

Exercise 5.1

Q1

If a matrix has 8 elements, what are the possible orders it can have? What if it has 5 elements?

Solution

We know that if a matrix is of the order $m \times n$, it has mn elements. Thus, to find all the possible orders of a matrix having 8 elements, we have to find all the ordered pairs of natural numbers whose products is 8.

The ordered pairs are: (1×8) , (8×1) , (2×4) , (4×2)

$(1, 5)$ and $(5, 1)$ are the ordered pairs of natural numbers whose product is 5.

Hence, the possible orders of a matrix having 5 elements are 1×5 and 5×1

Q2

If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$ then find

- (i) $a_{22} + b_{21}$
- (ii) $a_{11}b_{11} + a_{22}b_{22}$

Solution

If $A = [a_{ij}] = \begin{bmatrix} 2 & 3 & -5 \\ 1 & 4 & 9 \\ 0 & 7 & -2 \end{bmatrix}$ and $B = [b_{ij}] = \begin{bmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 2 \end{bmatrix}$

$$(i) \quad a_{22} + b_{21} = 4 + (-3) = 1$$

$$\text{Hence, } a_{22} + b_{21} = 1$$

$$(ii) \quad a_{11}b_{11} + a_{22}b_{22} = (2)(2) + (4)(4) = 4 + 16 = 20$$

Hence,

$$a_{11}b_{11} + a_{22}b_{22} = 20$$

Q3

Let A be a matrix of order 3×4 . If R_1 denotes the first row of A and C_2 denotes its second column, then determine the orders of matrices R_1 and C_2 .

Solution

Here, $A = [a_{ij}]_{3 \times 4}$

R_1 = first row of $A = [a_{11} a_{12} a_{13} a_{14}]_{1 \times 4}$

So, order of $R_1 = 1 \times 4$

C_2 = Second column of A

$$= \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}_{3 \times 1}$$

Order of $C_2 = 3 \times 1$

Q4

Construct a 2×3 matrix whose elements a_{ij} are given by:

(i) $a_{ij} = i \cdot j$

(ii) $a_{ij} = 2i - j$

(iii) $a_{ij} = i + j$

(iv) $a_{ij} = \frac{(i+j)^2}{2}$

Solution

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Let $A = (a_{ij})_{2 \times 3}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \quad \dots (i)$$

$$(i) \quad a_{ij} = i \cdot j$$

$$a_{11} = 1 \cdot 1 = 1, \quad a_{12} = 1 \cdot 2 = 2, \quad a_{13} = 1 \cdot 3 = 3$$

$$a_{21} = 2 \cdot 1 = 2, \quad a_{22} = 2 \cdot 2 = 4, \quad a_{23} = 2 \cdot 3 = 6$$

So, using equation (i)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix}$$

$$(ii) \quad a_{ij} = 2i - j$$

$$a_{11} = 2(1) - 1 = 1, \quad a_{12} = 2(1) - 2 = 0, \quad a_{13} = 2(1) - 3 = -1$$

$$a_{21} = 2(2) - 1 = 3, \quad a_{22} = 2(2) - 2 = 2, \quad a_{23} = 2(2) - 3 = 1$$

Using equation (i)

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$(iii) \quad a_{ij} = i + j$$

$$a_{11} = 1 + 1 = 2, \quad a_{12} = 1 + 2 = 3, \quad a_{13} = 1 + 3 = 4$$

$$a_{21} = 2 + 1 = 3, \quad a_{22} = 2 + 2 = 4, \quad a_{23} = 2 + 3 = 5$$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

$$(iv) \quad a_{ij} = \frac{(i+j)^2}{2}$$

$$a_{11} = \frac{(1+1)^2}{2} = 2, \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}, \quad a_{13} = \frac{(1+3)^2}{2} = 8$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, \quad a_{22} = \frac{(2+2)^2}{2} = 8, \quad a_{23} = \frac{(2+3)^2}{2} = \frac{25}{2}$$

Using equation (i),

$$A = \begin{bmatrix} 2 & \frac{9}{2} & 8 \\ \frac{9}{2} & 8 & \frac{25}{2} \end{bmatrix}$$

Q5

Construct a 2×2 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

$$(i) \quad \frac{(i+j)^2}{2} \qquad (ii) \quad a_{ij} = \frac{(i-j)^2}{2}$$

$$(iii) \quad a_{ij} = \frac{(i-2j)^2}{2} \qquad (iv) \quad a_{ij} = \frac{(2i+j)^2}{2}$$

$$(v) \quad a_{ij} = \frac{|2i-3j|}{2} \qquad (vi) \quad a_{ij} = \frac{|-3i+j|}{2}$$

Solution

Here,

$$A = [a_{ij}]_{2 \times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{--- (i)}$$

$$(i) \quad a_{ij} = \frac{(i+j)^2}{2}$$

$$a_{11} = \frac{(1+1)^2}{2} = 2, \quad a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}, \quad a_{22} = \frac{(2+2)^2}{2} = 8,$$

Using equation (i)

$$A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

$$(ii) \quad a_{ij} = \frac{(i-j)^2}{2}$$

$$a_{11} = \frac{(1-1)^2}{2} = 0, \quad a_{12} = \frac{(1-2)^2}{2} = \frac{1}{2},$$

$$a_{21} = \frac{(2-1)^2}{2} = \frac{1}{2}, \quad a_{22} = \frac{(2-2)^2}{2} = 0,$$

Using equation (i)

$$A = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{bmatrix}$$

$$(iii) \quad a_{ij} = \frac{(i-2j)^2}{2}$$

$$a_{11} = \frac{(1-2(1))^2}{2} = \frac{1}{2}, \quad a_{12} = \frac{(1-2(2))^2}{2} = \frac{9}{2},$$

$$a_{21} = \frac{(2-2(1))^2}{2} = 0, \quad a_{22} = \frac{(2-2(2))^2}{2} = 2,$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & \frac{9}{2} \\ 0 & 2 \end{bmatrix}$$

$$(iv) \quad a_{ij} = \frac{(2i+j)^2}{2}$$

$$a_{11} = \frac{(2(1)+1)^2}{2} = \frac{9}{2}, \quad a_{12} = \frac{(1(1)+2)^2}{2} = 9,$$

$$a_{21} = \frac{(2(2)+2)^2}{2} = \frac{25}{2}, \quad a_{22} = \frac{(2(2)+2)^2}{2} = 18$$

Using equation (i)

$$A = \begin{bmatrix} \frac{9}{2} & 9 \\ \frac{25}{2} & 18 \end{bmatrix}$$

$$(v) \quad a_{ij} = \frac{(2i-3j)^2}{2}$$

$$a_{11} = \frac{(2(1)-3(1))^2}{2} = \frac{1}{2}, \quad a_{12} = \frac{(2(1)-3(2))^2}{2} = 2$$

$$a_{21} = \frac{(2(2)-3(1))^2}{2} = \frac{1}{2}, \quad a_{22} = \frac{(2(2)-3(2))^2}{2} = 1$$

Using equation (i)

$$A = \begin{bmatrix} \frac{1}{2} & 2 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$(vi) a_{ij} = \frac{|-3i+j|}{2}$$

$$\text{Thus, } a_{11} = \frac{|-3 \times 1 + 1|}{2} = \frac{|-3+1|}{2} = 1$$

$$a_{12} = \frac{|-3 \times 1 + 2|}{2} = \frac{|-3+2|}{2} = \frac{1}{2}$$

$$a_{21} = \frac{|-3 \times 2 + 1|}{2} = \frac{|-6+1|}{2} = \frac{5}{2}$$

$$a_{22} = \frac{|-3 \times 2 + 2|}{2} = \frac{|-6+2|}{2} = 2$$

$$\text{Thus, } A = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$$

Q6

Construct a 2×2 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by:

$$A_{ij} = e^{2ix} \sin xj$$

Solution

We have to form a matrix A of order 2×2 as shown below,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

where $a_{ij} = e^{2ix} \sin xj$

so, a_{11} has $i = 1$ and $j = 1$

$$\therefore a_{11} = e^{2x} \sin x$$

similarly,

$$i = 1 \text{ and } j = 2, a_{12} = e^{4x} \sin 2x,$$

$$i = 2 \text{ and } j = 1, a_{21} = e^{4x} \sin x,$$

$$i = 2 \text{ and } j = 2, a_{22} = e^{8x} \sin 2x,$$

$$\text{So, } A = \begin{bmatrix} e^{2x} \sin x & e^{4x} \sin 2x \\ e^{4x} \sin x & e^{8x} \sin 2x \end{bmatrix}$$

Q7

Construct a 3×4 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by :

(i) $a_{ij} = i + j$

(ii) $a_{ij} = i - j$

(iii) $a_{ij} = 2i$

(iv) $a_{ij} = j$

(v) $a_{ij} = \frac{1}{2} |-3i + j|$

Solution

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Here, $A = (a_{ij})_{3 \times 4} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$ --- (i)

(i) $a_{ij} = i + j$

$a_{11} = 1+1 = 2, a_{12} = 1+2 = 3, a_{13} = 1+3 = 4, a_{14} = 1+4 = 5$

$a_{21} = 2+1 = 3, a_{22} = 2+2 = 4, a_{23} = 2+3 = 5, a_{24} = 2+4 = 6$

$a_{31} = 3+1 = 4, a_{32} = 3+2 = 5, a_{33} = 3+3 = 6, a_{34} = 3+4 = 7$

Using equation (i)

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$$

(ii) $a_{ij} = i - j$

$a_{11} = 1-1 = 0, a_{12} = 1-2 = -1, a_{13} = 1-3 = -2, a_{14} = 1-4 = -3$

$a_{21} = 2-1 = 1, a_{22} = 2-2 = 0, a_{23} = 2-3 = -1, a_{24} = 2-4 = -2$

$a_{31} = 3-1 = 2, a_{32} = 3-2 = 1, a_{33} = 3-3 = 0, a_{34} = 3-4 = -1$

Using equation (i)

$$A = \begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \end{bmatrix}$$

(iii) $a_{ij} = 2i$

$a_{11} = 2(1) = 2, a_{12} = 2(1) = 2, a_{13} = 2(1) = 2, a_{14} = 2(1) = 2$

$a_{21} = 2(2) = 4, a_{22} = 2(2) = 4, a_{23} = 2(2) = 4, a_{24} = 2(2) = 4$

$a_{31} = 2(3) = 6, a_{32} = 2(3) = 6, a_{33} = 2(3) = 6, a_{34} = 2(3) = 6$

Using Equation (i),

$$A = \begin{bmatrix} 2 & 2 & 2 & 2 \\ 4 & 4 & 4 & 4 \\ 6 & 6 & 6 & 6 \end{bmatrix}$$

$$(iv) \quad a_{ij} = j$$

$$a_{11} = 1, a_{12} = 2, a_{13} = 3, a_{14} = 4$$

$$a_{21} = 1, a_{22} = 2, a_{23} = 3, a_{24} = 4$$

$$a_{31} = 1, a_{32} = 2, a_{33} = 3, a_{34} = 4$$

Using Equation (i),

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{bmatrix}$$

(v)

$$a_{ij} = \frac{|-3i+j|}{2}$$

$$\text{Thus, } a_{11} = \frac{|-3 \times 1 + 1|}{2} = \frac{|-3 + 1|}{2} = 1$$

$$a_{12} = \frac{|-3 \times 1 + 2|}{2} = \frac{|-3 + 2|}{2} = \frac{1}{2}$$

$$a_{13} = \frac{|-3 \times 1 + 3|}{2} = \frac{|0|}{2} = 0$$

$$a_{14} = \frac{|-3 \times 1 + 4|}{2} = \frac{1}{2}$$

$$a_{21} = \frac{|-3 \times 2 + 1|}{2} = \frac{|-6 + 1|}{2} = \frac{5}{2}$$

$$a_{22} = \frac{|-3 \times 2 + 2|}{2} = \frac{|-6 + 2|}{2} = 2$$

$$a_{23} = \frac{|-3 \times 2 + 3|}{2} = \frac{|-3|}{2} = \frac{3}{2}$$

$$a_{24} = \frac{|-3 \times 2 + 4|}{2} = 1$$

$$a_{31} = \frac{|-3 \times 3 + 1|}{2} = \frac{|-9 + 1|}{2} = 4$$

$$a_{32} = \frac{|-3 \times 3 + 2|}{2} = \frac{|-9 + 2|}{2} = \frac{7}{2}$$

$$a_{33} = \frac{|-3 \times 3 + 3|}{2} = \frac{|-6|}{2} = 3$$

$$a_{34} = \frac{|-3 \times 3 + 4|}{2} = \frac{5}{2}$$

$$\text{Thus, } A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

Construct a 4×3 matrix whose elements are

(a) $a_{ij} = 2i + \frac{i}{j}$

(b) $a_{ij} = \frac{i-j}{i+j}$

(c) $a_{ij} = i$

Solution

Here,

$$A = [a_{ij}]_{4 \times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix}$$

(a) $a_{ij} = 2i + \frac{i}{j}$

$$a_{11} = 2(1) + \frac{1}{1} = 3, \quad a_{12} = 2(1) + \frac{1}{2} = \frac{5}{2}, \quad a_{13} = 2(1) + \frac{1}{3} = \frac{7}{3}$$

$$a_{21} = 2(2) + \frac{2}{1} = 6, \quad a_{22} = 2(2) + \frac{2}{2} = 5, \quad a_{23} = 2(2) + \frac{2}{3} = \frac{14}{3}$$

$$a_{31} = 2(3) + \frac{3}{1} = 9, \quad a_{32} = 2(3) + \frac{3}{2} = \frac{15}{2}, \quad a_{33} = 2(3) + \frac{3}{3} = 7$$

$$a_{41} = 2(4) + \frac{4}{1} = 12, \quad a_{42} = 2(4) + \frac{4}{2} = 10, \quad a_{43} = 2(4) + \frac{4}{3} = \frac{28}{3}$$

Using equation (i),

$$A = \begin{bmatrix} 3 & \frac{5}{2} & \frac{7}{3} \\ 6 & 5 & \frac{14}{3} \\ 9 & \frac{15}{2} & 7 \\ 12 & 10 & \frac{28}{3} \end{bmatrix}$$

(b) $a_{ij} = \frac{i-j}{i+j}$

$$a_{11} = \frac{1-1}{1+1} = 0, \quad a_{12} = \frac{1-2}{1+2} = \frac{-1}{3}, \quad a_{13} = \frac{1-3}{1+3} = \frac{-2}{4} = \frac{-1}{2}$$

$$a_{21} = \frac{2-1}{2+1} = \frac{1}{3}, \quad a_{22} = \frac{2-2}{2+2} = 0, \quad a_{23} = \frac{2-3}{2+3} = \frac{-1}{5}$$

$$a_{31} = \frac{3-1}{3+1} = \frac{1}{2}, \quad a_{32} = \frac{3-2}{3+2} = \frac{1}{5}, \quad a_{33} = \frac{3-3}{3+3} = 0$$

$$a_{41} = \frac{4-1}{4+1} = \frac{3}{5}, \quad a_{42} = \frac{4-2}{4+2} = \frac{1}{3}, \quad a_{43} = \frac{4-3}{4+3} = \frac{1}{7}$$

using equation (i)

$$A = \begin{bmatrix} 0 & \frac{-1}{3} & \frac{-1}{2} \\ \frac{1}{3} & 0 & \frac{-1}{5} \\ \frac{1}{2} & \frac{1}{5} & 0 \\ \frac{3}{5} & \frac{1}{3} & \frac{1}{7} \end{bmatrix}$$

$$(c) \quad a_{ij} = i$$

$$a_{11} = 1, a_{12} = 1, a_{13} = 1,$$

$$a_{21} = 2, a_{22} = 2, a_{23} = 2$$

$$a_{31} = 3, a_{32} = 3, a_{33} = 3$$

$$a_{41} = 4, a_{42} = 4, a_{43} = 4$$

Using equation (i)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}$$

Q9

Find x, y, a and b if $\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$

Solution

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Given,

$$\begin{bmatrix} 3x+4y & 2 & x-2y \\ a+b & 2a-b & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 4 \\ 5 & -5 & -1 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal.

So,

$$3x + 4y = 2 \quad \text{--- (i)}$$

$$x - 2y = 4 \quad \text{--- (ii)}$$

$$a + b = 5 \quad \text{--- (iii)}$$

$$2a - b = -5 \quad \text{--- (iv)}$$

Solving equation (i) and (ii)

$$3x + 4y = 2$$

$$\begin{array}{r} 3x + 4y = 2 \\ (-) \quad (-) \quad (-) \\ \hline 3x - 6y = 12 \end{array}$$

$$10y = -10$$

$$y = \frac{-10}{10} = -1$$

Put $y = -1$ in equation (ii)

$$x - 2y = 4$$

$$x - 2(-1) = 4$$

$$x = 4 - 2$$

$$x = 2$$

Now, solving equation (iii) and (iv),

$$2a + 2b = 10$$

$$\begin{array}{r} 2a + 2b = 10 \\ (-) \quad (-) \quad (-) \\ \hline 2a - b = -5 \end{array}$$

$$3b = 15$$

$$b = \frac{15}{3}$$

$$b = 5$$

Put the value of b in equation of (iii)

$$a + b = 5$$

$$a + 5 = 5$$

$$a = 5 - 5$$

$$a = 0$$

Hence,

$$x = 2, y = -1, a = 0, b = 5$$

Q10

Find x, y, a and b if $\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$

Solution

Given,

$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

Since corresponding entries of equal matrix are equal,

So,

$$2x - 3y = 1 \quad \text{--- (i)}$$

$$x - b = -2 \quad \text{--- (ii)}$$

$$x + 4y = 6 \quad \text{--- (iii)}$$

$$3a + 4b = 29 \quad \text{--- (iv)}$$

Solving equation (i) and (iii)

$$2x - 3y = 1$$

$$\begin{array}{r} 2x - 8y = 12 \\ (-) \quad (-) \quad (-) \\ \hline \end{array}$$

$$-11y = -11$$

$$y = \frac{-11}{-11}$$

$$y = 1$$

Put the value of y in equation (i),

$$2x - 3y = 1$$

$$2x - 3(1) = 1$$

$$2x - 3 = 1$$

$$2x = 1 + 3$$

$$2x = 4$$

$$x = 2$$

Solving equation (ii) and (iv)

$$4a - 4b = -8$$

$$\begin{array}{r} 3a - 4b = 29 \\ \hline \end{array}$$

$$7a = 21$$

$$a = \frac{21}{7}$$

$$a = 3$$

Put $a = 3$ in equation (ii),

$$3 - b = -2$$

$$b = 3 + 2$$

$$b = 5$$

Hence,

$$x = 2, y = 1, a = 3, b = 5$$

Q11

Find the values of a, b, c and d from the following equations:

$$\begin{bmatrix} 2a+b & a-2b \\ 5c-d & 4c+3d \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 11 & 24 \end{bmatrix}$$

Solution

As the given matrices are equal, therefore their corresponding elements must be equal.

Comparing the corresponding elements, we get

$$2a + b = 4 \quad \text{-----(i)}$$

$$a - 2b = -3 \quad \text{-----(ii)}$$

$$5c - d = 11 \quad \text{-----(iii)}$$

$$4c + 3d = 24 \quad \text{-----(iv)}$$

Multiplying (i) by 2 and adding to (ii)

$$5a = 5 \Rightarrow a = 1$$

$$(i) \Rightarrow b = 4 - 2.1 = 2$$

Multiplying (iii) by 3 and adding to (iv)

$$19c = 57 \Rightarrow c = 3$$

$$(iii) \Rightarrow d = 5.3 - 11 = 4$$

Hence, $a = 1$, $b = 2$, $c = 3$, $d = 4$

Q12

Find x , y and z so that $A=B$, where

$$A = \begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix}, B = \begin{bmatrix} y & z & 6 \\ 6y & x & 2y \end{bmatrix}$$

Solution

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Given,

$$A = B$$

$$\begin{bmatrix} x-2 & 3 & 2z \\ 18z & y+2 & 6z \end{bmatrix} = \begin{bmatrix} y & z & 6 \\ 6y & x & 2y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x-2 = y \quad \text{--- (i)}$$

$$3 = z \quad \text{--- (ii)}$$

$$2z = 6 \quad \text{--- (iii)}$$

$$18z = 6 \quad \text{--- (iv)}$$

$$y+2 = x \quad \text{--- (v)}$$

$$6z = 2y \quad \text{--- (vi)}$$

Equation (ii) gives, $z = 3$

Put the value of z in equation (iv),

$$18z = 6y$$

$$18(3) = 6y$$

$$54 = 6y$$

$$y = \frac{54}{6}$$

$$y = 9$$

Put $y = 9$ in equation (v)

$$y+2 = x$$

$$9+2 = x$$

$$11 = x$$

Hence,

$$x = 11, y = 9, z = 3$$

Q13

If $\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$, find x, y, z, w .

Solution

Given,

$$\begin{bmatrix} x & 3x-y \\ 2x+z & 3y-w \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 7 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x = 3 \quad \text{--- (i)}$$

$$3x - y = 2 \quad \text{--- (ii)}$$

$$2x + z = 4 \quad \text{--- (iii)}$$

$$3y - w = 7 \quad \text{--- (iv)}$$

Put the value of $x = 3$ from equation on (i) in equation (ii),

$$3x - y = 2$$

$$3(3) - y = 2$$

$$9 - y = 2$$

$$y = 9 - 2$$

$$y = 7$$

Put the value of $y = 7$ in equation (iv),

$$3y - w = 7$$

$$3(7) - w = 7$$

$$w = 21 - 7$$

$$w = 14$$

Put the value of $x = 3$ in equation (iii),

$$2x + z = 4$$

$$2(3) + z = 4$$

$$6 + z = 4$$

$$z = 4 - 6$$

$$z = -2$$

Hence,

$$x = 3, y = 7, z = -2, w = 14$$

Q14

If $\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$, find x, y, z, w

Solution

Given,

$$\begin{bmatrix} x-y & z \\ 2x-y & w \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 0 & 5 \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$\begin{aligned} x-y &= -1 & \text{---(i)} \\ z &= 4 & \text{---(ii)} \\ 2x-y &= 0 & \text{---(iii)} \\ w &= 5 & \text{---(iv)} \end{aligned}$$

Solving equation (i) and (iii)

$$\begin{array}{r} x-y = -1 \\ 2x-y = 0 \\ \hline (-) \quad (-) \quad (-) \\ -x = -1 \\ x = 1 \end{array}$$

Put $x = 1$ in equation (i),

$$\begin{aligned} x-y &= -1 \\ 1-y &= -1 \\ -y &= -1-1 \\ -y &= -2 \\ y &= 2 \end{aligned}$$

equation (ii) and (iv) give the values of z and w respectively, so

$$z = 4, w = 5$$

Hence,

$$x = 1, y = 2, z = 4, w = 5$$

Q15

$$\text{If } \begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

obtain the values of a, b, c, x, y and z

Solution

By the definition of equality of matrices we know that if two matrices

$$A = [a_{ij}]_{m \times n} \text{ and } B = [b_{ij}]_{m \times n}$$

are equal then $a_{ij} = b_{ij}$ for $i = 1, 2, 3, \dots, m$ and $j = 1, 2, 3, \dots, n$.

$$\text{Given that } \begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c+2 \\ 2b+4 & -21 & 0 \end{bmatrix}$$

$\therefore$ Equating the entries gives:

$$\begin{aligned} x+3 &= 0, \quad z+4 = 6 \text{ and } 2y-7 = 3y-2 \\ \Rightarrow x &= -3, \quad z = 2 \text{ and } 2y-3y = -2+7 \\ \Rightarrow x &= -3, \quad z = 2 \text{ and } -y = 5 \\ \Rightarrow x &= -3, \quad z = 2 \text{ and } y = -5 \end{aligned}$$

Similarly, $a-1 = -3$ and $2c+2 = 0$

$$\begin{aligned} \Rightarrow a &= -3+1 \text{ and } 2c = -2 \\ \Rightarrow a &= -2 \text{ and } c = -1 \end{aligned}$$

Lastly, $b-3 = 2b+4$

$$\begin{aligned} \Rightarrow b-2b &= 4+3 \\ \Rightarrow -b &= 7 \\ \Rightarrow b &= -7 \end{aligned}$$

The values of x, y, z, a, b, c are $-3, -5, 2, -2, -7, -1$ respectively.

Q16

$$\text{If } \begin{bmatrix} 2x+1 & 5x \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix},$$

Find the value of $(x+y)$.

Solution

$$\text{Given that } \begin{bmatrix} 2x+1 & 5x \\ 0 & y^2+1 \end{bmatrix} = \begin{bmatrix} x+3 & 10 \\ 0 & 26 \end{bmatrix}$$

The corresponding entries of the equal matrices are equal.

$$\begin{aligned} \Rightarrow 2x+1 &= x+3, \quad y^2+1 = 26, \\ \Rightarrow 2x-x &= 2, \quad y^2 = 25 \\ \Rightarrow x &= 2, \quad y = \pm 5 \\ \Rightarrow x &= 2, \quad y = 5 \text{ or } x = 2, \quad y = -5 \\ \therefore x+y &= 7 \text{ or } -3 \end{aligned}$$

Q17

$$\text{if } \begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix},$$

then find the values of x, y, z and w .

Solution

$$\begin{bmatrix} xy & 4 \\ z+6 & x+y \end{bmatrix} = \begin{bmatrix} 8 & w \\ 0 & 6 \end{bmatrix}$$

The corresponding entries of the two equal matrices are equal.

$$\Rightarrow xy = 8 \dots\dots(1),$$

$$w = 4 \dots\dots(2),$$

$$z + 6 = 0 \dots\dots(3),$$

$$\text{and } x + y = 6 \dots\dots(4)$$

from equation (2) and equation(3) we get $z = -6$ and $w = 4$.

from equation(4) we have,

$$x + y = 6,$$

$$\Rightarrow x = 6 - y,$$

substituting value of x in equation (1) we get,

$$\Rightarrow (6 - y)y = 8,$$

$$\Rightarrow y^2 - 6y + 8 = 0,$$

$$\Rightarrow (y - 2)(y - 4) = 0,$$

$$\Rightarrow y = 2, 4$$

substituting the value of y in equation(1) we get,

$$\Rightarrow x = 4, 2$$

Therefore, value of x, y, z, w are $2, 4, -6, 4$ or $4, 2, -6, 4$.

Q18

Give an example of

- (i) a row matrix which is also a column matrix,
- (ii) a diagonal matrix which is not scalar,
- (iii) a triangular matrix.

Solution

(i) We know that,

Order of a row matrix = $1 \times n$

order of a column matrix = $m \times 1$

So, order of a row as well as column matrix = 1×1

Therefore,

$$\text{Required matrix} = [a]_{1 \times 1}$$

(ii) A diagonal matrix has only a_{11}, a_{22}, a_{33} for a 3×3 matrix such that a_{11}, a_{22}, a_{33} are equal or different and all other entries zero while scalar matrix has

$a_{11} = a_{22} = a_{33} = m$ (say) So, A diagonal matrix which is not scalar must have,

$a_{11} \neq a_{22} \neq a_{33}$ and $a_{ij} = 0$ for $i \neq j$, So

$$\text{Required Matrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

(iii) A triangular matrix is a square matrix $A = [a_{ij}]$ such that $a_{ij} = 0$ for all $i > j$, so

$$A = \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 3 \\ 0 & 0 & -6 \end{bmatrix}$$

Q19

The sales figure of two car dealer during January 2013 showed that Dealer A sold 5 deluxe, 3 premium and 4 standard cars, while dealer B sold 7 deluxe, 2 premium and 3 standard cars. Total sales over the 2 month period of January - February revealed that dealer A sold 8 deluxe 7 premium and 6 standard cars. in the same 2 month period, dealer b sold 10 deluxe, 5 premium and 7 standard cars. Write 2×3 matrices summarizing for January and 2 - month period for each dealer.

Solution

Given data is,

For January 2013:

Dealer A	Deluxe	Premium	Standard Cars
	5	3	4
Dealer B	7	2	3

For January-February:

Dealer A	Deluxe	Premium	Standard Cars
	8	7	6
Dealer B	10	5	7

Hence,

$$A = \begin{matrix} & \begin{matrix} \text{Deluxe} & \text{Premium} & \text{Standard} \end{matrix} \\ \begin{matrix} \text{Dealer A} \\ \text{Dealer B} \end{matrix} & \begin{bmatrix} 5 & 3 & 4 \\ 7 & 2 & 3 \end{bmatrix} \end{matrix}$$

$$B = \begin{matrix} & \begin{matrix} \text{Deluxe} & \text{Premium} & \text{Standard} \end{matrix} \\ \begin{matrix} \text{Dealer A} \\ \text{Dealer B} \end{matrix} & \begin{bmatrix} 8 & 7 & 6 \\ 10 & 5 & 7 \end{bmatrix} \end{matrix}$$

Q20

For what values of x and y are the following matrices equal?

$$A = \begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix}, B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Solution

Given,

$$A = B$$

$$\begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix} = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Since equal matrices has all corresponding entries equal,

So,

$$2x+1 = x+3 \quad \text{--- (i)}$$

$$2y = y^2+2 \quad \text{--- (ii)}$$

$$y^2-5y = -6 \quad \text{--- (iii)}$$

Solving equation (i)

$$2x+1 = x+3$$

$$2x - x = 3 - 1$$

$$x = 2$$

Solving equation (ii)

$$2y = y^2+2$$

$$y^2 - 2y + 2 = 0$$

$$D = b^2 - 4ac$$

$$= (-2)^2 - 4(1)(2)$$

$$= 4 - 8$$

$$= -4$$

So, There is no real value of y from equation (ii).

Solving equation (iii)

$$y^2 - 5y = -6$$

$$y^2 - 5y + 6 = 0$$

$$y^2 - 3y - 2y + 6 = 0$$

$$y(y-3) - 2(y-3) = 0$$

$$(y-3)(y-2) = 0$$

$$y = 3 \quad \text{or} \quad y = 2$$

From solution of equation (i), (ii) and (iii), We can say that A and B can not be equal for any value of y .

Q21

Find the values of x and y if

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Solution

Given,

$$\begin{bmatrix} x+10 & y^2+2y \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 3x+4 & 3 \\ 0 & y^2-5y \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, So

$$x+10 = 3x+4 \quad \text{---(i)}$$

$$y^2+2y = 3 \quad \text{---(ii)}$$

$$-4 = y^2-5y \quad \text{---(iii)}$$

Solving equation (i),

$$x+10 = 3x+4$$

$$x-3x = 4-10$$

$$-2x = -6$$

$$x = \frac{6}{2}$$

Solving equation (ii),

$$y^2+2y = 3$$

$$y^2+2y-3 = 0$$

$$y^2+3y-y-3 = 0$$

$$y(y+3)(y-1) = 0$$

$$\Rightarrow y = -3 \text{ and } y = 1$$

Solving equation (iii)

$$-4 = y^2-5y$$

$$y^2-5y+4 = 0$$

$$y^2-4y-y+(y-4) = 0$$

$$y(y-4)-1(y-4) = 0$$

$$(y-4)(y-1) = 0$$

$$\Rightarrow y = 4 \text{ and } y = 1$$

From equation (ii) and (iii),

The common value of $y = 1$

So, $x = 3, y = 1$

Q22

Find the values of a and b if $A = B$, where

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Solution

$$A = \begin{bmatrix} a+4 & 3b \\ 8 & -6 \end{bmatrix}, B = \begin{bmatrix} 2a+2 & b^2+2 \\ 8 & b^2-5b \end{bmatrix}$$

Given that $A = B$

Corresponding element of two equal matrices are equal

$$\Rightarrow a + 4 = 2a + 2, 3b = b^2 + 2 \quad \text{and} \quad -6 = b^2 - 5b$$

$$\Rightarrow a - 2a = 2 - 4, b^2 - 3b + 2 = 0 \quad \text{and} \quad b^2 - 5b + 6 = 0$$

$$\Rightarrow -a = -2, (b-1)(b-2) = 0 \quad \text{and} \quad (b-2)(b-3) = 0$$

$$\Rightarrow a = 2, b = 1, 2 \quad \text{and} \quad b = 2, 3$$

So value of $a = 2$, $b = 2$ respectively.



Exercise 5.2

Q1

Compute the following:

$$(i) \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} \quad (ii) \begin{bmatrix} 2 & 1 & 2 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix}$$

Solution

$$\begin{aligned} (i) \quad & \begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 3-2 & -2+4 \\ 1+1 & 4+3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} -2 & 4 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 7 \end{bmatrix}$$

$$\begin{aligned} (ii) \quad & \begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2+1 & 1-2 & 3+3 \\ 0+2 & 3+6 & 5+1 \\ -1+0 & 2-3 & 5+1 \end{bmatrix} \\ &= \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & 3 & 5 \\ -1 & 2 & 5 \end{bmatrix} + \begin{bmatrix} 1 & -2 & 3 \\ 2 & 6 & 1 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 6 \\ 2 & 9 & 6 \\ -1 & -1 & 6 \end{bmatrix}$$

Q2

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, Find $2A - 3B$

Solution

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$2A - 3B$$

$$\begin{aligned} &= 2 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 3 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 9 \\ -6 & 15 \end{bmatrix} \\ &= \begin{bmatrix} 4-3 & 8-9 \\ 6+6 & 4-15 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix} \end{aligned}$$

Hence,

$$2A - 3B = \begin{bmatrix} 1 & -1 \\ 12 & -11 \end{bmatrix}$$

Q3

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, Find $B - 4C$

Solution

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$B - 4C$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - 4 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} - \begin{bmatrix} -8 & 20 \\ 12 & 16 \end{bmatrix} \\ &= \begin{bmatrix} 1+8 & 3-20 \\ -2-12 & 5-16 \end{bmatrix} \\ &= \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix} \end{aligned}$$

Hence,

$$B - 4C = \begin{bmatrix} 9 & -17 \\ -14 & -11 \end{bmatrix}$$

Q4

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, Find $3A - C$

Solution

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$3A - C$$

$$\begin{aligned} &= 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6+2 & 12-5 \\ 9-3 & 6-4 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix} \end{aligned}$$

Hence,

$$3A - C = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

Q5

Let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ and $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$, Find $3A - 2B + 3C$

Solution

Given, $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

$$3A - 2B + 3C$$

$$\begin{aligned} &= 3 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} + 3 \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} 2 & 6 \\ -4 & 10 \end{bmatrix} + \begin{bmatrix} -6 & 15 \\ 9 & 12 \end{bmatrix} \\ &= \begin{bmatrix} 6-2-6 & 12-6+15 \\ 9+4+9 & 6-10+12 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix} \end{aligned}$$

Hence,

$$3A - 2B + 3C = \begin{bmatrix} -2 & 21 \\ 22 & 8 \end{bmatrix}$$

Q6

If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$, Find

(i) $A+B$ and $B+C$

(ii) $2B+3A$ and $3C-4B$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix}$

(i) $A + B$

$A + B$ is not possible as order of A is 2×2 and order of B is 2×3 .

And we know that sum of matrix is possible only when their order is same.

Hence,

$A + B$ is not possible

$B + C$

$$\begin{aligned} &= \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -1-1 & 0+2 & 2+3 \\ 3+2 & 4+1 & 1+0 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix} \end{aligned}$$

So,

$$B + C = \begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$$

(ii)

We need to find $2B + 3A$ and $3C - 4B$

Thus, $2B + 3A$ does not exist as the order of A and B are different.

Let us find $3C - 4B = 3 \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix} - 4 \begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}$

$$\begin{aligned} &= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 0 & 8 \\ 12 & 16 & 4 \end{bmatrix} \\ &= \begin{bmatrix} -3 & 6 & 9 \\ 6 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & -8 \\ -12 & -16 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 6 & 1 \\ -6 & -13 & -4 \end{bmatrix} \end{aligned}$$

Q7

Let $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$. Compute $2A - 3B + 4C$.

Solution

Given, $A = \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix}$

$$2A - 3B + 4C$$

$$\begin{aligned} &= 2 \begin{bmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{bmatrix} - 3 \begin{bmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{bmatrix} + 4 \begin{bmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{bmatrix} \\ &= \begin{bmatrix} -2 & 0 & 4 \\ 6 & 2 & 8 \end{bmatrix} - \begin{bmatrix} 0 & -6 & 15 \\ 3 & -9 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -20 & 8 \\ 24 & 0 & -16 \end{bmatrix} \\ &= \begin{bmatrix} -2-0+4 & 0+6-20 & 4-15+8 \\ 6-3+24 & 2+9+0 & 8-3-16 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix} \end{aligned}$$

Hence,

$$2A - 3B + 4C = \begin{bmatrix} 2 & -14 & -3 \\ 27 & 11 & -11 \end{bmatrix}$$

Q8

if $A = \text{diag} (2 -5 9)$, $B = \text{diag} (1 1 -4)$ and $C = (-6 3 4)$ find

(i) $A - 2B$ (ii) $B + C - 2A$ (iii) $2A + 3B - 5C$

Solution

Given, $A = \text{diag} (2 -5 9)$, $B = \text{diag} (1 1 -4)$
and $C = \text{diag} (-6 3 4)$

$$\begin{aligned} \text{(i)} \quad A - 2B &= \text{diag} (2 -5 9) - 2 \text{diag} (1 1 -4) \\ &= \text{diag} (2 -5 9) - \text{diag} (2 2 -8) \\ &= \text{diag} (2-2 -5-2 9+8) \\ &= \text{diag} (0 -7 -17) \end{aligned}$$

$$\text{So, } A - 2B = \text{diag} (0 -7 -17)$$

$$\begin{aligned} \text{(ii)} \quad B + C - 2A &= \text{diag} (1 1 -4) + \text{diag} (-6 3 4) - 2 \text{diag} (2 -5 9) \\ &= \text{diag} (1 1 -4) + \text{diag} (-6 3 4) - \text{diag} (4 -10 18) \\ &= \text{diag} (1-6-4 1+3+10 -4+4-18) \\ &= \text{diag} (-9 14 -18) \end{aligned}$$

$$\text{So, } B + C - 2A = \text{diag} (-9 14 -18)$$

$$\begin{aligned} \text{(iii)} \quad 2A + 3B - 5C &= 2 \text{diag} (2 -5 9) + 3 \text{diag} (1 1 -4) - 5 \text{diag} (-6 3 4) \\ &= \text{diag} (4 -10 18) + \text{diag} (3 3 -12) - \text{diag} (-30 15 20) \\ &= \text{diag} (4+3+30 -10+3-15 18-12-20) \\ &= \text{diag} (37 -22 -14) \end{aligned}$$

So,

$$2A + 3B - 5C = \text{diag} (37 -22 -14)$$

Q9

Give the matrices:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \text{ and } C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

Verify that $(A+B)+C = A+(B+C)$.

Solution

Given,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix}, C = \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$\text{LHS} = (A+B)+C$$

$$= \left\{ \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} \right\} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+9 & 1+7 & 1-1 \\ 3+3 & -1+5 & 0+4 \\ 0+2 & 2+1 & 4+6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 8 & 0 \\ 6 & 4 & 4 \\ 2 & 3 & 10 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 11+2 & 8-4 & 0+3 \\ 6+1 & 4-1 & 4+0 \\ 2+9 & 3+4 & 10+5 \end{bmatrix}$$

$$\text{LHS} = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \quad \text{---(i)}$$

$$\text{RHS} = A+(B+C)$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \left\{ \begin{bmatrix} 9 & 7 & -1 \\ 3 & 5 & 4 \\ 2 & 1 & 6 \end{bmatrix} + \begin{bmatrix} 2 & -4 & 3 \\ 1 & -1 & 0 \\ 9 & 4 & 5 \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 9+2 & 7-4 & -1+3 \\ 3+1 & 5-1 & 4+0 \\ 2+9 & 1+4 & 6+5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 3 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 11 & 3 & 2 \\ 4 & 4 & 4 \\ 11 & 5 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 2+11 & 1+3 & 1+2 \\ 3+4 & -1+4 & 0+4 \\ 0+11 & 2+5 & 4+11 \end{bmatrix}$$

$$\text{RHS} = \begin{bmatrix} 13 & 4 & 3 \\ 7 & 3 & 4 \\ 11 & 7 & 15 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$(A+B)+C = A+(B+C)$$

Q10

Find X and Y , if $X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and $X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$.

Solution

We have

$$(X + Y) + (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2} \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$\text{Also, } (X + Y) - (X - Y) = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$$

$$\Rightarrow 2Y = \begin{bmatrix} 5-3 & 2-6 \\ 0-0 & 9+1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{2} \begin{bmatrix} 2 & -4 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

Q11

Find X , if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Solution

$$2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1-3 & 0-2 \\ -3-1 & 2-4 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

Q12

Find matrices X and Y , if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$

Solution

Given,

$$2x - y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} \quad \text{---(i)}$$

$$x + 2y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \quad \text{---(ii)}$$

Now find

$$2(2x - y) + (x + 2y) = 2 \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \quad \text{(using equation (i) and (ii))}$$

$$\Rightarrow 4x - 2y + x + 2y = \begin{bmatrix} 12 & -12 & 0 \\ -8 & 4 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$$

$$\Rightarrow 5x = \begin{bmatrix} 12+3 & -12+2 & 0+5 \\ -8-2 & 4+1 & 2-7 \end{bmatrix}$$

$$\Rightarrow 5x = \begin{bmatrix} 15 & -10 & 5 \\ -10 & 5 & -5 \end{bmatrix}$$

$$\Rightarrow 5x = 5 \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}$$

Now find,

$$(2x - y) - 2(x + 2y) = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix} \quad \text{(using equation (i) and (ii))}$$

$$\Rightarrow 2x - y - 2x - 4y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 6 & 4 & 10 \\ -4 & 2 & -14 \end{bmatrix}$$

$$\Rightarrow -y - 4y = \begin{bmatrix} 6-6 & -6-4 & 0-10 \\ -4+4 & 2-2 & 1+14 \end{bmatrix}$$

$$\Rightarrow -5y = \begin{bmatrix} 0 & -10 & -10 \\ 0 & 0 & 15 \end{bmatrix}$$

$$\Rightarrow -5y = -5 \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$$

Q13

If $X - Y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $X + Y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$, find X and Y .

Solution

Given,

$$x - y = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$x + y = \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

Now find,

$$(x - y) + (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix} 1+3 & 1+5 & 1+1 \\ 1-1 & 1+1 & 0+4 \\ 1+11 & 0+8 & 0+0 \end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix} 4 & 6 & 2 \\ 0 & 2 & 4 \\ 12 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow 2x = 2 \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}$$

Now find,

$$(x - y) - (x + y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 3 & 5 & 1 \\ -1 & 1 & 4 \\ 11 & 8 & 0 \end{bmatrix}$$

$$\Rightarrow x - y - x - y = \begin{bmatrix} 1-3 & 1-5 & 1-1 \\ 1-1 & 1-1 & 0-4 \\ 1-11 & 0-8 & 0-0 \end{bmatrix}$$

$$\Rightarrow -2y = \begin{bmatrix} -2 & -4 & 0 \\ 0 & 0 & -4 \\ -10 & -8 & 0 \end{bmatrix}$$

$$\Rightarrow -2y = -2 \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & 4 & 0 \end{bmatrix}, y = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 2 \\ 5 & 4 & 0 \end{bmatrix}$$

Q14

Find matrix A, if $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & 9 \end{bmatrix} + A = \begin{bmatrix} 9 & -1 & 4 \\ -2 & 1 & 3 \end{bmatrix}$

Solution

Given,

$$\begin{aligned} & \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix} + A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix} \\ \Rightarrow & A = \begin{bmatrix} 9 & -1 & 1 \\ 4 & -2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 9 \end{bmatrix} \\ \Rightarrow & A = \begin{bmatrix} 9-1 & -1-2 & 1-4 \\ 4+1 & -2-0 & 3-9 \end{bmatrix} \\ \Rightarrow & A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix} \end{aligned}$$

Hence,

$$A = \begin{bmatrix} 8 & -3 & -3 \\ 5 & -2 & -6 \end{bmatrix}$$

Q15

If $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find matrix C such that $5A + 3B + 2C$ is a null matrix.

Solution

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Given, $A = \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$

Let, $C = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$

Since, $5A + 3B + 2C$ is a null matrix, so

$$5A + 3B + 2C = 0$$

$$\Rightarrow 5 \begin{bmatrix} 9 & 1 \\ 7 & 8 \end{bmatrix} + 3 \begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix} + 2 \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 45 & 5 \\ 35 & 40 \end{bmatrix} + \begin{bmatrix} 3 & 15 \\ 21 & 36 \end{bmatrix} + \begin{bmatrix} 2x & 2y \\ 2z & 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 45 + 3 + 2x & 5 + 15 + 2y \\ 35 + 21 + 2z & 40 + 36 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 48 + 2x & 20 + 2y \\ 56 + 2z & 76 + 2w \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal.

$$48 + 2x = 0$$

$$x = -\frac{48}{2}$$

$$x = -24$$

$$20 + 2y = 0$$

$$y = -\frac{20}{2}$$

$$y = -10$$

$$56 + 2z = 0$$

$$z = -\frac{56}{2}$$

$$z = -28$$

$$76 + 2w = 0$$

$$w = -\frac{76}{2}$$

$$w = -38$$

Hence, $C = \begin{bmatrix} -24 & -10 \\ -28 & -38 \end{bmatrix}$

Q16

If $A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$, find matrix X such that $2A + 3X = 5B$.

Solution

Given,

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

And

$$2A + 3x = 5B$$

$$\Rightarrow 3x = 5B - 2A$$

$$\Rightarrow 3x = 5 \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix} - 2 \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 40 & 0 \\ 20 & -10 \\ 15 & 30 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ 8 & 4 \\ -10 & 2 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 40-4 & 0+4 \\ 20-8 & -10-4 \\ 15+10 & 30-2 \end{bmatrix}$$

$$\Rightarrow 3x = \begin{bmatrix} 36 & 4 \\ 12 & -14 \\ 25 & 28 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} \frac{36}{3} & \frac{4}{3} \\ \frac{12}{3} & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Hence,

$$x = \begin{bmatrix} 12 & \frac{4}{3} \\ 4 & \frac{-14}{3} \\ \frac{25}{3} & \frac{28}{3} \end{bmatrix}$$

Q17

If $A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, find the matrix C such that $A+B+C$ is zero matrix.

Solution

Given,

$$A = \begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

And

$$A + B + C = 0$$

$$\Rightarrow C = -A - B + 0$$

$$\Rightarrow C = -A - B$$

$$\Rightarrow C = -\begin{bmatrix} 1 & -3 & 2 \\ 2 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -1 & 3 & -2 \\ -2 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -1-2 & 3+1 & -2+1 \\ -2-1 & 0-0 & -2+1 \end{bmatrix}$$

$$\Rightarrow C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Hence,

$$C = \begin{bmatrix} -3 & 4 & -1 \\ -3 & 0 & -1 \end{bmatrix}$$

Q18

Find x,y satisfying the matrix equation $\begin{bmatrix} x-y & 2 & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$

Solution

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$$\begin{bmatrix} x-y & 2 & -2 \\ 4 & x & 6 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 2 \\ 1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 2-2 & -2+2 \\ 4+1 & x+0 & 6-1 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

$$\begin{bmatrix} x-y+3 & 0 & 0 \\ 5 & x & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 5 & 2x+y & 5 \end{bmatrix}$$

Use know that, corresponding entries of equal matrices are equal. So,

$$\begin{aligned} x-y+3 &= 6 \\ \Rightarrow x-y &= 3 \quad \text{---(i)} \end{aligned}$$

$$\begin{aligned} \text{and } x &= 2x+y \\ \Rightarrow 2x-x+y &= 0 \\ \Rightarrow x+y &= 0 \quad \text{---(ii)} \end{aligned}$$

Adding equation (i), (ii),

$$\begin{aligned} x-y+x+y &= 3+0 \\ \Rightarrow 2x &= 3 \\ \Rightarrow x &= \frac{3}{2} \end{aligned}$$

Put in equation (i),

$$\begin{aligned} x-y &= 3 \\ \Rightarrow \frac{3}{2} - y &= 3 \\ \Rightarrow -y &= \frac{3-3}{2} \\ \Rightarrow y &= \frac{-3}{2} \end{aligned}$$

Hence,

$$x = \frac{3}{2}, y = \frac{-3}{2}$$

Q19

Find x,y satisfying the matrix equation $\begin{bmatrix} x & y & 2 \\ z & -3 \end{bmatrix} + \begin{bmatrix} y & 4 & 5 \end{bmatrix} = \begin{bmatrix} 4 & 9 & 12 \end{bmatrix}$

Solution

$$\begin{aligned}
 [x \ y+2 \ z-3] + [y \ 4 \ 5] &= [4 \ 9 \ 12] \\
 \Rightarrow [x+y \ y+2+4 \ z-3+5] &= [4 \ 9 \ 12] \\
 \Rightarrow [x+y \ y+6 \ z+2] &= [4 \ 9 \ 12]
 \end{aligned}$$

We know that, corresponding entries, of equal matrices are equal, So

$$\begin{aligned}
 x+y &= 4 & \text{--- (i)} \\
 y+6 &= 9 & \text{--- (ii)} \\
 z+2 &= 12 & \text{--- (iii)}
 \end{aligned}$$

From equation (ii), We get

$$\begin{aligned}
 y &= 9 - 6 \\
 y &= 3
 \end{aligned}$$

Put the value of y in equation (i),

$$\begin{aligned}
 x+y &= 4 \\
 \Rightarrow x+3 &= 4 \\
 \Rightarrow x &= 4-3 \\
 \Rightarrow x &= 1
 \end{aligned}$$

From equation (iii)

$$\begin{aligned}
 z+2 &= 12 \\
 z &= 12-2 \\
 z &= 10
 \end{aligned}$$

Hence,

$$x = 1, y = 3, z = 10$$

Q20

Find x, y satisfying the matrix equations

$$x \begin{bmatrix} 2 \\ 1 \end{bmatrix} + y \begin{bmatrix} 3 \\ 5 \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solution

Let $A = [a_{ij}]_{m \times n}$ be a matrix and let k be any scalar then

$$k(A) = [k a_{ij}]_{m \times n}$$

using this we get,

$$\begin{bmatrix} 2x \\ x \end{bmatrix} + \begin{bmatrix} 3y \\ 5y \end{bmatrix} + \begin{bmatrix} -8 \\ -11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

adding the corresponding entries of the matrices by matrix algebra

$$\begin{bmatrix} 2x + 3y - 8 \\ x + 5y - 11 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The corresponding entries of equal matrices are equal

$$\begin{aligned}
 \Rightarrow 2x + 3y - 8 &= 0, \\
 x + 5y - 11 &= 0
 \end{aligned}$$

solving the two equations simultaneously we have values of $x = 1, y = 2$.

Q21

If $2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix}$, find x and y .

Solution

Given,

$$\begin{aligned} & 2\begin{bmatrix} 3 & 4 \\ 5 & x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} 6 & 8 \\ 10 & 2x \end{bmatrix} + \begin{bmatrix} 1 & y \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} 6+1 & 8+y \\ 10+0 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} 7 & 8+y \\ 10 & 2x+1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 10 & 5 \end{bmatrix} \end{aligned}$$

Since corresponding entries of equal matrices are equal, So

$$8+y=0$$

$$y=-8$$

And

$$2x+1=5$$

$$2x=5-1$$

$$x=\frac{4}{2}$$

$$x=2$$

Hence,

$$x=2, y=-8$$

Q22

Find the value of λ , a non-zero scalar, $\lambda\begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix}$

Solution

Given,

$$\begin{aligned} & \lambda\begin{bmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \end{bmatrix} + 2\begin{bmatrix} 1 & 2 & 3 \\ -1 & -3 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} \lambda & 0 & 2\lambda \\ 3\lambda & 4\lambda & 5\lambda \end{bmatrix} + \begin{bmatrix} 2 & 4 & 6 \\ -2 & -6 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix} \\ \Rightarrow & \begin{bmatrix} \lambda+2 & 4 & 2\lambda+6 \\ 3\lambda-2 & 4\lambda-6 & 5\lambda+4 \end{bmatrix} = \begin{bmatrix} 4 & 4 & 10 \\ 4 & 2 & 14 \end{bmatrix} \end{aligned}$$

Since corresponding entries of equal matrices are equal, So

$$\lambda+2=4$$

$$\Rightarrow \lambda=2$$

and

$$3\lambda-2=4$$

$$3\lambda=6$$

$$\Rightarrow \lambda=2$$

Hence,

$$\lambda=2$$

Q23

(i)

Find a matrix X such that $2A + B + X = 0$ where

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

(ii)

If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$, then find the matrix X of order 3×2

such that $2A + 3X = 5B$.**Solution**

(i)

Given,

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

and

$$2A + B + X = 0$$

$$\Rightarrow X = -2A - B$$

$$\Rightarrow X = -2 \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2 & -4 \\ -6 & -8 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 2-3 & -4+2 \\ -6-1 & -8-5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

Hence,

$$X = \begin{bmatrix} -1 & -2 \\ -7 & -13 \end{bmatrix}$$

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(ii)

Given, $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$

Also we have $2A + 3X = 5B$ Thus, we have, $3X = 5B - 2A$

$$\Rightarrow 3X = 5 \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix} - 2 \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} - \begin{bmatrix} 16 & 0 \\ 8 & -4 \\ 6 & 12 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} 10-16 & -10-0 \\ 20-8 & 10-(-4) \\ -25-6 & 5-12 \end{bmatrix}$$

$$\Rightarrow 3X = \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & \frac{-10}{3} \\ 4 & \frac{14}{3} \\ \frac{-31}{3} & \frac{-7}{3} \end{bmatrix}$$

Q24

Given $3 \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & y \\ z+t & 3 \end{bmatrix}$, find the values of x, y, z and t .

Solution

$$3 \begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} x+4 & 6+x+y \\ -1+z+t & 2t+3 \end{bmatrix}$$

Comparing the corresponding elements of these two matrices, we get:

$$3x = x + 4$$

$$\Rightarrow 2x = 4$$

$$\Rightarrow x = 2$$

$$3y = 6 + x + y$$

$$\Rightarrow 2y = 6 + x = 6 + 2 = 8$$

$$\Rightarrow y = 4$$

$$3t = 2t + 3$$

$$\Rightarrow t = 3$$

$$3z = -1 + z + t$$

$$\Rightarrow 2z = -1 + t = -1 + 3 = 2$$

$$\Rightarrow z = 1$$

$$\therefore x = 2, y = 4, z = 1, \text{ and } t = 3$$

Q25

Find x , y , z , and t , if

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Solution

$$2 \begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 10 \\ 14 & 2y-6 \end{bmatrix} + \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 14 \\ 15 & 2y-4 \end{bmatrix} = \begin{bmatrix} 7 & 14 \\ 15 & 14 \end{bmatrix}$$

Comparing the corresponding elements from both sides,

$$2x + 3 = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$$

$$2y - 4 = 14 \Rightarrow 2y = 18 \Rightarrow y = 9$$

Hence, $x = 2$, $y = 9$

Q26

If X and Y are 2×2 matrices, then solve the following matrix equations for X and Y.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}, 3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

Solution

Let us solve this problem using simultaneous linear equation and algebra of matrices.

$$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots (1)$$

$$3X + 2Y = \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \dots\dots\dots (2)$$

multiplying the first equation by 3 and second equation by 2 we get,

$$6X + 9Y = 3 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots (3),$$

$$6X + 4Y = 2 \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \dots\dots\dots (4)$$

Subtracting equation (4) from equation (3) we have,

$$5Y = 3 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 2 \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ 2 & -10 \end{bmatrix}$$

$$\Rightarrow 5Y = \begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \frac{1}{5} \begin{bmatrix} 10 & 5 \\ 10 & 10 \end{bmatrix}$$

$$\Rightarrow Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

Similarly, multiplying the equation (1) by 2 and equation (2) by 3 we get,

$$4X + 6Y = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \dots\dots\dots (5),$$

$$9X + 6Y = 3 \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix} \dots\dots\dots (6)$$

Subtracting equation (6) from equation (5) we have,

$$-5X = 2 \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3 \begin{bmatrix} -2 & 2 \\ 1 & -5 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} -6 & 6 \\ 3 & -15 \end{bmatrix}$$

$$\Rightarrow -5X = \begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow X = -\frac{1}{5} \begin{bmatrix} 10 & 0 \\ 5 & 15 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$$

Hence the value of $X = \begin{bmatrix} -2 & 0 \\ -1 & -3 \end{bmatrix}$ and $Y = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$.

Q27

In certain city there are 30 colleges. Each college has 15 peons, 6 clerks, 1 typist and 1 section officer. Express the given information as a column matrix. Using scalar multiplication, find the total number of posts of each kind in all the colleges.

Solution

Let A represent the post allocation matrix for a college, So

$$A = \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix} \begin{array}{l} \text{Peons} \\ \text{Clerks} \\ \text{Typist} \\ \text{Section officer} \end{array}$$

The total number of posts of each kind in 30 colleges is given by
 $= 30A$

$$= 30 \begin{bmatrix} 15 \\ 6 \\ 1 \\ 1 \end{bmatrix}$$
$$30A = \begin{bmatrix} 450 \\ 180 \\ 30 \\ 30 \end{bmatrix} \begin{array}{l} \text{Peons} \\ \text{Clerks} \\ \text{Typist} \\ \text{Section Officers} \end{array}$$

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Exercise 5.3

Q1

Compute the indicated products:

$$(i) \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$$

Solution

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$$\begin{aligned}
 \text{(i)} \quad & \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} \\
 &= \begin{bmatrix} (a)(a) + (b)(b) & (a)(-b) + (b)(a) \\ (-b)(a) + (a)(b) & (-b)(-b) + (a)(a) \end{bmatrix} \\
 &= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix} \\
 &= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}
 \end{aligned}$$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

$$\begin{aligned}
 \text{(ii)} \quad & \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} (1)(1) + (-2)(-3) & (1)(2) + (-2)(2) & (1)(3) + (-2)(-1) \\ (2)(1) + (3)(-3) & (2)(2) + (3)(2) & (2)(3) + (3)(-1) \end{bmatrix} \\
 &= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix} \\
 &= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}
 \end{aligned}$$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

$$\begin{aligned}
 \text{(iii)} \quad & \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} \\
 &= \begin{bmatrix} (2)(1) + (3)(0) + (4)(3) & (2)(-3) + (3)(2) + (4)(0) & (2)(5) + (3)(4) + (4)(5) \\ (3)(1) + (4)(0) + (5)(3) & (3)(-3) + (4)(2) + (5)(0) & (3)(5) + (4)(4) + (5)(5) \\ (4)(1) + (5)(0) + (6)(3) & (4)(-3) + (5)(2) + (6)(0) & (4)(5) + (5)(4) + (6)(5) \end{bmatrix} \\
 &= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix} \\
 &= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}
 \end{aligned}$$

Hence,

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Q2

Show that $AB \neq BA$ in $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Q3

Show that $AB \neq BA$ in $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ +0+0+1 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii), $AB \neq BA$

Q4

Show that $AB \neq BA$ in $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \quad \text{--- (i)}$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix} \quad \text{--- (ii)}$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Q5

Compute the products AB and BA whichever exists in $A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Solution

$$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

Since order of A is 2×2 and order of B is 2×3 ,

So AB is possible but BA is not possible order of AB is 2×3 .

$$\begin{aligned} AB &= \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} \\ &= \begin{bmatrix} (1)(1) + (-2)(2) & (1)(2) + (-2)(3) & (1)(3) + (-2)(1) \\ (2)(1) + (3)(2) & (2)(2) + (3)(3) & (2)(3) + (3)(1) \end{bmatrix} \\ &= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix} \\ AB &= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix} \end{aligned}$$

Hence,

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exist

Q6

Compute the products AB and BA whichever exists in $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Solution

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Here, $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Order of $A = 3 \times 2$ and order of $B = 2 \times 3$. So,

AB and BA Both exists and order of $AB = 3 \times 3$ and order of $BA = 2 \times 2$

$$\begin{aligned} AB &= \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} (3)(4) + (2)(0) & (3)(5) + (2)(1) & (3)(6) + (2)(2) \\ (-1)(4) + (0)(0) & (-1)(5) + (0)(1) & (-1)(6) + (0)(2) \\ (-1)(4) + (1)(0) & (-1)(5) + (1)(1) & (-1)(6) + (1)(2) \end{bmatrix} \\ &= \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+0 & -6+2 \end{bmatrix} \\ &= \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -5 & -4 \end{bmatrix} \\ BA &= \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} (4)(3) + (5)(-1) + (6)(-1) & (4)(2) + (5)(0) + (6)(1) \\ (0)(3) + (1)(-1) + (2)(-1) & (0)(2) + (1)(0) + (2)(1) \end{bmatrix} \\ &= \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix} \end{aligned}$$

Hence,

$$AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}, BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Q7

Compute the products AB and BA whichever exists in $A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

Solution

Here,

$$A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

Order of $A = 1 \times 4$ and order of $B = 4 \times 1$ So,

AB and BA both exist and order of $AB = 1 \times 1$ and order of $BA = 4 \times 4$, So

$$AB = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

$$= [(1)(0) + (-1)(1) + (2)(3) + (3)(2)]$$

$$= [0 - 1 + 6 + 6]$$

$$AB = [11]$$

$$BA = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} (0)(1) & (0)(-1) & (0)(2) & (0)(3) \\ (1)(1) & (1)(-1) & (1)(2) & (1)(3) \\ (3)(1) & (3)(-1) & (3)(2) & (3)(3) \\ (2)(1) & (2)(-1) & (2)(2) & (2)(3) \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

Hence,

$$AB = [11]$$

$$BA = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & -1 & 2 & 3 \\ 3 & -3 & 6 & 9 \\ 2 & -2 & 4 & 6 \end{bmatrix}$$

Q8

Compute the products AB and BA whichever exists in $\begin{bmatrix} a & b \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix} + \begin{bmatrix} a & b & c & d \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

Solution

$$\begin{aligned}
 [a \ b] \begin{bmatrix} c \\ d \end{bmatrix} + [a \ b \ c \ d] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \\
 = [ac + bd] + [a^2 + b^2 + c^2 + d^2] \\
 = [ac + bd + a^2 + b^2 + c^2 + d^2]
 \end{aligned}$$

Hence,

$$\begin{aligned}
 [a \ b] \begin{bmatrix} c \\ d \end{bmatrix} + [a \ b \ c \ d] \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \\
 = [ac + bd + a^2 + b^2 + c^2 + d^2]
 \end{aligned}$$

Q9

Show that $AB \neq BA$ in $A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Solution

$$\begin{aligned}
 A &= \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \\
 AB &= \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \\
 &= \begin{bmatrix} -2+3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix} \\
 AB &= \begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (i)} \\
 BA &= \begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix} \\
 &= \begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix} \\
 BA &= \begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -9 & 1 \end{bmatrix} \quad \text{--- (ii)}
 \end{aligned}$$

From equation (i) and (ii),

$$AB \neq BA$$

Q10

Show that $AB \neq BA$ in $A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$

Solution

$$A = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 10-12-1 & 20-16-3 & 10-8-2 \\ -11+15+0 & -22+20+0 & -11+10+0 \\ 9-15+1 & 18-20+3 & 9-10+2 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \quad \text{--- (i)}$$

$$BA = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-22+9 & -4+10-5 & -9+0+1 \\ 30-44+10 & -12+20-10 & -3+0+2 \\ 10-33+18 & -4+15-10 & -1+0+2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 1 & 0 \\ 4 & -2 & -1 \\ -5 & 1 & 1 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii)

$$AB \neq BA$$

Q11

Evaluate $\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

Solution

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 1+3 & 3-2 \\ -1-1 & -4+1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 4+2 & 12+4 & 20+6 \\ -2-6 & -6-12 & -10-18 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix} 1 & 3 \\ -1 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix} = \begin{bmatrix} 6 & 16 & 26 \\ -8 & -18 & -28 \end{bmatrix}$$

Q12

Evaluate $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

Solution

$$\begin{aligned} & \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 1+4+0 & 0+0+3 & 2+0+6 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 3 & 10 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \\ &= \begin{bmatrix} 10+12+60 \end{bmatrix} \\ &= \begin{bmatrix} 82 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} = \begin{bmatrix} 82 \end{bmatrix}$$

Q13

Evaluate $\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right)$

Solution

$$\begin{aligned} & \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1-0 & 0-1 & 2-2 \\ 2-1 & 0-0 & 1-2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1-1 & -1+0 & 0+1 \\ 0+2 & 0+0 & 0-2 \\ 2+3 & -2+0 & 0-3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix} \end{aligned}$$

Hence,

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 2 \end{bmatrix} \right) = \begin{bmatrix} 0 & -1 & 1 \\ 2 & 0 & -2 \\ 5 & -2 & -3 \end{bmatrix}$$

Q14

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,
then show that $A^2 = B^2 = C^2 = I_2$.

Solution

Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = I_2 \quad \text{--- (i)}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = I_2 \quad \text{--- (ii)}$$

$$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$C^2 = I_2 \quad \text{--- (iii)}$$

Hence,

From equation (i), (ii) and (iii)

$$A^2 = B^2 = C^2 = I_2$$

Q15

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$

Solution

Given, $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$$\begin{aligned}
 & 3A^2 - 2B + I \\
 &= 3 \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} - 2 \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= 3 \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= 3 \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 3 & -12 \\ 36 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 8 \\ -2 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 3-0+1 & -12+8+0 \\ 36+2+0 & 3-14+1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}
 \end{aligned}$$

Hence,

$$3A^2 - 2B + I = \begin{bmatrix} 4 & -20 \\ 38 & -10 \end{bmatrix}$$

Q16

If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, prove that $(A - 2I)(A - 3I) = O$

Solution

Given, $A = \begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix}$

$$\begin{aligned}
 & (A - 2I)(A - 3I) \\
 &= \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) \\
 &= \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \right) \left(\begin{bmatrix} 4 & 2 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \right) \\
 &= \begin{bmatrix} 4-2 & 2-0 \\ -1-0 & -1-2 \end{bmatrix} \begin{bmatrix} 4-3 & 2-0 \\ -1-0 & -1-3 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \\
 &= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & -2+2 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= O
 \end{aligned}$$

Hence,

$$(A - 2I)(A - 3I) = O$$

Q17

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, show that $A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+1 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+2 \\ 0+0 & 0+1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Hence,

$$A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

Q18

If $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$, show that $A^2 = O$

Solution

Given, $A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$

$$A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}$$

$$= \begin{bmatrix} a^2b^2 - a^2b^2 & ab^3 - ab^3 \\ -a^3b + a^3b & -a^2b^2 + a^2b^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= O$$

Hence,

$$A^2 = O$$

Q19

If $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$, find A^2

Solution

Given, $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin 2\theta + \cos 2\theta \sin 2\theta \\ -\cos 2\theta \sin 2\theta - \sin^2 2\theta \cos 2\theta & -\sin^2 2\theta + \cos^2 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 4\theta & 2 \sin^2 \theta \cos^2 \theta \\ -2 \sin^2 \theta \cos^2 \theta & \cos 4\theta \end{bmatrix}$$

$$\left\{ \text{since } \cos^2 \theta - \sin^2 \theta = \cos 2\theta \right\}$$

$$= \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$$\left\{ \text{since } \sin^2 \theta = 2 \sin \theta \cos \theta \right\}$$

Hence,

$$A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

Q20

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ show that $AB - BA = O_{3 \times 3}$

Solution

Copykitab
Same textbooks, black away

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 6+9-15 & 10+15+25 \\ 1+4-5 & -3-12+15 & -5-20+25 \\ -1-3+4 & 3+9-12 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3 \times 3} \quad \text{--- (i)}$$

$$BA = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} -2-3+5 & 3+12-15 & 5+15-20 \\ 2+3-5 & -3-12+15 & -5-15+20 \\ -2-3+5 & 3+12-15 & 5+15-20 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = O_{3 \times 3} \quad \text{--- (ii)}$$

From equation (i) and (ii),
 $AB = BA = O_{3 \times 3}$

Q21

If $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$ and $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$, show that $AB = BA = O_{3 \times 3}$.

Solution

Given, $A = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix}$, $B = \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & c & -b \\ -c & 0 & a \\ b & -a & 0 \end{bmatrix} \begin{bmatrix} a^2 & ab & ac \\ ab & b^2 & bc \\ ac & bc & c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0+a^2c-abc & 0+b^2c-b^2c & 0+bc^2-bc^2 \\ -a^2c+0+a^2c & -abc+0+abc & -ac^2+0+ac^2 \\ a^2b-a^2b+0 & ab^2-ab^2+0 & abc-abc+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$AB = O_{3 \times 3} \quad \text{--- (i)}$$

From equation (i) and (ii),
 $AB = BA = O_{3 \times 3}$

Q22

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$, show that $AB = A$ and $BA = B$.

Solution

$$\begin{aligned} \text{Given, } A &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}, B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \\ AB &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 4+3-5 & -4-9+10 & -8-12+15 \\ -2-4+5 & 2+12-10 & 4+16-15 \\ 2+3-4 & -2-9+18 & -4-12+12 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \\ AB &= A \\ BA &= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \\ &= \begin{bmatrix} 4+2-4 & -6-8+12 & -10-10+16 \\ -2-3+4 & 3+12-12 & 5+15-16 \\ 2+2-3 & -3-8+9 & -5-10+12 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \\ BA &= B \end{aligned}$$

Q23

Let $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$, compute $A^2 - B^2$.

Solution

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix} \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \quad \text{---(i)}$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix} \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{---(ii)}$$

Subtracting equation (ii) from equation (i),

$$A^2 - B^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & -9-0 & -1-0 \\ 3-0 & 27-1 & 3-0 \\ 35-0 & 15-0 & 35-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Q24

For the following matrices verify the associativity of matrix multiplication i.e.

$$(AB)C = A(BC): A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Solution

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$ and

$$C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & 0+0+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \text{---(i)}$$

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii) we get,

$$(AB)C = A(BC)$$

Q25

For the following matrices verify the associativity of matrix multiplication i.e.

$$(AB)C = A(BC): \quad A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution

(ii) Given,

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4+0+6 & -4+2-3 & 4+4+3 \\ 1+0+4 & -1+1-2 & 1+2+2 \\ 3+0+2 & -3+0-1 & 3+0+1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & -5 & 11 \\ 5 & -2 & 5 \\ 5 & -4 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10-15+0 & 20+0+0 & -10+5+11 \\ 5-6+0 & 10+0+0 & -5-2+5 \\ 5-12+0 & 10+0+0 & -5-4+4 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \quad \text{---(i)}$$

$$A(BC) = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1-3+0 & 2+0+0 & -1-1+1 \\ 0+3+0 & 0+0+0 & 0+1+2 \\ 2-3+0 & 4+0+0 & -2-1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 & 2 & -1 \\ 3 & 0 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -8+6-3 & 8+0+12 & -4+6-6 \\ -2+3-2 & 2+0+8 & -1+3-4 \\ -6+0-1 & 6+0+4 & -3+0-2 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 & 20 & -4 \\ -1 & 10 & -2 \\ -7 & 10 & -5 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$(AB)C = A(BC)$$

Q26

For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. $A(B+C) = AB+AC$:

$$A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

Solution

Given, $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \left[\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \right]$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1+0 & 0+1 \\ 2+1 & 1-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1-3 & 1+0 \\ 0+6 & 0+0 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \quad \text{--- (i)}$$

$$AB + AC = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-2 & 0-1 \\ 0+4 & 0+2 \end{bmatrix} + \begin{bmatrix} 0+1 & 1+1 \\ 0+2 & 0-2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -3-1 & -1+2 \\ 4+2 & 2-2 \end{bmatrix}$$

$$AB + AC = \begin{bmatrix} -4 & 1 \\ 6 & 0 \end{bmatrix} \quad \text{--- (ii)}$$

Using equation (i) and (ii),

$$A(B+C) = AB + AC$$

Q27

For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. $A(B+C) = AB + AC$:

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

Solution

Given, $A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$

$$A(B+C) = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \left(\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0+1 & 1-1 \\ 1+0 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 0+2 \\ 1+1 & 0+2 \\ -1+2 & 0+4 \end{bmatrix}$$

$$A(B+C) = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{--- (i)}$$

$$AB+AC = \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 2-1 \\ 0+1 & 1+1 \\ 0+2 & -1+2 \end{bmatrix} + \begin{bmatrix} 2+0 & -2-1 \\ 1+0 & -1+1 \\ -1+0 & 1+2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ -1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1+2 & 1-3 \\ 1+1 & 2+0 \\ 2-1 & 1+3 \end{bmatrix}$$

$$AB+AC = \begin{bmatrix} 1 & -2 \\ 2 & 2 \\ 1 & 4 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$A(B+C) = AB+AC$$

Q28

If $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, verify that $A(B-C) = AB-AC$.

Solution

Given,

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0-1 & 5-5 & -4-2 \\ -2+1 & 1-1 & 3-0 \\ -1-0 & 0+1 & 2-1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & -6 \\ -1 & 0 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+2 & 0+0-2 & -6+0-2 \\ -3+1+0 & 0+0+0 & -18-3+0 \\ 2-1-1 & 0+0+1 & 12+3+1 \end{bmatrix}$$

$$A(B - C) = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \quad \text{---(i)}$$

$$AB - AC = \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 5 & -4 \\ -2 & 1 & 3 \\ -1 & 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & -2 \\ 3 & -1 & 0 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 5 & 2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+2 & 5+0+0 & -4+0-4 \\ 0+2+0 & 15-1+0 & -12-3+0 \\ 0-2-1 & -10+1+0 & 8+3+2 \end{bmatrix} - \begin{bmatrix} 1+0+0 & 5+0+2 & 2+0-2 \\ 3+1+0 & 15-1+0 & 6+0+0 \\ 0-2-1 & -10+1+1 & -4+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 5 & -8 \\ 2 & 14 & -15 \\ -3 & -9 & 13 \end{bmatrix} - \begin{bmatrix} 1 & 7 & 0 \\ 4 & 14 & 6 \\ -3 & -10 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-1 & 5-7 & -8-0 \\ 2-4 & 14-14 & -14-6 \\ -3+3 & -9+10 & 13+3 \end{bmatrix}$$

$$AB - AC = \begin{bmatrix} 1 & -2 & -8 \\ -2 & 0 & -21 \\ 0 & 1 & 16 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$A(B - C) = AB - AC$$

Q29

Compute the elements a_{43} and a_{22} of the matrix:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

Solution

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 2 \\ 0 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -3 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0-3+0 & 0+2+0 \\ 4+0+8 & -2+0+6 \\ 0-9+8 & 0+6+6 \\ 8+0+16 & -4+0+12 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -3 & 2 \\ 12 & 4 \\ -1 & 12 \\ 24 & 8 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 & 2 & -2 \\ 3 & -3 & 4 & -4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 0+6 & -3-6 & 3+8 & -6-8 & 6+0 \\ 0+12 & 12-12 & -12+16 & 24-16 & -24+0 \\ 0+36 & -1-36 & 1+48 & -2-48 & 2+0 \\ 0+24 & 24-24 & -24+34 & 48-32 & -48+0 \end{bmatrix}$$

$$A = \begin{bmatrix} 6 & -9 & 11 & -14 & 6 \\ 12 & 0 & 4 & 8 & -24 \\ 36 & -37 & 49 & -50 & 2 \\ 24 & 0 & 8 & 16 & -48 \end{bmatrix}$$

Here, $a_{43} = 8, a_{22} = 0$

Q30

If $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$, and I is the identity matrix of order 3, show that

$$A^3 = pI + qA + rA^2$$

Solution

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \quad \text{--- (i)}$$

$$pI + qA + rA^2$$

$$= p \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p+0+0 & 0+q+0 & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+pq+pr^2 & 0+q^2+pr+qr^2 & p+qr+qr+r^2 \end{bmatrix}$$

$$pI + qA + rA^2$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

From equation (i) and (ii)

$$A^3 = pI + qA + rA^2$$

Q31

If w is a complex cube root of unity, show that:

$$\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Given, W is a complex cube root of unity,

$$\begin{aligned} & \left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \\ &= \begin{bmatrix} 1+w & w+w^2 & w^2+1 \\ w+w^2 & w^2+1 & 1+w \\ w^2+w & 1+w^2 & w+1 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \\ &= \begin{bmatrix} -w^2 & -1 & -w \\ -1 & -w & -w^2 \\ -1 & -w & -w^2 \end{bmatrix} \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} \quad \left\{ \begin{array}{l} \text{Since } 1+w+w^2 = 0 \\ \text{and } w^3 = 1 \end{array} \right\} \quad \text{---(i)} \end{aligned}$$

$$= \begin{bmatrix} -w^2 - w - w^3 \\ -1 - w^2 - w^4 \\ -1 - w^2 - w^2 \end{bmatrix}$$

$$= \begin{bmatrix} -w(1+w+w^2) \\ -1 - w^2 - w^3w \\ -1 - w^2 - w^3w \end{bmatrix}$$

$$= \begin{bmatrix} -w \cdot 0 \\ -1 - w^2 - w \\ -1 - w^2 - w \end{bmatrix} \quad \text{(using reason (i))}$$

$$= \begin{bmatrix} 0 \\ -(1+w+w^2) \\ -(1+w+w^2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ -(0) \\ -(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix} \right) \begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Q32

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$, show that $A^2 = A$

Solution

Given, $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3-5 & -6-12+15 & -10-15+20 \\ -2-4+5 & 3+16-15 & 5+20-20 \\ 2+3-4 & -3-12+12 & -5-15+16 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$$

$$= A$$

Hence,

$$A^2 = A$$

Q33

If $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$, show that $A^2 = I_3$

Solution

Given, $A = \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$

$$A^2 = A \cdot A$$

$$= \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix} \begin{bmatrix} 4 & -1 & -4 \\ 3 & 0 & -4 \\ 3 & -1 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 16-3-12 & -4+0+4 & -16+4+12 \\ 12+0-12 & -3+0+4 & -12+0+12 \\ 12-3-9 & -3+0+3 & -12+4+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= I_3$$

Hence,

$$A^2 = I_3$$

Q34

If $\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$, find x .

Solution

Given,

$$\begin{bmatrix} 1 & 1 & x \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0+2x & 0+2+x & 2+1+0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+1 & 2+x & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [2x+1+2+x+3] = 0$$

$$\Rightarrow 3x+6=0$$

$$\Rightarrow x = -\frac{6}{3}$$

$$\Rightarrow x = -2$$

Q35

If $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$ find x

Solution

Given that $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 1 & -3 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$

By multiplication of matrices, we have,

$$\begin{bmatrix} 2 \times 1 + 3 \times (-2) & 2 \times (-3) + 3 \times 4 \\ 5 \times 1 + 7 \times (-2) & 5 \times (-3) + 7 \times 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 6 \\ -9 & 13 \end{bmatrix} = \begin{bmatrix} -4 & 6 \\ -9 & x \end{bmatrix}$$

$$\Rightarrow x = 13$$

Q36

If $\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$, find x .

Solution

Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+4+0 & x+0+2 & 2x+8-4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow [(2x+4)x + 4(x+2) - 1(2x+4)] = 0$$

$$\Rightarrow 2x^2 + 4x + 4x + 8 - 2x - 4 = 0$$

$$\Rightarrow 2x^2 + 6x + 4 = 0$$

$$\Rightarrow 2x^2 + 2x + 4x + 4 = 0$$

$$\Rightarrow 2x(x+1) + 4(x+1) = 0$$

$$\Rightarrow (x+1)(2x+4) = 0$$

$$\Rightarrow x+1 = 0 \text{ or } 2x+4 = 0$$

$$\Rightarrow x = -1 \text{ or } x = -2$$

Hence, $x = -1$ or -2

Q37

If $\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$, find x .

Solution

Given,

$$\begin{bmatrix} 1 & -1 & x \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0-2+x & 1-1+x & -1-3+x \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & x & x-4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow [0(x-2) + x \cdot 1 + 1 \cdot (x-4)] = 0$$

$$\Rightarrow 0 + x + x - 4 = 0$$

$$\Rightarrow 2x - 4 = 0$$

$$\Rightarrow x = 2$$

Hence,

$$x = 2$$

Q38

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then prove that $A^2 - A + 2I = O$.

Solution

$$\begin{aligned}
 \text{Given, } A &= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 A^2 - A + 2I &= \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 9-8 & -6+4 \\ 12-8 & -8+4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} - \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 1-3+2 & -2+2+0 \\ 4-4+0 & -4+2+2 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= O
 \end{aligned}$$

Hence,

$$A^2 - A + 2I = O$$

Q39

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ so that $A^2 = 5A + \lambda I$.

Solution

$$\begin{aligned}
 \text{Given, } A &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 \text{And} \\
 A^2 &= 5A + \lambda I \\
 \Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} &= 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 9-1 & 3+2 \\ -3+2 & -1+4 \end{bmatrix} &= \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \\
 \Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} &= \begin{bmatrix} 15+\lambda & 5 \\ -5 & 10+\lambda \end{bmatrix}
 \end{aligned}$$

Since, Corresponding entries of equal matrices are equal, So

$$8 = 15 + \lambda$$

$$\lambda = 8 - 15$$

$$\lambda = -7$$

Q40

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I_2 = O$.

Solution

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\begin{aligned}
 A^2 - 5A + 7I_2 &= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\
 &= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= 0
 \end{aligned}$$

Hence, $A^2 - 5A + 7I_2 = 0$

Q41

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, show that $A^2 - 2A + 3I_2 = 0$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$\begin{aligned}
 A^2 - 2A + 3I_2 &= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2 \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2+2+0 & -3+0+3 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\
 &= 0
 \end{aligned}$$

Hence,

$$A^2 - 2A + 3I_2 = 0$$

Q42

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^3 - 4A^2 + A = 0$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$$

$$A^3 = A^2 \cdot A$$

$$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$$

Hence, $A^3 - 4A^2 + A$

$$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4 \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So, $A^3 - 4A^2 + A = 0$

Q43

Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ is root of the equation $A^2 - 12A - I = 0$

Solution

Given, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$$A^2 - 12A - I$$

$$= \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12 \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

Since $A^2 - 12A - I = 0$

So,

$$A \text{ is a root of the equation } A^2 - 12A - I = 0$$

Q44

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, find that $A^2 - 5A - 14I$

Solution

Given, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$

$$A^2 = 5A - 14I$$

$$\begin{aligned} &= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 5 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} \\ &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} \\ &= \begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20-0 & 24-10-14 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$

So,

$$A^2 - 5A - 14I = 0$$

Q45

If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

Use this to find A^{-1}

Solution

It is given that $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$\begin{aligned} \therefore A^2 &= A \cdot A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 3(3)+1(-1) & 3(1)+1(2) \\ -1(3)+2(-1) & -1(1)+2(2) \end{bmatrix} \\ &= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} \end{aligned}$$

$$\therefore \text{L.H.S.} = A^2 - 5A + 7I$$

$$\begin{aligned} &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= O = \text{R.H.S.} \end{aligned}$$

$$\therefore A^2 - 5A + 7I = O$$

Since $A^2 - 5A + 7I = O$, we have

$$A^2 = 5A - 7I$$

$$\text{Therefore, } A^4 = A^2 \times A^2 = (5A - 7I)(5A - 7I)$$

$$\Rightarrow A^4 = 25A^2 - 35A - 35A + 49I$$

$$\Rightarrow A^4 = 25A^2 - 70A + 49I$$

$$\Rightarrow A^4 = 25(5A - 7I) - 70A + 49I$$

$$\Rightarrow A^4 = 125A - 175I - 70A + 49I$$

$$\Rightarrow A^4 = 55A - 126I$$

$$\Rightarrow A^4 = 55 \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 126 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165 & 55 \\ -55 & 110 \end{bmatrix} - \begin{bmatrix} 126 & 0 \\ 0 & 126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 165-126 & 55-0 \\ -55-0 & 110-126 \end{bmatrix}$$

$$\Rightarrow A^4 = \begin{bmatrix} 39 & 55 \\ -55 & -16 \end{bmatrix}$$

If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Solution

$$\begin{aligned} A^2 &= A \cdot A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 3(3) + (-2)(4) & 3(-2) + (-2)(-2) \\ 4(3) + (-2)(4) & 4(-2) + (-2)(-2) \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} \end{aligned}$$

Now $A^2 = kA - 2I$

$$\begin{aligned} \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \\ \Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} &= \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix} \end{aligned}$$

Comparing the corresponding elements, we have:

$$\begin{aligned} 3k-2 &= 1 \\ \Rightarrow 3k &= 3 \\ \Rightarrow k &= 1 \end{aligned}$$

Thus, the value of k is 1.

Q47

If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$, find k such that $A^2 - 8A + kI = 0$.

Solution

Here,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

And

$$A^2 - 8A + kI = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8 \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since,

corresponding entries of equal matrices are equal, so

$$-7+k = 0$$

$$k = 7$$

Q48

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$.

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } f(x) = x^2 - 2x - 3$$

$$f(A) = A^2 - 2A - 3I$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$

So,

$$f(A) = 0$$

Q49

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ & μ so that $A^2 = \lambda A + \mu I$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given,

$$A^2 = \lambda A + \mu I$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda \\ \lambda & 2\lambda + \mu \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, so

$$2\lambda + \mu = 7 \quad \text{--- (i)}$$

$$\lambda = 4 \quad \text{--- (ii)}$$

Put λ from equation (ii) in equation (i),

$$2(4) + \mu = 7$$

$$\mu = 7 - 8$$

$$\mu = -1$$

Hence, $\lambda = 4, \mu = -1$

Q50

Find the value of x for which the matrix product

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$$

equals an identity matrix.

Solution

Given,

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2x+0+7x & 28x+0-28x & 14x+0-14x \\ 0+0+0 & 0+1+0 & 0+0+0 \\ -x+0+x & 14x-2-4x & 7x+0-2x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 10x-2 & 5x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x = 1 \quad \text{and} \quad 10x - 2 = 0$$

$$\Rightarrow x = \frac{1}{5} \quad \text{and} \quad x = \frac{1}{5}$$

Hence, $x = \frac{1}{5}$

Q51

Solve the matrix equation $\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$

Solution

Here,

$$\begin{bmatrix} x & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} x-2 & 0-3 \end{bmatrix} \begin{bmatrix} x \\ 5 \end{bmatrix} = 0$$

$$\Rightarrow [(x-2)x - 15] = 0$$

$$\Rightarrow x^2 - 2x - 15 = 0$$

$$\Rightarrow x^2 - 5x + 3x - 15 = 0$$

$$\Rightarrow x(x-5) + 3(x-5) = 0$$

$$\Rightarrow (x-5)(x+3) = 0$$

$$\Rightarrow x-5 = 0 \quad \text{or} \quad x+3 = 0$$

$$\Rightarrow x = 5 \quad \text{or} \quad x = -3$$

So,

$$x = 5 \text{ or } -3$$

Q52

Solve the matrix equation $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0?$

Solution

We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow [6(0) + 2(2) + 4(x)] = 0$$

$$\Rightarrow [4 + 4x] = [0]$$

$$\therefore 4 + 4x = 0$$

$$\Rightarrow x = -1$$

Thus, the required value of x is -1 .

Q53

Solve the matrix equation $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

Solution

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow [x(x-2) - 40 + 2x - 8] = 0$$

$$\Rightarrow [x^2 - 2x - 40 + 2x - 8] = [0]$$

$$\Rightarrow [x^2 - 48] = [0]$$

$$\therefore x^2 - 48 = 0$$

$$\Rightarrow x^2 = 48$$

$$\Rightarrow x = \pm 4\sqrt{3}$$

Q54

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$, Compute $A^2 - 4A + 3I_3$.

Solution

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$

$$\begin{aligned}
 & A^2 - 4A + 3I_3 \\
 &= \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1+6+0 & 2-8+0 & 0+10+0 \\ 3-12+0 & 6+16-5 & 0-20+15 \\ 0-3+0 & 0+4-3 & 0-5+9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \\
 &= \begin{bmatrix} 7-4+3 & -6-8+0 & 10-0+0 \\ -9-12+0 & 17+16+3 & -5-20+0 \\ -3-0+0 & 1+4+0 & 4-12+3 \end{bmatrix} \\
 &= \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}
 \end{aligned}$$

Hence,

$$A^2 - 4A + 3I_3 = \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Q55

If $f(x) = x^2 - 2x$, find $f(A)$ where $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

And, $f(x) = x^2 - 2x$

$\Rightarrow f(A) = A^2 - 2A$

$\Rightarrow f(A) = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix} - 2 \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 0+4+0 & 0+5+4 & 0+0+6 \\ 0+20+0 & 4+25+0 & 8+0+0 \\ 0+8+0 & 0+10+6 & 0+0+9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 4 & 9 & 6 \\ 20 & 29 & 8 \\ 8 & 16 & 9 \end{bmatrix} - \begin{bmatrix} 0 & 2 & 4 \\ 8 & 10 & 0 \\ 0 & 4 & 6 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 4-0 & 9-2 & 6-4 \\ 20-8 & 29-10 & 8-0 \\ 8-0 & 16-4 & 9-6 \end{bmatrix}$

$\Rightarrow f(A) = \begin{bmatrix} 4 & 7 & 2 \\ 12 & 19 & 8 \\ 8 & 12 & 3 \end{bmatrix}$

Q56

If $f(x) = x^2 + 4x^2 - x$, find $f(A)$, where $A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$

Solution

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Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

And $f(x) = x^3 + 4x^2 - x$

$$\Rightarrow f(A) = A^3 + 4A^2 - A \quad \text{---(i)}$$

$$A^2 = A \times A$$

$$\begin{aligned} &= \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+2+2 & 0-3-2 & 0+0+0 \\ 0-6+0 & 2+9+0 & 4+0+0 \\ 0-2+0 & 1+3+0 & 0+0+0 \end{bmatrix} \end{aligned}$$

$$A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$\begin{aligned} &= \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0-10+0 & 4+15+0 & 8+0+0 \\ 0+22+4 & -6-33-4 & -12+0+0 \\ 0+8+2 & -2-12-2 & -4+0+0 \end{bmatrix} \end{aligned}$$

$$A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix}$$

Put the value of A, A^2, A^3 in equation (i)

$$f(A) = A^3 + 4A^2 - A$$

$$\begin{aligned} &= \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4 \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix} \\ &= \begin{bmatrix} -10+16-0 & 19-20-1 & 8+0-2 \\ 26-24-2 & -43+44+3 & -12+16+0 \\ 10-8-1 & -16+16+1 & -4+8-0 \end{bmatrix} \\ &= \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix} \end{aligned}$$

Hence,

$$f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ then show that A is a root of the polynomial

$$f(x) = x^3 - 6x^2 + 7x + 2$$

Solution

Given that, $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ and $f(x) = x^3 - 6x^2 + 7x + 2$

Therefore, $f(A) = A^3 - 6A^2 + 7A + 2I_3$

First find A^2 :

$$A^2 = A \times A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

Now, Let us find A^3 :

$$A^3 = A^2 \times A = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Thus,

$$\begin{aligned} f(A) &= A^3 - 6A^2 + 7A + 2I_3 \\ &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6 \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 21-30+7+2 & 0 & 34-48+14+0 \\ 12-12+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0 & 55-78+21+2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O \end{aligned}$$

Thus, A is a root of the polynomial.

Q58

If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then prove that $A^2 - 4A - 5I = O$.

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$A^2 - 4A - 5I$$

$$\begin{aligned} &= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &= 0 \end{aligned}$$

Hence,

$$A^2 - 4A - 5I = 0$$

Q59

If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, show that $A^2 - 7A + 10I_3 = 0$

Solution

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^2 - 7A + 10I_3$$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7 \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Q60

Without using the concept of inverse of a matrix, find the matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$ such that

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Solution

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$5x - 7z = -16 \quad \text{--- (i)}$$

$$-2x + 3z = 7 \quad \text{--- (ii)}$$

$$5y - 7u = -6 \quad \text{--- (iii)}$$

$$-2y + 3u = 2 \quad \text{--- (iv)}$$

Solving equation (i) and (ii)

$$10x - 14z = -32$$

$$\underline{-10x + 15z = 35}$$

$$z = 3$$

Put the value of z in equation (i)

$$5x - 7(3) = -16$$

$$\Rightarrow 5x = 16 + 21$$

$$\Rightarrow 5x = 5$$

$$\Rightarrow x = 1$$

Solving equation (iii) and (iv)

$$10y - 14u = -12$$

$$\underline{-10y + 15u = 10}$$

$$u = -2$$

Put the value of u in equation (iii)

$$5y - 7u = -6$$

$$\Rightarrow 5y - 7(-2) = -6$$

$$\Rightarrow 5y + 14 = -6$$

$$\Rightarrow 5y = -20$$

$$\Rightarrow y = -4$$

So,

$$\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

Q61

Find the matrix A such that

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Solution

Given,

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}_{2 \times 2} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}_{2 \times 3}$

$\Rightarrow$ A is a matrix of order 2×3 .

So,

$$\text{Let } A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+d & b+e & c+f \\ 0+d & 0+e & 0+f \end{bmatrix} = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$d = 1, e = 0, f = 1$$

And $a + d = 3$

$$a + 1 = 3$$

$$a = 3 - 1$$

$$a = 2$$

$$b + e = 3$$

$$b + 0 = 3$$

$$b = 3$$

And $c + f = 5$

$$c + 1 = 5$$

$$c = 4$$

Hence,

$$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

Q62

Find the matrix A so that $A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Solution

It is given that:

$$A \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

$$\text{Now, let } X = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$$

Therefore, we have:

$$\begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4c & 2a+5c & 3a+6c \\ b+4d & 2b+5d & 3b+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$$a+4c = -7, \quad 2a+5c = -8, \quad 3a+6c = -9$$

$$b+4d = 2, \quad 2b+5d = 4, \quad 3b+6d = 6$$

$$\text{Now, } a+4c = -7 \Rightarrow a = -7-4c$$

$$\therefore 2a+5c = -8 \Rightarrow -14-8c+5c = -8$$

$$\Rightarrow -3c = 6$$

$$\Rightarrow c = -2$$

$$\therefore a = -7-4(-2) = -7+8 = 1$$

$$\text{Now, } b+4d = 2 \Rightarrow b = 2-4d$$

$$\therefore 2b+5d = 4 \Rightarrow 4-8d+5d = 4$$

$$\Rightarrow -3d = 0$$

$$\Rightarrow d = 0$$

$$\therefore b = 2-4(0) = 2$$

$$\text{Thus, } a = 1, b = 2, c = -2, d = 0$$

$$\text{Hence, the required matrix } X \text{ is } \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}.$$

Q63

Find the matrix A such that

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} A = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}$$

Solution

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e., we can assume $A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$.

Therefore,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3 \times 1} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}_{1 \times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

$$\Rightarrow \begin{bmatrix} 4x(x_1) & 4x(x_2) & 4x(x_3) \\ 1x(x_1) & 1x(x_2) & 1x(x_3) \\ 3x(x_1) & 3x(x_2) & 3x(x_3) \end{bmatrix}_{3 \times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

$$\Rightarrow \begin{bmatrix} 4x_1 & 4x_2 & 4x_3 \\ x_1 & x_2 & x_3 \\ 3x_1 & 3x_2 & 3x_3 \end{bmatrix}_{3 \times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3 \times 3}$$

$$\Rightarrow 4x_1 = -4, 4x_2 = 8, 4x_3 = 4$$

$$\text{Solving } x_1 = -1, x_2 = 2, x_3 = 1$$

$$\text{So, matrix } A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$$

Q64

Find the matrix A such that

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

Solution

Using matrix multiplication,

$$\text{Let, } A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \text{ and } A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } A_1 A_2 &= \begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \\ &= \begin{bmatrix} (2 \times -1) + (1 \times -1) + (3 \times 0) & (2 \times 0) + (1 \times 1) + (3 \times 1) & (2 \times -1) + (1 \times 0) + (3 \times 1) \end{bmatrix} \\ &= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } (A_1 A_2) A_3 &= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} (-3 \times 1) + (4 \times 0) + (1 \times -1) \end{bmatrix} \end{aligned}$$

$$(A_1 A_2) A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating $(A_2 A_3)$ first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

Q65

Find a 2×2 matrix A such that

$$A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I_2$$

Solution

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Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$$A \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$a+b=6 \quad \text{---(i)}$$

$$-2a+4b=0 \quad \text{---(ii)}$$

$$c+d=0 \quad \text{---(iii)}$$

$$-2c+4d=6 \quad \text{---(iv)}$$

Solving equation (i) and (ii)

$$4a+4b=24$$

$$-2a+4b=0$$

$$\begin{array}{r} (+) \quad (-) \\ \hline 6a = 24 \end{array}$$

$$\Rightarrow a = \frac{24}{6}$$

$$a = 4$$

Put $a = 4$ in equation (i)

$$a+b=6$$

$$4+b=6$$

$$b=6-4$$

$$b=2$$

Solving equation (iii) and (iv)

$$2c+2d=0$$

$$-2c+4d=6$$

$$\begin{array}{r} 2c+2d=0 \\ -2c+4d=6 \\ \hline 6d=6 \end{array}$$

$$d = \frac{6}{6}$$

$$d = 1$$

Put $d = 1$ in equation (iii)

$$c+d=0$$

$$c=-1$$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Q66

If $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find A^{16} .

Solution

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$A^4 = A^2 \times A^2$$

$$= 0 \times 0$$

$$= 0$$

$$A^{16} = A^4 \times A^4$$

$$= 0 \times 0$$

$$= 0$$

So,
 A^{16} is null matrix.

Q67

If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$,

Then show that $(A + B)^2 = A^2 + B^2$.

Solution

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Solving the LHS of the given equation we have ,

$$\Rightarrow A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix} \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} (0 \times 0) + ((-x+1) \times (x+1)) & (0 \times (-x+1)) + ((-x+1) \times 0) \\ ((x+1) \times 0) + (0 \times (x+1)) & ((x+1) \times (-x+1)) + (0 \times 0) \end{bmatrix}$$

$$(A+B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$$

Solving the RHS we get,

$$\Rightarrow A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} (0 \times 0) + ((-x) \times (x)) & (0 \times (-x)) + ((-x) \times 0) \\ ((x) \times 0) + (0 \times (x)) & ((x) \times (-x)) + (0 \times 0) \end{bmatrix} + \begin{bmatrix} (0 \times 0) + (1 \times 1) & (0 \times 1) + (1 \times 0) \\ (1 \times 0) + (0 \times 1) & (1 \times 1) + (0 \times 0) \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$$

Substituting the value of $x^2 = -1$ in the LHS and RHS above,

$$\Rightarrow (A+B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \text{ and}$$

$$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow (A+B)^2 = A^2 + B^2.$$

Q68

If $A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A+I)$,

where I is the identity matrix.

Solution

Solving the LHS i.e.

$$\begin{aligned} A^2 + A &= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix} \end{aligned}$$

Solving the RHS i.e.

$$\begin{aligned} A(A+I) &= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix} \end{aligned}$$

So, LHS = RHS verified.

Q69

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3 .

Solution

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Now, } A^2 = AA &= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3 \times 3) + (-5 \times -4) & (3 \times -5) + (-5 \times 2) \\ (-4 \times 3) + (2 \times -4) & (-4 \times -5) + (2 \times 2) \end{bmatrix} \\ &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}, \end{aligned}$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \text{ and } -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\begin{aligned} \therefore A^2 - 5A - 14I &= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix} \\ &= \begin{bmatrix} 29 - 15 - 14 & -25 + 25 + 0 \\ -20 + 20 + 0 & 24 - 10 - 14 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Now,

$$A^2 - 5A - 14I = 0$$

$$\Rightarrow A^2 = 5A + 14I$$

$$\Rightarrow A^3 = A^2 \cdot A = (5A + 14I) \cdot A$$

$$\Rightarrow A^3 = A^2 \cdot A = 5A^2 + 14A \quad \left[\begin{array}{l} \text{By using dist. of matrices over} \\ \text{matrix addition} \end{array} \right]$$

$$\Rightarrow A^3 = 5 \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14 \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Q70

If $p(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x) P(y) = P(x+y) = P(y) P(x)$.

Solution

We have,

$$P(x), P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$\Rightarrow P(x), P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \sin y \cos x + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$\Rightarrow P(x), P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

Now,

$$P(y), P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow P(y), P(x) = \begin{bmatrix} \cos y \cos x - \sin y \sin x & \sin x \cos y + \sin y \cos x \\ -\sin y \cos x - \cos y \sin x & -\sin y \sin x + \cos y \cos x \end{bmatrix}$$

$$\Rightarrow P(y), P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

$$\Rightarrow P(x), P(y) = P(x+y) = P(y), P(x)$$

Q71

if $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, Prove that $PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP$

Solution

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We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\begin{aligned} \text{So, } PQ &= \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \\ &= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix} \\ &= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } QP &= \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix} \\ &= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix} \\ &= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & cz \end{bmatrix} \end{aligned}$$

$$\text{as, } xa = ax, yb = by, zc = cz$$

$$\therefore PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Q72

$$\text{If } A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}, \text{ find } A^2 - 5A + 4I \text{ and hence find a matrix } X \text{ such that}$$

$$A^2 - 5A + 4I + X = 0.$$

Solution

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then,

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + 0 \times 2 + 1 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times -1 & 2 \times 1 + 0 \times 3 + 1 \times 0 \\ 2 \times 2 + 1 \times 2 + 3 \times 1 & 2 \times 0 + 0 \times 1 + 1 \times -1 & 2 \times 1 + 1 \times 3 + 3 \times 0 \\ 1 \times 2 + -1 \times 2 + 0 \times 1 & 1 \times 0 + -1 \times 1 + 0 \times -1 & 1 \times 1 + -1 \times 3 + 0 \times 0 \end{bmatrix},$$

$$= \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, \quad 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\text{Hence, } A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{aligned} A^2 - 5A + 4I &= \begin{bmatrix} 5-10+4 & -1+0+0 & 5-5+0 \\ 9-10+0 & -2-5+4 & 5-15+0 \\ 0-5-0 & -1+5+0 & -2+0+4 \end{bmatrix} \\ &= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix} \end{aligned}$$

Now, given is $A^2 - 5A + 4I + X = 0$

$$\Rightarrow X = -(A^2 - 5A + 4I)$$

$$X = - \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Q73

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all positive integers n .

Solution

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, then

$$A^k = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \quad \text{--- (i)}$$

Step 3: We have to show that $A^{k+1} = \begin{bmatrix} 1 & k+1 \\ 0 & 1 \end{bmatrix}$

So,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \quad \text{(using equation (i) and given)} \\ &= \begin{bmatrix} 1+0 & 1+k \\ 0+0 & 0+1 \end{bmatrix} \\ A^{k+1} &= \begin{bmatrix} 1 & 1+k \\ 0 & 1 \end{bmatrix} \end{aligned}$$

This shows that A^n is true for $n = k + 1$ whenever it is true for $n = k$.

Hence, by the principle of mathematical induction A^n is true for all positive integer.

Q74

If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$ for every positive integer n .

Solution

Given,

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} a^1 & \frac{b(a^1 - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n is true for $n = k$, so,

$$A^k = \begin{bmatrix} a^k & \frac{b(a^k - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \quad \text{--- (i)}$$

Step 3: We have to show that

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b(a^{k+1} - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} a^k & \frac{b(a^k - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \quad (\text{using equation (i) and given}) \\ &= \begin{bmatrix} a^{k+1} + 0 & a^k b + \frac{b(a^k - 1)}{a - 1} \\ 0 + 0 & 0 + 1 \end{bmatrix} \\ &= \begin{bmatrix} a^{k+1} & \frac{a^{k+1}b - a^k b + a^k b - b}{a - 1} \\ 0 & 1 \end{bmatrix} \\ A^{k+1} &= \begin{bmatrix} a^{k+1} & \frac{b(a^{k+1} - 1)}{a - 1} \\ 0 & 1 \end{bmatrix} \end{aligned}$$

So,

A^n is true for $n = k + 1$ whenever it is true $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer n .

Q75

If $A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$, then prove by principle of mathematical induction that

$$A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Solution

Given,

$$A = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

To show that,

$$A^n = \begin{bmatrix} \cos n\theta & i \sin n\theta \\ i \sin n\theta & \cos n\theta \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Put $n = 1$

$$A^1 = \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

So,

$$A^n \text{ is true for } n = 1$$

Let, A^n is true for $n = k$, so

$$A^k = \begin{bmatrix} \cos k\theta & i \sin k\theta \\ i \sin k\theta & \cos k\theta \end{bmatrix} \quad \text{---(i)}$$

Now, we have to show that,

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\theta & i \sin(k+1)\theta \\ i \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

Now, $A^{k+1} = A^k \times A$

$$= \begin{bmatrix} \cos k\theta & i \sin k\theta \\ i \sin k\theta & \cos k\theta \end{bmatrix} \begin{bmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta \cos \theta + i^2 \sin k\theta \sin \theta & i^2 \cos k\theta \sin \theta + i \sin k\theta \cos \theta \\ i \sin k\theta \cos \theta + i \cos k\theta \sin \theta & i^2 \sin k\theta \sin \theta + \cos \theta \cos k\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos k\theta \cos \theta - \sin k\theta \sin \theta & i(\cos k\theta \sin \theta + \sin k\theta \cos \theta) \\ i(\sin k\theta \cos \theta + \cos k\theta \sin \theta) & \cos k\theta \cos \theta - \sin k\theta \sin \theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(k+1)\theta & i \sin(k+1)\theta \\ i \sin(k+1)\theta & \cos(k+1)\theta \end{bmatrix}$$

So, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Q76

If $A = \begin{bmatrix} \cos \alpha + i \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - i \sin \alpha \end{bmatrix}$, prove that

$$A^n = \begin{bmatrix} \cos n\alpha + i \sin n\alpha & \sqrt{2} \sin n\alpha \\ -\sqrt{2} \sin n\alpha & \cos \alpha - i \sin n\alpha \end{bmatrix} \text{ for all } n \in \mathbb{N}.$$

Solution

Given,

$$A = \begin{bmatrix} \cos \alpha + \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - \sin \alpha \end{bmatrix}$$

To prove $P(n)$: $A^n = \begin{bmatrix} \cos n\alpha + \sin n\alpha & \sqrt{2} \sin n\alpha \\ -\sqrt{2} \sin n\alpha & \cos n\alpha - \sin n\alpha \end{bmatrix}$ we use mathematical induction.

Step 1: To show $P(1)$ is true.

A^1 is true for $n = 1$

Step 2: Let, $P(k)$ be true, so

$$A^k = \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2} \sin k\alpha \\ -\sqrt{2} \sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \quad \text{--- (i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1} = \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2} \sin(k+1)\alpha \\ -\sqrt{2} \sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix}$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} \cos k\alpha + \sin k\alpha & \sqrt{2} \sin k\alpha \\ -\sqrt{2} \sin k\alpha & \cos k\alpha - \sin k\alpha \end{bmatrix} \begin{bmatrix} \cos \alpha + \sin \alpha & \sqrt{2} \sin \alpha \\ -\sqrt{2} \sin \alpha & \cos \alpha - \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos k\alpha + \sin k\alpha)(\cos \alpha + \sin \alpha) - 2 \sin k\alpha \sin \alpha & (\cos k\alpha + \sin k\alpha)\sqrt{2} \sin \alpha + \sqrt{2} \sin k\alpha (\cos \alpha - \sin \alpha) \\ (\cos \alpha + \sin \alpha)(-\sqrt{2} \sin k\alpha) - \sqrt{2} \sin \alpha (\cos k\alpha - \sin k\alpha) & -2 \sin k\alpha \sin \alpha + (\cos k\alpha - \sin k\alpha)(\cos \alpha - \sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos k\alpha \cos \alpha + \sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha + \sin k\alpha \sin \alpha - 2 \sin k\alpha \sin \alpha & \sqrt{2} \cos k\alpha \sin \alpha + \sqrt{2} \sin k\alpha \cos \alpha + \sqrt{2} \sin k\alpha \cos \alpha - \sqrt{2} \sin k\alpha \sin \alpha \\ -\sqrt{2} \cos \alpha \sin \alpha - \sqrt{2} \sin \alpha \sin k\alpha - \sqrt{2} \sin \alpha \cos k\alpha + 2 \sin k\alpha \sin \alpha + \cos k\alpha \cos \alpha - \cos \alpha \sin k\alpha + \sqrt{2} \sin \alpha \sin k\alpha & \sin k\alpha - \sin \alpha \cos k\alpha \sin \alpha \sin k\alpha \end{bmatrix} \\ &= \begin{bmatrix} \cos \alpha \cos k\alpha + \sin \alpha \sin k\alpha & \sqrt{2} (\sin k\alpha \cos \alpha + \cos k\alpha \sin \alpha) \\ \sin \alpha \cos k\alpha + \sin k\alpha \cos \alpha & \cos k\alpha \cos \alpha - \sin k\alpha \sin \alpha - (\sin k\alpha \cos \alpha + \sin \alpha \cos k\alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos(k+1)\alpha + \sin(k+1)\alpha & \sqrt{2} \sin(k+1)\alpha \\ -\sqrt{2} \sin(k+1)\alpha & \cos(k+1)\alpha - \sin(k+1)\alpha \end{bmatrix} \end{aligned}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

Q77

If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then use the principle of mathematical induction to show that

$$A^n = \begin{bmatrix} 1 & n & n(n+1)/2 \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix} \text{ for every positive integer } n.$$

Solution

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To prove, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$, we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: We will prove that A^n be true for $n = k + 1$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad (\text{using equation (i) and given}) \\ &= \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & (k+1) & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

Hence, A^n is true for $n = k + 1$ whenever it is true for $n = k$.

So, by principle of mathematical induction A^n is true for all positive integer n .

Q78

If B, C are n rowed square matrices and if $A = B + C$, $BC = CB$, $C^2 = 0$, then show that for every $n \in N$, $A^{n+1} = B^n (B + (n+1)C)$.

Solution

We will prove $P(n): A^{n+1} = B^n [B + (n+1)C]$ is true for all natural numbers using mathematical induction.

Given,

$$A = B + C, \quad BC = CB, \quad C^2 = 0$$

$$A = B + C$$

Squaring both the sides, so

$$\begin{aligned} A^2 &= (B + C)^2 \\ \Rightarrow A^2 &= (B + C)(B + C) \\ \Rightarrow A^2 &= B \times B + BC + CB + C \times C && \text{(using distributive property)} \\ \Rightarrow A^2 &= B^2 + BC + BC + C^2 && \text{(using } BC = CB \text{ given)} \\ \Rightarrow A^2 &= B^2 + 2BC + 0 && \text{(since, given } C^2 = 0) \\ \Rightarrow A^2 &= B^2 + 2BC && \text{---(1)} \\ A^2 &= B(B + 2C) \end{aligned}$$

Now, consider

$$P(n): A^{n+1} = B^n [B + (n+1)C]$$

Step 1: To prove $P(1)$ is true, put $n = 1$

$$\begin{aligned} A^{1+1} &= B^1 [B + (1+1)C] \\ A^2 &= B [B + 2C] \\ A^2 &= B^2 + 2BC \end{aligned}$$

From equation (i), $P(1)$ is true.

Step 2: Suppose $P(k)$ is true.

$$A^{k+1} = B^k [B + (k+1)C] \quad \text{---(2)}$$

Step 3: Now, we have to show that $P(k+1)$ is true.

That is we need to prove that,

$$A^{k+2} = B^{k+1} [B + (k+2)C]$$

Now,

$$\begin{aligned} A^{k+2} &= A^k \times A^2 \\ &= B^{(k-1)} [B + kC] \times [B + 2C] \\ &= B^k [B + kC] \times [B + 2C] \\ &= B^k [B \times B + B \times 2C + kC \times B + 2kC^2] \\ &= B^k [B^2 + 2BC + kBC + 2k \times 0] && \text{(since } BC = CB, \quad C^2 = 0) \\ &= B^k [B^2 + BC(2+k)] \\ &= B^k \times B [B + (k+2)C] \\ &= B^{k+1} [B + (k+2)C] \end{aligned}$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Therefore by principle of mathematical induction $P(n)$ is true for all natural number.

Q79

If $A = \text{diag}(a, b, c)$, show that $A^n = \text{diag}(a^n, b^n, c^n)$ for all positive integer n .

Solution

Given,

$$A = \text{diag}(a, b, c)$$

Show that,

$$A^n = \text{diag}(a^n, b^n, c^n)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}(a^1, b^1, c^1)$$

$$A = \text{diag}(a, b, c)$$

So,

$$A^n \text{ is true for } n = 1$$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}(a^k, b^k, c^k) \quad \text{--- (i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}(a^{k+1}, b^{k+1}, c^{k+1})$$

Now,

$$\begin{aligned} A^{k+1} &= A^k \times A \\ &= \text{diag}(a^k, b^k, c^k) \times \text{diag}(a, b, c) \quad \text{(using equation (i) and given)} \end{aligned}$$

$$\begin{aligned} A^{k+1} &= \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix} \\ &= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix} \\ &= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix} \end{aligned}$$

$$A^{k+1} = \text{diag}(a^{k+1}, b^{k+1}, c^{k+1})$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Q80

A matrix X has $a+b$ rows $a+2$ columns the matrix Y has $b+1$ rows and $a+3$ columns. Both matrices XY and YX exist. Find a and b .
Can you say XY and YX are of the same type? Are they equal.

Solution

Given,

$$\text{order of matrix } X = (a+b) \times (a+2)$$

$$\text{order of matrix } Y = (b+1) \times (a+3)$$

Given, $X_{(a+b) \times (a+2)} \cdot Y_{(b+1) \times (a+3)}$ exist,

$$\Rightarrow a+2 = b+1$$

$$\Rightarrow a-b = -1 \quad \dots (i)$$

And

$$Y_{(b+1) \times (a+3)} \cdot X_{(a+b) \times (a+2)} \text{ exists.}$$

$$\Rightarrow a+3 = a+b$$

$$\Rightarrow b = 3$$

Put $b = 3$ in equation (i),

$$a-b = -1$$

$$a-3 = -1$$

$$a = 3-1$$

$$a = 2$$

$$\text{So, } a = 2, b = 3$$

So,

$$\text{Order of } X = (a+b) \times (a+2)$$

$$= (2+3) \times (2+2)$$

$$= 5 \times 4$$

$$\text{Order of } Y = (b+1) \times (a+3)$$

$$= (3+1) \times (2+3)$$

$$= 4 \times 5$$

$$\text{Order of } X_{5 \times 4} \cdot Y_{4 \times 5} = 5 \times 5$$

$$\text{Order of } X_{4 \times 5} \cdot Y_{5 \times 4} = 4 \times 4$$

So, order of XY and YX are not same and they are not equal but both are square matrices.

Q81

Give an example of matrices A and B such that $AB \neq BA$

Solution

$$\begin{aligned}
 \text{Let, } A &= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \\
 AB &= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix} \\
 AB &= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad \text{---(i)} \\
 BA &= \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\
 BA &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
 \end{aligned}$$

From equation (i) and (ii)

$$AB \neq BA$$

$$\text{when } A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

Q82

Given an example of matrices A and B such that $AB = 0$ but $A \neq 0, B \neq 0$.

Solution

$$\begin{aligned}
 \text{Let, } A &= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0 \\
 B &= \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0 \\
 AB &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\
 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}
 \end{aligned}$$

Hence,

$$AB = 0$$

When,

$$\begin{aligned}
 A &= \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0 \\
 B &= \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0
 \end{aligned}$$

Q83

Give an example of matrices A and B such that $AB = 0$ but $BA \neq 0$.

Solution

Let, $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{aligned} AB &= \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$AB = 0$$

$$\begin{aligned} BA &= \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \\ BA &\neq 0 \end{aligned}$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$

Q84

Given an example of matrices A, B and C such that $AB = AC$ but $B \neq C$, $A \neq 0$.

Solution

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

$$A \neq 0, B \neq C$$

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} &= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$\text{LHS} = \text{RHS}$$

So,

for $A \neq 0$, $B \neq C$ but $AB = AC$

We have,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Q85

Let A and B be square matrices of the same order. Does $(A+B)^2 = A^2 + 2AB + B^2$ hold? If not, why?

Solution

Given,

A and B are square matrices of same order

$$\begin{aligned}(A+B)^2 &= (A+B)(A+B) \\ &= A(A+B) + B(A+B) && \text{(using distributive property)} \\ &= A \times A + AB + BA + B^2 \\ &= A^2 + AB + BA + B^2\end{aligned}$$

But,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ is possible only when } AB = BA$$

Here, we can not say that $AB = BA$

So,

$$(A+B)^2 = A^2 + 2AB + B^2 \text{ does not hold.}$$

Q86

If A and B are square matrices of the same order, explain, why in general

$$(i) (A+B)^2 \neq A^2 + 2AB + B^2 \quad (ii) (A-B)^2 \neq A^2 - 2AB + B^2$$

$$(iii) (A+B)(A-B) \neq A^2 - B^2$$

Solution

Given, A and B are square matrices of same order.

$$\begin{aligned}
 \text{(i)} \quad (A+B)^2 &= (A+B)(A+B) \\
 &= A(A+B) + B(A+B) \quad \text{(using distributive property)} \\
 &= A \times A + AB + BA + B \times B \\
 &= A^2 + AB + BA + B^2 \\
 &\neq A^2 + 2AB + B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$).

$$\text{So, } (A+B)^2 \neq A^2 + 2AB + B^2$$

$$\begin{aligned}
 \text{(ii)} \quad (A-B)^2 &= (A-B)(A-B) \\
 &= A(A-B) - B(A-B) \quad \text{(using distributive property)} \\
 &= A \times A - AB - BA + B \times B \\
 &= A^2 - AB - BA + B^2 \\
 &\neq A^2 - 2AB + B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$), so

$$\text{So, } (A-B)^2 \neq A^2 - 2AB + B^2$$

$$\begin{aligned}
 \text{(iii)} \quad (A+B)(A-B) &= A(A-B) + B(A-B) \quad \text{(using distributive property)} \\
 &= A \times A - AB + BA - B \times B \\
 &= A^2 - AB + BA - B^2 \\
 &\neq A^2 - B^2
 \end{aligned}$$

Since, in general matrix multiplication is not commutative ($AB \neq BA$),

$$\text{So, } (A+B)(A-B) \neq A^2 - B^2$$

Q87

Let A and B be square matrices of the order 3×3 .
Is $(AB)^2 = A^2 B^2$? Give reasons.

Solution

The given equality is true only when we choose A and B to be a square matrix in such a way that $AB = BA$ else the result is not true in general.

Example: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{aligned} \text{Here } AB &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 \times 0 + 0 \times 2 + 0 \times 0 & 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 1 \times 0 + 1 \times 2 + 0 \times 0 & 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{and } BA &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 1 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$AB \neq BA$$

$$\begin{aligned} \text{Now, } (AB)^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \times 0 + 1 \times 2 + 0 \times 0 & 0 \times 1 + 1 \times 2 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 1 \times 0 + 2 \times 2 + 0 \times 0 & 1 \times 1 + 2 \times 2 + 0 \times 0 & 1 \times 0 + 2 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 2 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 2 & 0 \\ 5 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 A^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 1 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 1 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 1 \times 1 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 1 + 0 \times 0 & 1 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 B^2 &= \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 0 \times 0 + 1 \times 2 + 0 \times 0 & 0 \times 1 + 1 \times 1 + 0 \times 0 & 0 \times 0 + 1 \times 0 + 0 \times 1 \\ 2 \times 0 + 1 \times 2 + 0 \times 0 & 2 \times 1 + 1 \times 1 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 0 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 1 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 A^2 B^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 1 \times 2 + 0 \times 2 + 0 \times 0 & 1 \times 1 + 0 \times 2 + 0 \times 0 & 1 \times 0 + 0 \times 0 + 0 \times 1 \\ 2 \times 2 + 1 \times 2 + 0 \times 0 & 2 \times 1 + 1 \times 2 + 0 \times 0 & 2 \times 0 + 1 \times 0 + 0 \times 1 \\ 0 \times 2 + 0 \times 2 + 1 \times 0 & 0 \times 1 + 0 \times 2 + 1 \times 0 & 0 \times 0 + 0 \times 0 + 1 \times 1 \end{bmatrix} \\
 &= \begin{bmatrix} 2 & 1 & 0 \\ 6 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2 B^2$.

Q88

If A and B be square matrices of the same order such that $AB = BA$, then show that $(A + B)^2 = A^2 + 2AB + B^2$.

Solution

Given,

A and B two square matrices of same order such that
 $AB = BA$.

To prove : $(A+B)^2 = A^2 + 2AB + B^2$

Now, solving LHS gives,

$$\begin{aligned}
 (A+B)^2 &= (A+B)(A+B) \\
 &= A(A+B) + B(A+B) && \left[\begin{array}{l} \text{by dist. of matrix multiplication} \\ \text{over addition} \end{array} \right] \\
 &= A^2 + AB + BA + B^2 && \left[\begin{array}{l} \text{by dist. of matrix multiplication} \\ \text{over addition} \end{array} \right] \\
 &= A^2 + 2AB + B^2 && [As, AB = BA] \\
 &= RHS
 \end{aligned}$$

Hence proved.

Q89

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$

Verify that $AB = AC$ though $B \neq C$, $A \neq O$.

Solution

Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix}$

$$\begin{aligned}
 AB &= \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 5 & 2 \\ -2 & 4 \end{bmatrix} \\
 &= \begin{bmatrix} 3+5-2 & 1+2+4 \\ 9+15-6 & 3+6+12 \end{bmatrix} \\
 AB &= \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} && \text{---(i)} \\
 AC &= \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix} \begin{bmatrix} 4 & 2 \\ -3 & 5 \\ 5 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 4-3+5 & 2+5+0 \\ 12-9+15 & 6+15+0 \end{bmatrix} \\
 AC &= \begin{bmatrix} 6 & 7 \\ 18 & 21 \end{bmatrix} && \text{---(ii)}
 \end{aligned}$$

From equation (i) and (ii)
 $AB = AC$

Q90

Three shopkeepers A, B and C go to a store to buy stationary. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs Rs. 1.25 and a pencil costs 35 paise. Use matrix multiplication to calculate each individual's bill.

Solution

The number of items purchased by A, B and C are represented in matrix form as,

$$X = \begin{matrix} & \begin{matrix} \text{Notebook} & \text{Pens} & \text{Pencils} \end{matrix} \\ \begin{matrix} A \\ B \\ C \end{matrix} & \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \end{matrix}$$

Now, matrix formed by the cost of each items is given by,

$$Y = \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \begin{matrix} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{matrix}$$

Individual bill can be calculated by

$$XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix}$$

$$XY = \begin{bmatrix} 57.60 + 75.00 + 25.20 \\ 48.00 + 90.00 + 29.40 \\ 52.80 + 195.00 + 33.60 \end{bmatrix}$$

$$XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

So,

$$\begin{aligned} \text{Bill of A} &= \text{Rs } 157.80 \\ \text{Bill of B} &= \text{Rs } 167.40 \\ \text{Bill of C} &= \text{Rs } 281.40 \end{aligned}$$

Q91

The cooperative stores of a particular school has 10 dozen physics books, 8 dozen chemistry books and 5 dozen mathematics books. Their selling prices are Rs. 8.30, Rs. 3.45 and Rs. 4.50 each respectively. Find the total amount the store will receive from selling all the items.

Solution

Matrix representation of stock of various types of book in the store is given by,

$$X = \begin{matrix} & \begin{matrix} \text{Physics} & \text{Chemistry} & \text{Mathematics} \end{matrix} \\ \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \end{matrix}$$

Matrix representation of selling price (Rs.) of each book is given by

$$Y = \begin{matrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} & \begin{matrix} \text{Physics} \\ \text{Chemistry} \\ \text{Mathematics} \end{matrix} \end{matrix}$$

So, total amount received by the store from selling all the items is given by,

$$\begin{aligned} XY &= \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \\ &= [(120)(8.30) + (96)(3.45) + (60)(4.50)] \\ &= [996 + 331.20 + 270] \\ &= [1597.20] \end{aligned}$$

Required amount = Rs 1597.20

Q92

In a legislative assembly election, a political group hired a public relations firm to promote its candidates in three ways: telephone, house calls and letters. The cost per contact (in paise) is given in matrix A as

$$A = \begin{matrix} \text{cost per contact} \\ \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{House call} \\ \text{Letter} \end{matrix} \end{matrix}$$

The number of contacts of each type made in two cities X and Y is given in matrix B as

$$B = \begin{matrix} \begin{matrix} \text{Telephone} & \text{Housecall} & \text{Letter} \end{matrix} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{matrix} \rightarrow X \\ \rightarrow Y \end{matrix} \end{matrix}$$

Solution

Given,

The cost per contact (in paise) is given by

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{bmatrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{bmatrix}$$

The number of contact of each type made in two cities X and Y is given by:

$$B = \begin{bmatrix} \text{Telephone} & \text{Housecall} & \text{Letter} \\ 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$$

Total amount spent by the group in the two cities X and Y can be given by

$$\begin{aligned} BA &= \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \\ &= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix} \\ &= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix} \end{aligned}$$

Hence,

Amount spend on X = Rs 3400

Amount spend on Y = Rs 7200

Q93

A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) Rs 1,800 (b) Rs 2,000

Solution

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.

Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} 5 \\ 100 \\ 7 \\ 100 \end{bmatrix} = 1800 \quad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100} \right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 1800$$

$$\Rightarrow 5x + 210000 - 7x = 180000$$

$$\Rightarrow 210000 - 2x = 180000$$

$$\Rightarrow 2x = 210000 - 180000$$

$$\Rightarrow 2x = 30000$$

$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.

(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000 - x)$.

Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000 - x) \end{bmatrix} \begin{bmatrix} 5 \\ 100 \\ 7 \\ 100 \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000 - x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow 210000 - 2x = 200000$$

$$\Rightarrow 2x = 210000 - 200000$$

$$\Rightarrow 2x = 10000$$

$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond.

To promote making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) letters and (iii) announcements. The cost for each mode per attempt is given below:

- i. Rs. 50
- ii. Rs. 20
- iii. Rs. 40

The number of attempts made in three villages X, Y, and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Find the total cost incurred by the organization for three villages separately, using matrices.

Solution

The cost for each mode per attempt is represented by 3×1 matrix:

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

The number of attempts made in the three villages X, Y, and Z are represented by a 3×3 matrix:

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

The total cost incurred by the organization for the three villages separately is given by matrix multiplication.

$$\begin{aligned}
 BA &= \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} \\
 BA &= \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix} \\
 &= \begin{bmatrix} 30,000 \\ 23,000 \\ 39,000 \end{bmatrix}
 \end{aligned}$$

Note: The answer given in the book is incorrect.

Q95

There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommended daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using matrix. Using matrix multiplication, Calculate the total requirement of calories and proteins for each of the families. What awareness can you create among people about the planned diet from this question?

Solution

Let F be the family matrix and R be the requirement matrix. Then,

$$F = \begin{matrix} & \begin{matrix} \text{Men} & \text{Women} & \text{Children} \end{matrix} \\ \begin{matrix} \text{Family A} \\ \text{Family B} \end{matrix} & \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \end{matrix}$$

$$R = \begin{matrix} & \begin{matrix} \text{Calories} & \text{Protein} \end{matrix} \\ \begin{matrix} \text{Men} \\ \text{Women} \\ \text{Children} \end{matrix} & \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{matrix}$$

The requirement of calories and protein of each of the two families is given by the product matrix FR, as matrix F has number of columns equal to number of rows of R, thus,

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$FR = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$FR = \begin{matrix} & \begin{matrix} \text{Calories} & \text{Protein} \end{matrix} \\ \begin{matrix} \text{Family A} \\ \text{Family B} \end{matrix} & \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix} \end{matrix}$$

we can say that balanced diet having the required amount of calories and protein must be taken by each of the family.

Q96

In a parliament election, a political party hired a public relations firm to promote its candidates in three ways - telephone, house calls and letters. The cost per contact (in paisa) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{matrix}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{matrix} \begin{matrix} \text{Telephone} & \text{House calls} & \text{Letters} \end{matrix} & \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{matrix} \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix}$$

Find the total amount spent by the party in the two cities.

What should one consider before casting his/her vote - party's promotional activity or their social activities?

Solution

The cost per contact (in paisa) is given in matrix A as:

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{matrix} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{matrix}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{matrix} & \begin{matrix} \text{Telephone} & \text{House calls} & \text{Letters} \end{matrix} \\ \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix} & \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{matrix}$$

The total amount of money spent by party in each of the city for the election is given by the matrix multiplication :

$$\begin{aligned} BA &= \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \\ &= \begin{bmatrix} 1000 \times 140 + 500 \times 200 + 5000 \times 150 \\ 3000 \times 140 + 1000 \times 200 + 10000 \times 150 \end{bmatrix} \\ &= \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix} \begin{bmatrix} 990000 \\ 2120000 \end{bmatrix} \end{aligned}$$

The total amount of money spent by party in each of the city for the election in rupees is given by

$$\begin{aligned} &= \left(\frac{1}{100} \right) \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix} \begin{bmatrix} 990000 \\ 2120000 \end{bmatrix} \\ &= \begin{matrix} \text{City X} \\ \text{City Y} \end{matrix} \begin{bmatrix} 9900 \\ 21200 \end{bmatrix} \end{aligned}$$

One should consider social activities before casting his/her vote to the party.

Exercise 5.4

Q1

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(2A)^T = 2A^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(2A)^T = 2 \times A^T$$

$$\Rightarrow \left(2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -6 \\ -14 & 10 \end{bmatrix}^T = 2 \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix} = \begin{bmatrix} 4 & -14 \\ -6 & 10 \end{bmatrix}$$

$$\text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2A^T$$

Q2

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(A+B)^T = A^T + B^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T + \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & -3 \\ -5 & 1 \end{bmatrix}^T = \begin{bmatrix} 2+1 & -3+0 \\ -7+2 & 5-4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -3 & 1 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Q3

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(A-B)^T = A^T - B^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(A-B)^T = A^T - B^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2-1 & -3-0 \\ -7-2 & 5+4 \end{bmatrix}^T = \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}^T - \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1 & -3 \\ -9 & 9 \end{bmatrix}^T = \begin{bmatrix} 2-1 & -7-2 \\ -3-0 & 5+4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix} = \begin{bmatrix} 1 & -9 \\ -3 & 9 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(A-B)^T = A^T - B^T$$

Q4

Let $A = \begin{bmatrix} 2 & -3 \\ -7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \right)^T = \left(\begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -7 & -5 \end{bmatrix} \right)^T$$

$$\Rightarrow \begin{bmatrix} 2-6 & 0+12 \\ -7+10 & 0-20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & -7 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 12 \\ 3 & -20 \end{bmatrix}^T = \begin{bmatrix} 2-6 & -7+10 \\ 0+12 & 0-20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 12 & -20 \end{bmatrix}$$

$$\Rightarrow \text{HS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Q5

If $A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$, verify that $(AB)^T = B^T A^T$

Solution

Given;

$$A = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 4 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \right)^T = \left(\begin{bmatrix} 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix} \right)^T$$

$$\Rightarrow \begin{bmatrix} 3 & 0 & 12 \\ 5 & 0 & 20 \\ 2 & 0 & 8 \end{bmatrix}^T = \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 5 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix} = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 0 & 0 \\ 12 & 20 & 8 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Q6

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that $(A+B)^T = A^T + B^T$

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(A+B)^T = A^T + B^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1+1 & -1+2 & 0+3 \\ 2+2 & 1+1 & 3+3 \\ 1+0 & 2+1 & 1+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 6 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 1+1 & 2+2 & 1+0 \\ -1+2 & 1+1 & 2+1 \\ 0+3 & 3+3 & 1+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 1 \\ 1 & 2 & 3 \\ 3 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(A+B)^T = A^T + B^T$$

Q7

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T, B^T and verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1-2+0 & 2-1+0 & 3-3+0 \\ 2+2+0 & 4+1+3 & 6+3+3 \\ 1+4+0 & 2+2+1 & 3+6+1 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}^T \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 0 \\ 4 & 8 & 12 \\ 5 & 5 & 10 \end{bmatrix}^T = \begin{bmatrix} 1-2+0 & 2+2+0 & 1+4+0 \\ 2-1+0 & 4+1+3 & 2+2+1 \\ 3-3+0 & 6+3+3 & 3+6+1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix} = \begin{bmatrix} -1 & 4 & 5 \\ 1 & 8 & 5 \\ 0 & 12 & 10 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Q8

Let $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$. Find A^T , B^T and verify that $(2A)^T = 2A^T$.

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$$

$$(2A)^T = 2A^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & -1 & 0 \\ 2 & 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \right)^T = 2 \begin{bmatrix} 1 & -1 & 0 \\ 2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 & 0 \\ 4 & 2 & 6 \\ 2 & 4 & 2 \end{bmatrix} = 2 \begin{bmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 2 \\ -2 & 2 & 4 \\ 0 & 6 & 2 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times A^T$$

Q9

If $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$, $B = [1 \ 3 \ -6]$, verify that $(AB)^T = B^T A^T$

Solution

Given,

$$A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}, B = [1 \ 3 \ -6]$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix} [1 \ 3 \ -6] \right)^T = [1 \ 3 \ -6]^T \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(2A)^T = 2 \times A^T$$

Q10

If $A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$, find $(AB)^T$

Solution

Given,

$$A = \begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$(AB)^T$$

$$\begin{aligned} &= \left(\begin{bmatrix} 2 & 4 & -1 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 2 & 1 \end{bmatrix} \right)^T \\ &= \begin{bmatrix} 6-4-2 & 8+8-1 \\ -3+0+4 & -4+0+2 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 15 \\ 1 & -2 \end{bmatrix}^T \\ &= \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix} \end{aligned}$$

So,

$$(AB)^T = \begin{bmatrix} 0 & 1 \\ 15 & -2 \end{bmatrix}$$

Q11

For two matrices A and B , $A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$ verify that

$$(AB)^T = B^T A^T$$

Solution

Given,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 5 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 0 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 2+0+15 & -2+20 \\ 4+0+0 & -4+2+0 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}^T \begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 17 & 0 \\ 4 & -2 \end{bmatrix}^T = \begin{bmatrix} 2+0+15 & 4+0+0 \\ -2+2+0 & -4+2+0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 17 & 4 \\ 0 & -2 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Q12

For the matrices A and B , verify that $(AB)^T = B^T A^T$, where

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

Solution

Given,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}$$

$$(AB)^T = B^T A^T$$

$$\Rightarrow \left(\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix} \right)^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 1+6 & 4+15 \\ 2+8 & 8+20 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 4 & 5 \end{bmatrix}^T \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}^T$$

$$\Rightarrow \begin{bmatrix} 7 & 19 \\ 10 & 28 \end{bmatrix}^T = \begin{bmatrix} 1+6 & 2+8 \\ 4+15 & 8+20 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 19 & 28 \end{bmatrix}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

So,

$$(AB)^T = B^T A^T$$

Q13

If $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find $A^T - B^T$.

Solution

Given that $A^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.

We need to find $A^T - B^T$.

Given that, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$$\Rightarrow B^T = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

Let us find $A^T - B^T$:

$$A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 3+1 & 4-1 \\ -1-2 & 2-2 \\ 0-1 & 1-3 \end{bmatrix}$$

$$\Rightarrow A^T - B^T = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$$

Q14

If $A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$, then verify that $A'A = I$

Solution

$$A = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$$

$$\begin{aligned} A'A &= \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix} \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\cos \alpha)(\cos \alpha) + (-\sin \alpha)(-\sin \alpha) & (\cos \alpha)(\sin \alpha) + (-\sin \alpha)(\cos \alpha) \\ (\sin \alpha)(\cos \alpha) + (\cos \alpha)(-\sin \alpha) & (\sin \alpha)(\sin \alpha) + (\cos \alpha)(\cos \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \alpha + \sin^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \sin^2 \alpha + \cos^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Q15

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}, \text{ then verify that } A'A = I$$

Solution

$$A = \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}$$

$$A'A = \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix}$$

$$\begin{aligned} &= \begin{bmatrix} \sin \alpha & -\cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix} \begin{bmatrix} \sin \alpha & \cos \alpha \\ -\cos \alpha & \sin \alpha \end{bmatrix} \\ &= \begin{bmatrix} (\sin \alpha)(\sin \alpha) + (-\cos \alpha)(-\cos \alpha) & (\sin \alpha)(\cos \alpha) + (-\cos \alpha)(\sin \alpha) \\ (\cos \alpha)(\sin \alpha) + (\sin \alpha)(-\cos \alpha) & (\cos \alpha)(\cos \alpha) + (\sin \alpha)(\sin \alpha) \end{bmatrix} \\ &= \begin{bmatrix} \sin^2 \alpha + \cos^2 \alpha & \sin \alpha \cos \alpha - \sin \alpha \cos \alpha \\ \sin \alpha \cos \alpha - \sin \alpha \cos \alpha & \cos^2 \alpha + \sin^2 \alpha \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \end{aligned}$$

Hence, we have verified that $A'A = I$.

Q16

If l_i, m_i, n_i ; $i = 1, 2, 3$ denote the direction cosines of three mutually perpendicular vectors in space, prove that $AA^T = I$,

Where $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$.

Solution

Given,

$$\begin{aligned} & l_i, m_i, n_i \text{ are direction cosines of three mutually perpendicular vectors} \\ \Rightarrow & \begin{cases} l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \\ l_2 l_3 + m_2 m_3 + n_2 n_3 = 0 \\ l_1 l_3 + m_1 m_3 + n_1 n_3 = 0 \end{cases} \quad \text{--- (A)} \end{aligned}$$

And,

$$\begin{cases} l_1^2 + m_1^2 + n_1^2 = 1 \\ l_2^2 + m_2^2 + n_2^2 = 1 \\ l_3^2 + m_3^2 + n_3^2 = 1 \end{cases} \quad \text{--- (B)}$$

Given,

$$\begin{aligned} A &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \\ AA^T &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \\ &= \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} \begin{bmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \\ n_1 & n_2 & n_3 \end{bmatrix} \\ &= \begin{bmatrix} l_1^2 + m_1^2 + n_1^2 & l_1 l_2 + m_1 m_2 + n_1 n_2 & l_1 l_3 + m_1 m_3 + n_1 n_3 \\ l_2 l_1 + m_2 m_1 + n_2 n_1 & l_2^2 + m_2^2 + n_2^2 & l_2 l_3 + m_2 m_3 + n_2 n_3 \\ l_3 l_1 + m_3 m_1 + n_3 n_1 & l_3 l_2 + m_3 m_2 + n_3 n_2 & l_3^2 + m_3^2 + n_3^2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{(Using (A) and (B))} \\ &= I \end{aligned}$$

Hence,

$$AA^T = I$$

Exercise 5.5

Q1

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\begin{aligned} (A - A^T) &= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T \right) \\ &= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} \right) \\ &= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix} \end{aligned}$$

$$(A - A^T) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$\begin{aligned} -(A - A^T)^T &= -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T \\ &= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \end{aligned}$$

$$-(A - A^T)^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii),

$$(A - A^T) = -(A - A^T)^T$$

We know that, X is a skew symmetric matrix if $X = -X^T$

So, $(A - A^T)$ is skew symmetric.

Q2

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $A - A^T$ is a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$\begin{aligned} A - A^T &= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T \\ &= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 3-3 & -4-1 \\ 1-4 & -1+1 \end{bmatrix} \end{aligned}$$

$$A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{--- (i)}$$

$$\begin{aligned} -(A - A^T)^T &= -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T \\ &= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix} \end{aligned}$$

$$-(A - A^T)^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$(A - A^T) = -(A - A^T)^T$$

We know that X is skewsymmetric matrix if $X = -X^T$.

So, $(A - A^T)$ is skewsymmetric matrix.

Q3

If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t .

Solution

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [a_{ij}]_{m \times n}$ is a symmetric matrix if $a_{ij} = a_{ji}$

$$\text{So, } x = a_{13} = a_{31} = 4$$

$$y = a_{21} = a_{12} = 2$$

$$z = a_{22} = a_{22} = z$$

$$t = a_{32} = a_{23} = -3$$

Hence,

$$x = 4, y = 2, t = -3 \text{ and } z \text{ can have any value.}$$

Q4

Let $A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$. Find matrix X and Y such that $X + Y = A$, where

X is a symmetric and Y is a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$$

$$\begin{aligned} \therefore X &= \frac{1}{2}(A + A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} + \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 3+3 & 2+1 & 7-2 \\ 1+2 & 4+4 & 3+5 \\ -2+7 & 5+3 & 8+8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 6 & 3 & 5 \\ 3 & 8 & 8 \\ 5 & 8 & 16 \end{bmatrix} \\ \Rightarrow X &= \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{Now, } Y &= \frac{1}{2}(A - A^T) \\ &= \frac{1}{2} \left(\begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} - \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix} \right) \\ &= \frac{1}{2} \begin{bmatrix} 3-3 & 2-1 & 7+2 \\ 1-2 & 4-4 & 3-5 \\ -2-7 & 5-3 & 8-8 \end{bmatrix} \\ &= \frac{1}{2} \begin{bmatrix} 0 & 1 & 9 \\ -1 & 0 & -2 \\ -9 & 2 & 0 \end{bmatrix} \\ Y &= \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ -\frac{1}{2} & 0 & -1 \\ -\frac{9}{2} & 1 & 0 \end{bmatrix} \end{aligned}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix.

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$\Rightarrow -Y^T = Y$

$\therefore Y$ is skew symmetric matrix.

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}+\frac{-1}{2} & 4+0 & 4-1 \\ \frac{5}{2}+\frac{-9}{2} & 4+1 & 8+0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ \frac{1}{2} & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$

$$= A$$

Hence,

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}, Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$$

Let $X = \frac{1}{2}(A + A^T)$

$$= \frac{1}{2} \left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 4+4 & 2+3 & -1+1 \\ 3+2 & 5+5 & 7-2 \\ 1-1 & -2+7 & 1+1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} = X$$

$\Rightarrow$ X is symmetric matrix

Now,

$$Y = \frac{1}{2}(A - A^T)$$

$$= \frac{1}{2} \left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 4-4 & 2-3 & -1-1 \\ 3-2 & 5-5 & 7+2 \\ 1+1 & -2-7 & 1-1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}$$

$$\Rightarrow -Y^T = - \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric matrix.

Now,

$$\begin{aligned}
 X+Y &= \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 4+0 & \frac{5}{2}-\frac{1}{2} & 0-1 \\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2} \\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} = A
 \end{aligned}$$

Q6

Define a symmetric matrix. Prove that for $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, $A + A^T$ is a symmetric matrix where A^T is the transpose of A .

Solution

A square matrix A is called a symmetric matrix, if $A^T = A$.

Here,

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$\begin{aligned}
 A + A^T &= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}^T \\
 &= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} \\
 &= \begin{bmatrix} 2+2 & 4+4 \\ 5+5 & 6+6 \end{bmatrix}
 \end{aligned}$$

$$A + A^T = \begin{bmatrix} 4 & 8 \\ 10 & 12 \end{bmatrix} \quad \text{--- (i)}$$

$$(A + A^T)^T = \begin{bmatrix} 4 & 8 \\ 10 & 12 \end{bmatrix}^T$$

$$(A + A^T)^T = \begin{bmatrix} 4 & 8 \\ 10 & 12 \end{bmatrix} \quad \text{--- (ii)}$$

From equation (i) and (ii),

$$(A + A^T)^T = (A + A^T)$$

So,

$(A + A^T)$ is a symmetric matrix.

Q7

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Solution

Here,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$\text{Let, } X = \frac{1}{2}(A + A^T) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$\text{Now, } X^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} = X$$

$\Rightarrow$ X is symmetric matrix

$$\text{Let, } Y = \frac{1}{2}(A - A^T) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

$$\text{Now, } -Y^T = -\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^T = -\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is skew symmetric

$$\text{Now, } X + Y = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{-3}{2}+\frac{-5}{2} \\ \frac{-3}{2}+\frac{5}{2} & -1+0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Q8

Express the following matrix as the sum of a symmetric and skew-

symmetric matrix and verify your result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Solution

Let,

$$A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$\begin{aligned} \text{Let, } X &= \frac{1}{2}(A + A^T) = \frac{1}{2} \left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3+3 & -2+3 & -4-1 \\ 3-2 & -2-2 & -5+1 \\ -1-4 & 1-5 & 2+2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} \end{aligned}$$

$$\text{Now, } X^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} = X$$

$\Rightarrow$ X is a symmetric matrix

$$\begin{aligned} \text{Let, } Y &= \frac{1}{2}(A - A^T) = \frac{1}{2} \left(\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3-3 & -2-3 & -4+1 \\ 3-2 & -2+2 & -5-1 \\ -1+4 & 1+5 & 2-2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -5 & -3 \\ 1 & 0 & -6 \\ 3 & 6 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{1}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} \end{aligned}$$

$$-Y^T = - \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{1}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{1}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric matrix

$$X + Y = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{1}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{1}{2}+\frac{-5}{2} & \frac{-5}{2}+\frac{-3}{2} \\ \frac{1}{2}+\frac{1}{2} & -2+0 & -2-3 \\ \frac{-5}{2}+\frac{3}{2} & -2+3 & 2+0 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 1 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} = A$$

$$\text{Hence, Symmetric matrix } X = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

Exercise MCQ

Q1

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

- a. a null matrix
- b. a unit matrix
- c. $-A$
- d. A

Solution

Correct option: (b)

Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$$\begin{aligned} A^2 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ which is a unit matrix.} \end{aligned}$$

Q2

If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in \mathbb{N}$, then A^n equals

- a. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$
- b. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- d. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

Solution

Correct option: (c)

$$\text{Given } A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^4 = I^4 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Q3

If A and B are two matrices such that $AB = A$ and $BA = B$, then B^2 is equal to

- a. B
- b. A
- c. 1
- d. 0

Solution

Correct option: (a)

Given $AB = A$ and $BA = B$, then

$$\Rightarrow BAB = B^2$$

$$\Rightarrow BA = B^2$$

$$\Rightarrow B = B^2$$

Q4

If $AB = A$ and $BA = B$, where A and B are square matrices, then

- a. $B^2 = B$ and $A^2 = A$
- b. $B^2 \neq B$ and $A^2 = A$
- c. $A^2 \neq A, B^2 = B$
- d. $A^2 \neq A, B^2 \neq B$

Solution

Correct option: (a)

Given $AB = A$ and $BA = B$, then

$$\Rightarrow BAB = B^2 \text{ and } ABA = A^2$$

$$\Rightarrow BA = B^2 \text{ and } AB = A^2$$

$$\Rightarrow B = B^2 \text{ and } A = A^2$$

Q5

If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ is equal to

- a. $2AB$
- b. $2BA$
- c. $A + B$
- d. AB

Solution

Correct option: (c)

Given $AB = B$ and $BA = A$, then

$$\Rightarrow BAB = B^2 \text{ and } ABA = A^2$$

$$\Rightarrow BA = B^2 \text{ and } AB = A^2$$

$$\Rightarrow B = B^2 \text{ and } A = A^2$$

$$\Rightarrow A^2 + B^2 = A + B$$

Q6

If $\begin{bmatrix} \cos \frac{2\pi}{7} & -\sin \frac{2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the least positive integral value of

- k is
- a. 3
 - b. 4
 - c. 6
 - d. 7

Solution

Correct option: (d)

$$\text{Let } A = \begin{bmatrix} \cos \frac{2\pi}{7} & \sin \frac{-2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix} = \begin{bmatrix} \operatorname{Re} e^{\frac{i2\pi}{7}} & \operatorname{Im} e^{\frac{-i2\pi}{7}} \\ \operatorname{Im} e^{\frac{i2\pi}{7}} & \operatorname{Re} e^{\frac{i2\pi}{7}} \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} \cos \frac{2\pi}{7} & \sin \frac{-2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix} \begin{bmatrix} \cos \frac{2\pi}{7} & \sin \frac{-2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 \frac{2\pi}{7} - \sin^2 \frac{2\pi}{7} & -2 \cos \frac{2\pi}{7} \sin \frac{2\pi}{7} \\ 2 \cos \frac{2\pi}{7} \sin \frac{2\pi}{7} & \cos^2 \frac{2\pi}{7} - \sin^2 \frac{2\pi}{7} \end{bmatrix} \\ &= \begin{bmatrix} \cos \frac{4\pi}{7} & -\sin \frac{4\pi}{7} \\ \sin \frac{4\pi}{7} & \cos \frac{4\pi}{7} \end{bmatrix} = \begin{bmatrix} \operatorname{Re} e^{\frac{i4\pi}{7}} & \operatorname{Im} e^{\frac{-i4\pi}{7}} \\ \operatorname{Im} e^{\frac{i4\pi}{7}} & \operatorname{Re} e^{\frac{i4\pi}{7}} \end{bmatrix} \end{aligned}$$

So for any k we have

$$A^k = \begin{bmatrix} \operatorname{Re} e^{\frac{ik\pi}{7}} & \operatorname{Im} e^{\frac{-ik\pi}{7}} \\ \operatorname{Im} e^{\frac{ik\pi}{7}} & \operatorname{Re} e^{\frac{ik\pi}{7}} \end{bmatrix}$$

Now

$$\begin{bmatrix} \operatorname{Re} e^{\frac{ik\pi}{7}} & \operatorname{Im} e^{\frac{-ik\pi}{7}} \\ \operatorname{Im} e^{\frac{ik\pi}{7}} & \operatorname{Re} e^{\frac{ik\pi}{7}} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \operatorname{Re} e^{\frac{ik\pi}{7}} = 1$$

$$\Rightarrow \cos \frac{2k\pi}{7} = 1$$

$$\Rightarrow \frac{2k\pi}{7} = 2n\pi \text{ for } n \in \mathbb{Z}$$

$$\Rightarrow k = 7n$$

So positive integral values of k can be 0, 7, 14 etc

Q7

If the matrix AB is zero, then

- It is not necessary that either $A = 0$ or $B = 0$
- $A = 0$ or $B = 0$
- $A = 0$ and $B = 0$
- All the above statements are wrong

Solution

Correct option: (a)

$$\text{Let } A = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$A \neq 0$, $B \neq 0$ but $AB = 0$

Q8

Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to

a. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$

b. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

c. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

d. $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

Solution

Correct option: (c)

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} = a \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^n = a^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

Q9

If A, B are square matrices of order 3, A is non-singular and $AB = 0$, then B is a

- a. Null matrix
- b. Singular matrix
- c. Unit matrix
- d. Non-singular matrix

Solution

Correct option: (a)

A is non singular that means determinant of A is non zero.

$$AB=0 \Rightarrow B=0$$

Q10

If $A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix}$ and $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then AB is equal to

- a. B
- b. nB
- c. B^n
- d. $A + B$

Solution

Correct option: (b)

$$\text{Given } A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix} = n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}, \text{ then}$$

$$\begin{aligned} AB &= n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \\ &= n \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} \\ &= nB \end{aligned}$$

Q11

If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then A^n (where $n \in \mathbb{N}$) equals

- a. $\begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$
- b. $\begin{bmatrix} 1 & n^2a \\ 0 & 1 \end{bmatrix}$
- c. $\begin{bmatrix} 1 & na \\ 0 & 0 \end{bmatrix}$
- d. $\begin{bmatrix} n & na \\ 0 & n \end{bmatrix}$

Solution

Correct option: (a)

$$\text{Given } A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 A \\ = \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \\ = \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}$$

On generalising we get

$$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$$

Q12

If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $x + y$ equals

- a. 0
b. -1
c. 2
d. None of these

Solution

Correct option: (a)

$$\text{Given } A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AB = I_3$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow x + y = 0$$

Q13

If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ and $(A+B)^2 = A^2 + B^2$,
then values of a and b are

- a. $a = 4, b = 1$
- b. $a = 1, b = 4$
- c. $a = 0, b = 4$
- d. $a = 2, b = 4$

Solution

Correct answer: (b)

Given $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$

$$(A+B)^2 = A^2 + B^2$$

$$\Rightarrow A^2 + B^2 + AB + BA = A^2 + B^2$$

$$\Rightarrow AB + BA = 0$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a-b & 2 \\ 2a-b & 3 \end{bmatrix} + \begin{bmatrix} a+2 & -a-1 \\ b-2 & -b+1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2a-b+2 & 2-a-1 \\ 2a-2 & 4-b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow a = 1 \text{ and } b = 4$$

Q14

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

- a. $1 + \alpha^2 + \beta\gamma = 0$
- b. $1 - \alpha^2 + \beta\gamma = 0$
- c. $1 - \alpha^2 - \beta\gamma = 0$
- d. $1 + \alpha^2 - \beta\gamma = 0$

Solution

Correct option: (c)

Given $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = I$, then

$$A^2 = I$$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \alpha^2 + \beta\gamma = 1$$

$$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$$

Q15

If $S = [s_{ij}]$ is a scalar matrix such that $s_{ij} = k$ and A is a square matrix of the same order, then $AS = SA = ?$

- a. A^k
- b. $k + A$
- c. kA
- d. kS

Solution

Correct option: (c)

$$S = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$SA = k \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A = kA$$

Q16

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

- a. A
- b. $I - A$
- c. I
- d. $3A$

Solution

Correct option: (c)

$$\begin{aligned} & (I + A)^3 \\ &= I^3 + A^3 + 3I^2A + 3IA^2 \\ &= I + A^2A + 3A + 3A^2 \\ &= I + A^2 + 3A + 3A \\ &= I + A + 6A \\ &= I + 7A \\ & (I + A)^3 - 7A \\ &= I \end{aligned}$$

Q17

If a matrix A is both symmetric and skew-symmetric, then

- a. A is a diagonal matrix
- b. A is a zero matrix
- c. A is a scalar matrix
- d. A is a square matrix

Solution

Correct option: (b)

A is symmetric $\Rightarrow a_{ij} = a_{ji} \rightarrow (1)$

A is skew-symmetric

$\Rightarrow a_{ij} = -a_{ji} \rightarrow (2)$ and

$a_{ii} = -a_{ii}$

$\Rightarrow a_{ii} = 0$ means the diagonal entries are zero.

From (1) and (2) we can write

$a_{ij} = a_{ji} = 0$ which means all the off diagonal entries are zero.

So A is a null matrix.

Q18

The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is

- a. A skew-symmetric matrix
- b. A symmetric matrix
- c. A diagonal matrix
- d. An upper triangular matrix

Solution

Correct option: (a)

The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is skew symmetric because $a_{ij} = -a_{ji}$
for $i, j = 1, 2, 3$.

Q19

If A is a square matrix, then AA is a

- a. Skew-symmetric matrix
- b. Symmetric matrix
- c. Diagonal matrix
- d. None of these

Solution

Correct option: (d)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$AA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$$

Q20

If A and B are symmetric matrices, then ABA is

- a. Symmetric matrix
- b. Skew-symmetric matrix
- c. Diagonal matrix
- d. Scalar matrix

Solution

Correct option : (a)

$$\text{Let } A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 8 & 7 \end{bmatrix}$$

$$ABA = \begin{bmatrix} 7 & 8 \\ 8 & 7 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 23 & 22 \\ 22 & 23 \end{bmatrix}$$

Q21

If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then

- a. $x = 0, y = 0$
- b. $x + y = 5$
- c. $x = y$
- d. None of these

Solution

Correct option : (c)

$A = A^T \Rightarrow A$ is symmetric, so $a_{12} = a_{21}$

Q22

If A is 3×4 matrix and B is a matrix such that $A^T B$ and BA^T are both defined. Then, B is of the type

- a. 3×4
- b. 3×3
- c. 4×4
- d. 4×3

Solution

Correct option : (a)

A is 3×4 matrix so A^T is 4×3 matrix

$A^T B$ is defined, so no of columns in A^T = no of rows in B = 3

BA^T is defined, so no of columns in B = no of rows in A^T = 4

So B is 3×4 matrix.

Q23

If $A = [a_{ij}]$ is a square matrix of even order such that $a_{ij} = i^2 - j^2$, then

- a. A is a skew - symmetric matrix and $|A| = 0$
- b. A is symmetric matrix and $|A|$ is a square
- c. A is symmetric matrix and $|A| = 0$
- d. None of these

Solution

Correct option: (d)

$$\text{Let } A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \quad [\because a_{ij} = i^2 - j^2]$$

$$|A| = 0 - (-9) = 9 \neq 0$$

Q24

If $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$, then $A^T + A = I_2$, if

- a. $\theta = n\pi, n \in \mathbb{Z}$
- b. $\theta = (2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$
- c. $\theta = 2n\pi + \frac{\pi}{3}, n \in \mathbb{Z}$
- d. none of these

Solution

Correct option: (c)

$$\text{If } A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$A^T + A = I_2$$

$$\Rightarrow \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos \theta & 0 \\ 0 & 2\cos \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow 2\cos \theta = 1$$

$$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in \mathbb{Z}$$

Q25

If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew-symmetric matrix,

then the symmetric matrix is

a. $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution

Correct option: (a)

$$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$$

The symmetric matrix is

$$\frac{1}{2}(A + A^T)$$

$$= \frac{1}{2} \left(\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix} \right)$$

$$= \frac{1}{2} \begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$$

Q26

Out of the following matrices, choose that matrix which is a scalar matrix:

- a. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 b. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$
 c. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
 d. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution

Correct option: (a)

A diagonal matrix with all diagonal elements are equal is a scalar matrix.

Q27

The number of all possible matrices of order 3×3 with each entry 0 or 1 is

- a. 27
 b. 18
 c. 81
 d. 512

Solution

Correct option: (d)

Let us consider a matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

The element a can have two values 0 or 1 in two ways.

Similarly all other elements can also have two values 0 or 1 in two ways each.

So the total number of combinations is $2^9 = 512$.

So total no of matrices will be 512.

Q28

Which of the given values of x and y make the following pairs of matrices equal?

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} \text{ and } \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

- a. $x = -\frac{1}{3}, y = 7$
 b. $y = 7, x = -\frac{2}{3}$
 c. $x = -\frac{1}{3}, y = -\frac{2}{5}$
 d. Not possible to find

Solution

Correct option: (d)

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow 3x+7=0$$

$$\Rightarrow x = -\frac{7}{3}$$

$$5 = y-2$$

$$\Rightarrow y = 7$$

$$y+1=8$$

$$\Rightarrow y = 7$$

$$2-3x=4$$

$$\Rightarrow x = -\frac{2}{3}$$

We are getting two values of x . So it is not possible to find.

Q29

If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values

of k, a, b , are respectively

- a. -6, -12, -18
 b. -6, 4, 9
 c. -6, -4, -9
 d. -6, 12, 18

Solution

Correct option: (c)

$$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$$

$$kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow -4k = 24$$

$$\Rightarrow k = -6$$

$$2k = 3a$$

$$\Rightarrow a = -4$$

$$3k = 2b$$

$$\Rightarrow b = -9$$

Q30

If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then B equals

- a. $I \cos \theta + J \sin \theta$
- b. $I \sin \theta + J \cos \theta$
- c. $I \cos \theta - J \sin \theta$
- d. $-I \cos \theta + J \sin \theta$

Solution

Correct option: (a)

$$I \cos \theta + J \sin \theta$$

$$= \begin{bmatrix} \cos \theta & 0 \\ 0 & \cos \theta \end{bmatrix} + \begin{bmatrix} 0 & \sin \theta \\ -\sin \theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$$

$$= B$$

Q31

The trace of the matrix $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is

- a. 17
- b. 25
- c. 3
- d. 12

Solution

Correct option: (a)

Trace = sum of diagonal elements

$$= 1 + 7 + 9 = 17$$

Q32

If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all i , then trace of A is equal to

- a. nk
- b. $n + k$
- c. $\frac{n}{k}$
- d. none of these

Solution

Correct option: (a)

$$\text{Trace} = \sum_{i=1}^n a_{ii} = nk$$

Q33

The matrix $A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

- a. square matrix
- b. diagonal matrix
- c. unit matrix
- d. none of these

Solution

Correct option: (a)

No of rows = no of columns.

Q34

The number of possible matrices of order 3×3 with each entry 2 or 0 is

- a. 9
- b. 27
- c. 81
- d. none of these

Solution

Correct option: (d)

Let us consider a matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

The element a can have two values 0 or 2 in two ways.

Similarly all other elements can also have two values 0 or 2 in two ways each.

So the total number of combinations is 2^9 .

So total no of matrices will be 2^9 .

Q35

If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x and y is

- a. $x=3, y=1$
- b. $x=2, y=3$
- c. $x=2, y=3$
- d. $x=3, y=3$

Solution

Correct option: (c)

Given $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$

Equating the terms, we get

$$4x = x + 6$$

$$\Rightarrow x = 2$$

and

$$2x + y = 7$$

$$\Rightarrow y = 3$$

Q36

If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

- a. A
- b. $I - A$
- c. $I + A$
- d. $3A$

Solution

Correct option: (a)

$$(A-I)^3 + (A+I)^3 - 7A$$

$$= (A-I+A+I)\left((A-I)^2 + (A+I)^2 - (A-I)(A+I)\right) - 7A$$

$$= 2A(2A^2 + 2I - A^2 + I^2) - 7A$$

$$= 2A(A^2 + 3I) - 7A$$

$$= 2A(I + 3I) - 7A$$

$$= 8A - 7A$$

$$= A$$

Q37

If A and B are two matrix of order $3 \times m$ and $3 \times n$ respectively and $m = n$, then the order of $5A - 2B$ is

- a. $m \times n$
- b. 3×3
- c. $m \times n$
- d. $3 \times n$

Solution

Correct option: (d)

In scalar multiplication and in addition or subtraction of matrices the order doesn't change.

Q38

If A is a matrix of order $m \times n$ and B is a matrix such that AB^T and $B^T A$ are both defined, then the order of matrix B is

- a. $m \times n$
- b. $n \times n$
- c. $n \times m$
- d. $3 \times n$

Solution

Correct option: (a)

A is $m \times n$ matrix and AB^T is defined then

number of columns in A = number of rows in $B^T = n$

$B^T A$ is also defined then

number of columns in B^T = number of rows in A = m

Order of B is $m \times n$

Q39

If A and B are matrices of the same order, then $(AB^T - BA^T)^T$ is a

- a. skew-symmetric matrix
- b. null matrix
- c. unit matrix
- d. symmetric matrix

Solution

Correct option: (a)

$$\begin{aligned}
 & (AB^T - BA^T)^T \\
 &= (AB^T)^T - (BA^T)^T \\
 &= BA^T - AB^T \\
 &= -(AB^T - BA^T)
 \end{aligned}$$

Q40

If matrix $A = [a_{ij}]_{2 \times 2}$, where $a_{ij} = \begin{cases} 1, & \text{if } i \neq j \\ 0, & \text{if } i = j \end{cases}$, then A^2 is equal to

- a. I
- b. A
- c. O
- d. -I

Solution

Correct option: (a)

$$\begin{aligned}
 A &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
 A^2 &= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\
 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I
 \end{aligned}$$

Q41

$$\text{If } A = \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}, B = \frac{1}{\pi} \begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix},$$

then $A - B$ is equal to

- a. I
- b. O
- c. 2I
- d. $\frac{1}{2}I$

Solution

Correct option: (d)

$$\begin{aligned} \text{Given } A &= \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(x\pi) \end{bmatrix} \quad B = \frac{1}{\pi} \begin{bmatrix} -\cos^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(x\pi) \end{bmatrix} \\ A - B &= \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(x\pi) + \cos^{-1}(x\pi) & 0 \\ 0 & \cot^{-1}(x\pi) + \tan^{-1}(x\pi) \end{bmatrix} \\ &= \frac{1}{\pi} \begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} I \end{aligned}$$

Q42

If A and B are square matrices of the same order, then $(A + B)(A - B)$ is equal to

- a. $A^2 - B^2$
- b. $A^2 - BA - AB - B^2$
- c. $A^2 - B^2 + BA - AB$
- d. $A^2 - BA + B^2 + AB$

Solution

Correct option: (c)

$$\begin{aligned} (A+B)(A-B) \\ = A^2 - AB + BA - B^2 \end{aligned}$$

Q43

If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then

- a. Only AB is defined
- b. Only BA is defined
- c. AB and BA both are defined
- d. AB and BA both are not defined

Solution

Correct option: (c)

$$\text{Given } A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix} \\ &= \begin{bmatrix} 3 & 23 \\ 13 & -17 \end{bmatrix} \end{aligned}$$

So AB is defined as no of columns in A is equal to number of rows in B.

$$\begin{aligned} BA &= \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -8 & 13 & 9 \\ 16 & -14 & 10 \\ -18 & 24 & 8 \end{bmatrix} \end{aligned}$$

So BA is also defined no of columns in B is equal to number of rows in A.

Q44

The matrix $A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

- a. Diagonal matrix
- b. Symmetric matrix
- c. Skew-symmetric matrix
- d. Scalar matrix

Solution

Correct option: (c)

$$\text{Given } A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$$

$$\Rightarrow A = -A^T$$

So A is skew-symmetric matrix.

Q45

The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

- a. Identity matrix
- b. Symmetric matrix
- c. Skew-symmetric matrix
- d. Diagonal matrix

Solution

Correct option: (d)

A matrix is called Diagonal matrix if all the elements, except those in the leading diagonal, are zero.



Exercise 5VSAQ

Q1

If A is an $m \times n$ matrix and B is $n \times p$ matrix does AB exist?
If yes, write its order.

Solution

Given,

$$\text{Order of } A = m \times n$$

$$\text{Order of } B = n \times p$$

Since number of columns of $A = n =$ Number of rows of B

$$\Rightarrow AB \text{ exists}$$

$$\text{and order of } AB = \text{number of rows of } A \times \text{Number of columns of } B = m \times p.$$

Q2

If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, Write the orders of AB and BA .

Solution

$$\text{Order of } A = 2 \times 3$$

$$\text{Order of } B = 3 \times 2$$

So,

$$A_{2 \times 3} \times B_{3 \times 2} \text{ has order} = 2 \times 2$$

$$B_{3 \times 2} \times A_{2 \times 3} \text{ has order} = 3 \times 3$$

Hence,

$$\text{Order of } AB = 2 \times 2$$

$$\text{Order of } BA = 3 \times 3.$$

Q3

If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, write AB .

Solution

Given,

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$\begin{aligned} AB &= \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -4 \\ 3 \end{bmatrix} \\ &= \begin{bmatrix} -16 + 9 \\ -4 + 6 \end{bmatrix} \\ &= \begin{bmatrix} -7 \\ 2 \end{bmatrix} \end{aligned}$$

Hence,

$$AB = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$$

Q4

If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, write AA^T .

Solution

Given,

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$\begin{aligned} AA^T &= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix} \end{aligned}$$

Hence,

$$AA^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Q5

Given an example of two non-zero 2×2 matrices A and B such that $AB = O$.

Solution

Let,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq O$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \neq O$$

$$\begin{aligned} AB &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \\ &= O \end{aligned}$$

So,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Q6

If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A^T$.

Solution

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$$

$$\begin{aligned} A + A^T &= \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix} \\ &= \begin{bmatrix} 2+2 & 3+5 \\ 5+3 & 7+7 \end{bmatrix} \\ &= \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix} \end{aligned}$$

Hence,

$$A + A^T = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$$

Q7

If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, write A^2 .

Solution

Given,

$$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$= \begin{bmatrix} i^2 + 0 & 0 + 0 \\ 0 + 0 & 0 + i^2 \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \quad \{\text{Since, } i^2 = -1\}$$

Hence,

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Q8

If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x satisfying $0 < x < \frac{\pi}{2}$ when $A + A^T = I$.

Solution

Given,

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow A^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$A + A^T = I$$

$$\Rightarrow \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos x + \cos x & \sin x - \sin x \\ -\sin x + \sin x & \cos x + \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 \cos x & 0 \\ 0 & 2 \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

$$\text{since } 0 < x < \frac{\pi}{2}$$

So,

$$x = \frac{\pi}{3}$$

Q9

If $A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$, find AA^T

Solution

$$\begin{aligned} A &= \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \\ \Rightarrow A^T &= \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \\ AA^T &= \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} \\ &= \begin{bmatrix} \cos^2 x + \sin^2 x & \cos x \sin x - \sin x \cos x \\ \cos x \sin x - \sin x \cos x & \sin^2 x + \cos^2 x \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= I \end{aligned}$$

So,

$$AA^T = I$$

Q10

If $\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I$, where I is 2×2 unit matrix. Find x and y .

Solution

Given,

$$\begin{aligned} &\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I \\ \Rightarrow &\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + \begin{bmatrix} 2x & 0 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow &\begin{bmatrix} 1+2x & 0 \\ y+2 & 5-4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow &\begin{bmatrix} 1+2x & 0 \\ y+2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}$$

Since, corresponding matrices of equal matrices are equal, so

$$1+2x = 1$$

$$\Rightarrow x = 0$$

$$\text{And } y+2 = 0$$

$$\Rightarrow y = -2$$

Hence,

$$x = 0, y = -2$$

Q11

If $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value of λ .

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

and

$$A^2 = kA$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so
 $k = 2$

Q12

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value of λ .

Solution

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

And

$$A^4 = \lambda A$$

$$\Rightarrow \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^2 = \lambda \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \left(\begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix} \right)^2 = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+4 & 4+4 \\ 4+4 & 4+4 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so
 $\lambda = 8$

Q13

If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^2 .

Solution

Given,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^2 = A \times A$$

$$\begin{aligned} &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$A^2 = I$$

Hence,

$$A^2 = I$$

Thus, we can say that, $A^2 = - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = -A = I_3$

Q14

If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^3 .

Solution

Given,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$\begin{aligned} A^2 &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} A^3 &= A^2 \times A \\ &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \\ &= \begin{bmatrix} -1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0-1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0-1 \end{bmatrix} \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \end{aligned}$$

$$A^3 = A$$

Hence, $A^3 = A$

Q15

If $A = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$, find A^4 .

Solution

Copykitab
Same textbooks, black away

Given,

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$A^2 = A \times A$$

$$\begin{aligned} &= \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \\ &= \begin{bmatrix} 9+0 & 0+0 \\ 0+0 & 0+9 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \end{aligned}$$

$$A^4 = A^2 \times A^2$$

$$\begin{aligned} &= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix} \\ &= \begin{bmatrix} 81+0 & 0+0 \\ 0+0 & 0+81 \end{bmatrix} \\ &= \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix} \end{aligned}$$

Hence,

$$A^4 = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$$

Q16

If $\begin{bmatrix} x & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$, find x .

Solution

Given,

$$\begin{aligned} &\begin{bmatrix} x & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2 \\ \Rightarrow &\begin{bmatrix} 2x+6 \end{bmatrix} = 2 \\ \Rightarrow &2x+6 = 2 \\ \Rightarrow &2x = 2-6 \\ \Rightarrow &x = \frac{-4}{2} \\ \Rightarrow &x = -2 \end{aligned}$$

Q17

If $A = [a_{ij}]$ is a 2×2 matrix such that $a_{ij} = i + 2j$, write A .

Solution

Here,

$$a_{ij} = i + 2j$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$= \begin{bmatrix} 1+2(1) & 1+2(2) \\ 2+2(1) & 2+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Hence,

$$A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

Q18

Write matrix A satisfying $A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.

Solution

Given,

$$A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 3-2 & -6-3 \\ -3+1 & 8-4 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$$

Hence,

$$A = \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$$

Q19

If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric.

Solution

Given,

$$A = [a_{ij}]$$

Such that $a_{ij} = i^2 - j^2$

$$\Rightarrow a_{ji} = (j)^2 - i^2$$

$$\Rightarrow a_{ji} = j^2 - i^2$$

$$\Rightarrow a_{ji} = - (i^2 - j^2)$$

$$\Rightarrow a_{ji} = -a_{ij}$$

We know that, A square matrix $A = [a_{ij}]$ is skew-symmetric if $a_{ji} = -a_{ij}$.

So,

A is a skew symmetric matrix.

Q20

For any square matrix write whether AA^T is symmetric or skew-symmetric.

Solution

$$\begin{aligned} (AA^T)^T &= (A^T)^T \times A^T && \left\{ \text{since, } (AB)^T = B^T A^T \right\} \\ &= (AA^T)^T = (AA^T) && \text{--- (i) } \left\{ \text{since, } (A^T)^T = A \right\} \end{aligned}$$

We know that, a square matrix A is symmetric if $A^T = A$.

So, from equation (i)

(AA^T) is a symmetric matrix.

Q21

If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i a_{ii}$.

Solution

Given,

$A = [a_{ij}]$ is skew symmetric

$$\Rightarrow a_{ji} = -a_{ij}$$

$$\Rightarrow a_{ii} = -a_{ii}$$

$$\Rightarrow a_{ii} + a_{ii} = 0$$

$$\Rightarrow 2a_{ii} = 0$$

$$\Rightarrow a_{ii} = 0$$

$$\sum_i a_{ii} = 0 + 0 + \dots + 0 \text{ (i times)}$$

$$= 0$$

So,

$$\sum_i a_{ii} = 0$$

Q22

If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i \sum_j a_{ij}$.

Solution

Given,

$A = [a_{ij}]$ is a skew symmetric

$$\Rightarrow a_{ij} = -a_{ji}$$

$$\Rightarrow a_{ii} = 0$$

$$\begin{aligned}\sum_i \sum_j a_{ij} &= a_{11} + a_{12} + a_{13} + \dots + a_{21} + a_{22} + a_{23} + \dots + a_{31} + a_{32} + a_{33} + \dots \\ &= 0 + a_{12} + a_{13} + \dots - a_{12} + 0 + a_{23} + \dots - a_{31} - a_{23} + 0 + \dots \\ &= 0\end{aligned}$$

So,

$$\sum_i \sum_j a_{ij} = 0$$

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