $\frac{17}{-30} = \frac{17\times(-1)}{-30\times(-1)} = \frac{-17}{30}$

Thus, the given numbers are $\frac{-3}{10}, \frac{-7}{15}, \frac{-11}{20}$ and $\frac{-17}{30}$

LCM of 10, 15, 20 and 30 is 60

Now,

$\frac{-3}{10} = \frac{-3\times6}{10\times6} = \frac{-18}{60}$

$\frac{-7}{15} = \frac{-7\times4}{15\times4} = \frac{-28}{60}$

$\frac{-11}{20} = \frac{-11\times3}{20\times3} = \frac{-33}{60}$

and

$\frac{-17}{30} = \frac{-17\times2}{30\times2} = \frac{-34}{60}$

Clearly,

$\frac{-18}{60} > \frac{-28}{60} > \frac{-33}{60} > \frac{-34}{60}$

$\therefore \frac{-3}{10} > \frac{-7}{15} > \frac{-11}{20} > \frac{-17}{30}$. i.e $\frac{-3}{10} > \frac{7}{-15} > \frac{-11}{20} > \frac{17}{-30}$

(iii) We will first write each of the given numbers with positive denominators. We have:

$\frac{23}{-24} = \frac{23\times(-1)}{-24\times(-1)} = \frac{-23}{24}$

Thus, the given numbers are $\frac{-5}{6}, \frac{-7}{12}, \frac{-13}{18}$ and $\frac{-23}{24}$

LCM of 6, 12, 18 and 24 is 72

Now,

Now,

$\frac{-5}{6} = \frac{-5\times12}{6\times12} = \frac{-60}{72}$

$\frac{-7}{12} = \frac{-7\times6}{12\times6} = \frac{-42}{72}$

$\frac{-13}{18} = \frac{-13\times4}{18\times4} = \frac{-52}{72}$

and

$\frac{-23}{24} = \frac{-23\times3}{24\times3} = \frac{-69}{72}$

Clearly,

$\frac{-42}{72} > \frac{-52}{72} > \frac{-60}{72} > \frac{-69}{72}$

$\therefore \frac{-7}{12} > \frac{-13}{18} > \frac{-5}{6} > \frac{-23}{24}$. i.e $\frac{-7}{12} > \frac{-13}{18} > \frac{-5}{6} > \frac{23}{-24}$

(iv) The given numbers are $\frac{-10}{11}$, $\frac{-19}{22}$, $\frac{-23}{33}$ and $\frac{-39}{44}$

LCM of 11, 22, 33 and 44 is 132

Now,

$\frac{-10}{11} = \frac{-10\times12}{11\times12} = \frac{-120}{132}$

$\frac{-19}{22} = \frac{-19\times6}{22\times6} = \frac{-114}{132}$

$\frac{-23}{33} = \frac{-23\times4}{33\times4} = \frac{-92}{132}$

and

$\frac{-39}{44} = \frac{-39\times3}{44\times3} = \frac{-117}{132}$

Clearly,

$\frac{-92}{132} > \frac{-114}{132} > \frac{-117}{132} > \frac{-120}{132}$

Q10.

Answer :

1. True
A whole number can be expressed as $\frac{a}{b}$, with $b = 1$ and $a \geq 0$. Thus, every whole number is rational.

2. True
Every integer is a rational number because any integer can be expressed as $\frac{a}{b}$, with $b = 1$ and $0 > a \geq 0$. Thus, every integer is a rational number.

3. False
$0 = \frac{a}{b}$, for $a = 0$ and $b \neq 0$. Thus, 0 is a rational and whole number.