Exercise 5.5

Q1

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^T$ is a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\left(A - A^T\right) = \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}^T\right)$$

$$= \left(\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2-2 & 3-4 \\ 4-3 & 5-5 \end{bmatrix}$$

$$\left(A - A^T\right) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is a skew symmetric matrix if $x = -x^T$

So, $\left(A - A^T\right)$ is skew symmetric.

Q2

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, show that $A - A^T$ is a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$-\left(A - A^T\right)^T = -\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}^T$$

$$= -\begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

$$-\left(A - A^T\right)^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$\left(A - A^T\right) = -\left(A - A^T\right)^T$$

We know that, x is skewsymmetric matrx if $x = -x^T$

So, $\left(A - A^T\right)$ is skewsymmetric matrix.

Q3

If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t.

Solution

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix} \text{ is a symmetric matrix.}$$

We know that $A = [aij]_{m \times n}$ is a symmetric matrix if $aij = aji$

So,
$$x = a_{13} = a_{31} = 4$$
$$y = a_{21} = a_{12} = 2$$
$$z = a_{22} = a_{22} = z$$
$$t = a_{32} = a_{32} = -3$$

Hence,

$x = 4, y = 2, t = -3$ and z can have any value.

Q4

Let $A=\begin{bmatrix}3 & 2 & 7\\ 1 & 4 & 3\\ -2 & 5 & 8\end{bmatrix}$. Find matrix X and Y such that $X+Y=A$, where X is a symmetric and Y is a skew-symmetric matrix.

Solution

Given,

$$A=\begin{bmatrix}3 & 2 & 7\\ 1 & 4 & 3\\ -2 & 5 & 8\end{bmatrix} \quad \Rightarrow \quad A^T=\begin{bmatrix}3 & 1 & -2\\ 2 & 4 & 5\\ 7 & 3 & 8\end{bmatrix}$$

$$\therefore X=\frac{1}{2}\left(A+A^T\right)$$

$$=\frac{1}{2}\left(\begin{bmatrix}3 & 2 & 7\\ 1 & 4 & 3\\ -2 & 5 & 8\end{bmatrix}+\begin{bmatrix}3 & 1 & -2\\ 2 & 4 & 5\\ 7 & 3 & 8\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}3+3 & 2+1 & 7-2\\ 1+2 & 4+4 & 3+5\\ -2+7 & 5+3 & 8+8\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}6 & 3 & 5\\ 3 & 8 & 8\\ 5 & 8 & 16\end{bmatrix}$$

$$\Rightarrow X=\begin{bmatrix}3 & \frac{3}{2} & \frac{5}{2}\\ \frac{3}{2} & 4 & 4\\ \frac{5}{2} & 4 & 8\end{bmatrix}$$

Now, $Y=\frac{1}{2}\left(A-A^T\right)$

$$=\frac{1}{2}\left(\begin{bmatrix}3 & 2 & 7\\ 1 & 4 & 3\\ -2 & 5 & 8\end{bmatrix}-\begin{bmatrix}3 & 1 & -2\\ 2 & 4 & 5\\ 7 & 3 & 8\end{bmatrix}\right)$$

$$=\frac{1}{2}\begin{bmatrix}3-3 & 2-1 & 7+2\\ 1-2 & 4-4 & 3-5\\ -2-7 & 5-3 & 8-8\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}0 & 1 & 9\\ -1 & 0 & -2\\ -9 & 2 & 0\end{bmatrix}$$

$$Y=\begin{bmatrix}0 & \frac{1}{2} & \frac{9}{2}\\ \frac{-1}{2} & 0 & -1\\ \frac{-9}{2} & 1 & 0\end{bmatrix}$$

Now,

$$X^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} = X$$

$\Rightarrow$ x is a symmetric matrix

Now,

$$-Y^T = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$

$$= -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$\Rightarrow$ $-Y^T = Y$

$\therefore$ Y is skew symmetric matrix.

$$X + Y = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3+0 & \frac{3}{2}+\frac{1}{2} & \frac{5}{2}+\frac{9}{2} \\ \frac{3}{2}-\frac{1}{2} & 4+0 & 4-1 \\ \frac{5}{2}-\frac{9}{2} & 4+1 & 8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$

$$= A$$

Hence,

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{4} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}, \; Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Q5

Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} \quad \Rightarrow \quad A^T = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$$

Let $X = \frac{1}{2}\left(A + A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix} 4+4 & 2+3 & -1+1 \\ 3+2 & 5+5 & 7-2 \\ 1-1 & -2+7 & 1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^T = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} = X$$

$\therefore$ X is symmetric matrix

Now,

$Y = \frac{1}{2}\left(A - A^T\right)$

$$= \frac{1}{2}\left(\begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 4-4 & 2-3 & -1-1 \\ 3-2 & 5-5 & 7+2 \\ 1+1 & -2-7 & 1-1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}$$

$$\Rightarrow -Y^T = -\begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is a skew symmetric maatrix.

Now,

$$X+Y=\begin{bmatrix}4 & \frac{5}{2} & 0\\ \frac{5}{2} & 5 & \frac{5}{2}\\ 0 & \frac{5}{2} & 1\end{bmatrix}+\begin{bmatrix}0 & \frac{-1}{2} & -1\\ \frac{1}{2} & 0 & \frac{9}{2}\\ 1 & \frac{-9}{2} & 0\end{bmatrix}$$

$$=\begin{bmatrix}4+0 & \frac{5}{2}-\frac{1}{2} & 0-1\\ \frac{5}{2}+\frac{1}{2} & 5+0 & \frac{5}{2}+\frac{9}{2}\\ 0+1 & \frac{5}{2}-\frac{9}{2} & 1+0\end{bmatrix}$$

$$=\begin{bmatrix}4 & 2 & -1\\ 3 & 5 & 7\\ 1 & -2 & 1\end{bmatrix}=A$$

Q6

Define a symmetric matrix. Prove that for $A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$, $A+A^T$ is a symmetric matrix where A^T is the transpose of A.

Solution

A square matrix A is called a symmetric matrix, if $A^T=A$

Here,

$$A=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$$

$$A+A^T=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}+\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}^T$$

$$=\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}+\begin{bmatrix}2 & 4\\ 5 & 6\end{bmatrix}$$

$$=\begin{bmatrix}2+2 & 4+5\\ 5+4 & 6+6\end{bmatrix}$$

$$A+A^T=\begin{bmatrix}4 & 9\\ 9 & 12\end{bmatrix} \quad \text{---(i)}$$

$$\left(A+A^T\right)^T=\begin{bmatrix}4 & 9\\ 9 & 12\end{bmatrix}^T$$

$$\left(A+A^T\right)^T=\begin{bmatrix}4 & 9\\ 9 & 12\end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$\left(A+A^T\right)^T=\left(A+A^T\right)$$

So,

$\left(A+A^T\right)$ is a symmetric matrix.

Q7

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Solution

Here,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \quad \Rightarrow A^T = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

$$Let,\ X = \frac{1}{2}\left(A + A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3+3 & -4+1 \\ 1-4 & -1-1 \end{bmatrix} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

$$Now,\ X^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^T = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} = X$$

$\Rightarrow$ X is symmetric matrix

$$Let\quad Y = \frac{1}{2}\left(A - A^T\right) = \frac{1}{2}\left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right) = \frac{1}{2}\begin{bmatrix} 3-3 & -4-1 \\ 1+4 & -1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

$$Now,\ -Y^T = -\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^T = \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$$

$\Rightarrow$ Y is skew symmetric

$$Now,\ X + Y = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3+0 & \frac{-3}{2} - \frac{5}{2} \\ \frac{-3}{2} + \frac{5}{2} & -1+0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$$

Q8

Express the following matrix as the sum of a symmetric and skey-symmetric matrix and verify your result: $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$.

Solution

Let,

$$A=\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}\quad\Rightarrow\quad A^T=\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}$$

$$\text{Let, } X=\frac{1}{2}\left(A+A^T\right)=\frac{1}{2}\left(\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}+\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}3+3&-2+3&-4-1\\3-2&-2-2&-5+1\\-1-4&1-5&2+2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}6&1&-5\\1&-4&-4\\-5&-4&4\end{bmatrix}$$

$$=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}$$

$$\text{Now, } X^T=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}^T=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}=X$$

$\Rightarrow$ X is a symmetric matrix

$$\text{Let, } Y=\frac{1}{2}\left(A-A^T\right)=\frac{1}{2}\left(\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}-\begin{bmatrix}3&3&-1\\-2&-2&1\\-4&-5&2\end{bmatrix}\right)=\frac{1}{2}\begin{bmatrix}3-3&-2-3&-4+1\\3+2&-2+2&-5-1\\-1+4&1+5&2-2\end{bmatrix}=\frac{1}{2}\begin{bmatrix}0&-5&-3\\5&0&-6\\3&6&0\end{bmatrix}$$

$$=\begin{bmatrix}0&\frac{-5}{2}&\frac{-3}{2}\\\frac{5}{2}&0&-3\\\frac{3}{2}&3&0\end{bmatrix}$$

$$-Y^T=-\begin{bmatrix}0&\frac{-5}{2}&\frac{-3}{2}\\\frac{5}{2}&0&-3\\\frac{3}{2}&3&0\end{bmatrix}^T=\begin{bmatrix}0&\frac{-5}{2}&\frac{-3}{2}\\\frac{5}{2}&0&-3\\\frac{3}{2}&3&0\end{bmatrix}=Y$$

$\Rightarrow$ Y is a skew symmetric matrix

$$X+Y=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}+\begin{bmatrix}0&\frac{-5}{2}&\frac{-3}{2}\\\frac{5}{2}&0&-3\\\frac{3}{2}&3&0\end{bmatrix}=\begin{bmatrix}3+0&\frac{1}{2}-\frac{5}{2}&\frac{-5}{2}-\frac{3}{2}\\\frac{1}{2}+\frac{5}{2}&-2+0&-2-3\\\frac{-5}{2}+\frac{3}{2}&-2+3&2+0\end{bmatrix}=\begin{bmatrix}3&-2&-4\\3&-2&-5\\-1&1&2\end{bmatrix}=A$$

$$\text{Hence, Symmetric matrix } X=\begin{bmatrix}3&\frac{1}{2}&\frac{-5}{2}\\\frac{1}{2}&-2&-2\\\frac{-5}{2}&-2&2\end{bmatrix}$$

Exercise MCQ

Q1

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$, then A^2 is equal to

a. a null matrix
b. a unit matrix
c. -A
d. A

Solution

Correct option: (b)

Given $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ which is a unit matrix.

Q2

If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, $n \in N$, then A^{4n} equals

a. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

d. $\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

Solution

Correct option: (c)

Given $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = i\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^4 = i^4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{4n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q3

If A and B are two matrices such that AB = A and BA = B, then B^2 is equal to

a. B
b. A
c. 1
d. 0

Solution

Correct option: (a)

Given AB=A and BA=B, then

$\Rightarrow BAB = B^2$

$\Rightarrow BA = B^2$

$\Rightarrow B = B^2$

Q4

If AB = A and BA = B, where A and B are square matrices, then

a. $B^2 = B$ and $A^2 = A$
b. $B^2 \neq B$ and $A^2 = A$
c. $A^2 \neq A, B^2 = B$
d. $A^2 \neq A$, $B^2 \neq B$

Solution

Correct option: (a)

Given AB=A and BA=B, then

$\Rightarrow BAB = B^2$ and $ABA=A^2$

$\Rightarrow BA = B^2$ and $AB=A^2$

$\Rightarrow B = B^2$ and $A=A^2$

Q5

If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to

a. 2 AB
b. 2 BA
c. A + B
d. AB

Solution

Correct option: (c)

Given AB=A and BA=B, then

$\Rightarrow BAB = B^2$ and $ABA = A^2$

$\Rightarrow BA = B^2$ and $AB = A^2$

$\Rightarrow B = B^2$ and $A = A^2$

$\Rightarrow A^2 + B^2 = A + B$

Q6

If $\begin{bmatrix} \cos\frac{2\pi}{7} & -\sin\frac{2\pi}{7} \\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the least positive integeral value of k is

a. 3
b. 4
c. 6
d. 7

Solution

Correct option: (d)

Let $A=\begin{bmatrix}\cos\frac{2\pi}{7} & \sin\frac{-2\pi}{7}\\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{bmatrix}=\begin{bmatrix}\operatorname{Re} e^{\frac{i2\pi}{7}} & \operatorname{Im} e^{\frac{-i2\pi}{7}}\\ \operatorname{Im} e^{\frac{i2\pi}{7}} & \operatorname{Re} e^{\frac{i2\pi}{7}}\end{bmatrix}$

$A^2=\begin{bmatrix}\cos\frac{2\pi}{7} & \sin\frac{-2\pi}{7}\\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{bmatrix}\begin{bmatrix}\cos\frac{2\pi}{7} & \sin\frac{-2\pi}{7}\\ \sin\frac{2\pi}{7} & \cos\frac{2\pi}{7}\end{bmatrix}$

$=\begin{bmatrix}\cos^2\frac{2\pi}{7}-\sin^2\frac{2\pi}{7} & -2\cos\frac{2\pi}{7}\sin\frac{2\pi}{7}\\ 2\cos\frac{2\pi}{7}\sin\frac{2\pi}{7} & \cos^2\frac{2\pi}{7}-\sin^2\frac{2\pi}{7}\end{bmatrix}$

$=\begin{bmatrix}\cos\frac{4\pi}{7} & -\sin\frac{4\pi}{7}\\ \sin\frac{4\pi}{7} & \cos\frac{4\pi}{7}\end{bmatrix}=\begin{bmatrix}\operatorname{Re} e^{\frac{i4\pi}{7}} & \operatorname{Im} e^{\frac{-i4\pi}{7}}\\ \operatorname{Im} e^{\frac{i4\pi}{7}} & \operatorname{Re} e^{\frac{i4\pi}{7}}\end{bmatrix}$

So for any k we have

$A^k=\begin{bmatrix}\operatorname{Re} e^{\frac{i2k\pi}{7}} & \operatorname{Im} e^{\frac{-i2k\pi}{7}}\\ \operatorname{Im} e^{\frac{i2k\pi}{7}} & \operatorname{Re} e^{\frac{i2k\pi}{7}}\end{bmatrix}$

Now

$\begin{bmatrix}\operatorname{Re} e^{\frac{i2k\pi}{7}} & \operatorname{Im} e^{\frac{-i2k\pi}{7}}\\ \operatorname{Im} e^{\frac{i2k\pi}{7}} & \operatorname{Re} e^{\frac{i2k\pi}{7}}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \operatorname{Re} e^{\frac{i2k\pi}{7}}=1$

$\Rightarrow \cos\frac{2k\pi}{7}=1$

$\Rightarrow \frac{2k\pi}{7}=2n\pi$ for $n\in Z$

$\Rightarrow k=7n$

So positive integral values of k can be 0, 7, 14 etc

Q7

If the matrix AB is zero , then

a. It is not necessary that either A = 0 or B = 0
b. A = 0 or B = 0
c. A = O and B = 0
d. All the above statements are wrong

Solution

Correct option: (a)

Let $A=\begin{bmatrix}0 & 3\\ 0 & 0\end{bmatrix}$ and $B=\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix}$

$AB=\begin{bmatrix}0 & 3\\ 0 & 0\end{bmatrix}\begin{bmatrix}2 & 0\\ 0 & 0\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$A\neq 0,\ B\neq 0$ but $AB=0$

Q8

Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then A^n is equal to

a. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a \end{bmatrix}$

b. $\begin{bmatrix} \begin{bmatrix} a^n & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \end{bmatrix}$

c. $\begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

d. $\begin{bmatrix} na & 0 & 0 \\ 0 & na & 0 \\ 0 & 0 & na \end{bmatrix}$

Solution

Correct option: (c)

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} = a\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^n = a^n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$$

Q9

If A, B are square matrices of order 3, A is non-singular and AB = 0, then B is a

a. Null matrix
b. Singular matrix
c. Unit matrix
d. Non-singular matrix

Solution

Correct option: (a)

A is non singular that means determinant of A is non zero.

$AB=0 \Rightarrow B=0$

Q10

If $A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix}$ and $B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then AB is equal to

a. B
b. nB
c. B^n
d. A + B

Solution

Correct option: (b)

Given $A = \begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix} = n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then

$AB = n\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$= n\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$= nB$

Q11

If $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$, then A^n (where $n \in N$) equals

a. $\begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & n^2a \\ 0 & 1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & na \\ 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} n & na \\ 0 & n \end{bmatrix}$

Solution

Correct option: (a)

Given $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$

$A^3 = A^2A$

$= \begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}$

On genaralising we get

$A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

Q12

If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $x + y$ equals

a. 0
b. -1
c. 2
d. None of these

Solution

Correct option: (a)

Given $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AB = I_3$

$\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & 0 & x+y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow x + y = 0$

Q13

If $A\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}, B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$ and $(A+B)^2=A^2+B^2$, then values of a and b are

a. a = 4, b = 1
b. a = 1, b = 4
c. a = 0, b = 4
d. a = 2, b = 4

Solution

Correct answer: (b)

Given $A=\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}$ $B=\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}$

$(A+B)^2=A^2+B^2,$

$\Rightarrow A^2+B^2+AB+BA=A^2+B^2$

$\Rightarrow AB+BA=0$

$\Rightarrow \begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}+\begin{bmatrix}a & 1\\ b & -1\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & -1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}a-b & 2\\ 2a-b & 3\end{bmatrix}+\begin{bmatrix}a+2 & -a-1\\ b-2 & -b+1\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2a-b+2 & 2-a-1\\ 2a-2 & 4-b\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$\Rightarrow a=1$ and $b=4$

Q14

If $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ is such that $A^2=I$, then

a. $1+\alpha^2+\beta\gamma=0$
b. $1-\alpha^2+\beta\gamma=0$
c. $1-\alpha^2-\beta\gamma=0$
d. $1+\alpha^2-\beta\gamma=0$

Solution

Correct option: (c)

Given $A=\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}$ and $A^2=I$, then

$A^2=I$

$\Rightarrow \begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \alpha^2+\beta\gamma\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \alpha^2+\beta\gamma=1$

$\Rightarrow 1-\alpha^2-\beta\gamma=0$

Q15

If $S = [s_{ij}]$ is a scalar matrix such that $s_{ii} = k$ and A is a square matrix of the same order, then AS = SA = ?

a. A^k
b. k + A
c. kA
d. kS

Solution

Correct option: (c)

$$S = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} = k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$SA = k\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}A = kA$$

Q16

If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

a. A
b. I - A
c. I
d. 3A

Solution

Correct option: (c)

$(I + A)^3$

$= I^3 + A^3 + 3I^2A + 3IA^2$

$= I + A^2A + 3A + 3A^2$

$= I + A^2 + 3A + 3A$

$= I + A + 6A$

$= I + 7A$

$(I + A)^3 - 7A$

$= I$

Q17

If a matrix A is both symmetric and skew-symmetric, then

a. A is a diagonal matrix
b. A is a zero matrix
c. A is a scalar matrix
d. A is a square matrix

Solution

Correct option: (b)

A is symmetric $\Rightarrow a_{ij} = a_{ji} \rightarrow (1)$

A is skew-symmetric

$\Rightarrow a_{ij} = -a_{ji} \rightarrow (2)$ and

$a_{ii} = -a_{ii}$

$\Rightarrow a_{ii} = 0$ means the diagonal entries are zero.

From (1) and (2) we can write

$a_{ij} = a_{ji} = 0$ which means all the off diagonal entries are zero.

So A is a null matrix.

Q18

The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is

a. A skew-symmetric matrix
b. A symmetric matrix
c. A diagonal matrix
d. An upper triangular matrix

Solution

Correct option: (a)

The matrix $\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$ is skew symmetric because $a_{ij} = -a_{ji}$

for i,j=1,2,3.

Q19

If A is a square matrix, then AA is a

a. Skew-symmetric matrix
b. Symmetric matrix
c. Diagonal matrix
d. None of these

Solution

Correct option: (d)

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$AA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

Q20

If A and B are symmetric matrices, then ABA is

a. Symmetric matrix
b. Skew-symmetric matrix
c. Diagonal matrix
d. Scalar matrix

Solution

Correct option: (a)

Let $A=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}$

$AB=\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}\begin{bmatrix}3 & 2\\ 2 & 3\end{bmatrix}=\begin{bmatrix}7 & 8\\ 8 & 7\end{bmatrix}$

$ABA=\begin{bmatrix}7 & 8\\ 8 & 7\end{bmatrix}\begin{bmatrix}1 & 2\\ 2 & 1\end{bmatrix}=\begin{bmatrix}23 & 22\\ 22 & 23\end{bmatrix}$

Q21

If $A=\begin{bmatrix}5 & x\\ y & 0\end{bmatrix}$ and $A=A^T$, then

a. X = 0, y = 0
b. X + y = 5
c. X = y
d. None of these

Solution

Correct option: (c)

$A=A^T \Rightarrow$ A is symmetric, so $a_{12}=a_{21}$

Q22

If A is 3 × 4 matrix and B is a matrix such that A^TB and BA^T are both defined. Then, B is of the type

a. 3 × 4
b. 3 × 3
c. 4 × 4
d. 4 × 3

Solution

Correct option: (a)

A is 3x4 matrix so A^T is 4x3 matrix

A^TB is defined, so no of columns in A^T=no of rows in B=3

BA^T is defined, so no of columns in B=no of rows in A^T=4

So B is 3x4 matrix.

Q23

If A = [a_{ij}] is a square matrix of even order such that $a_{ij} = i^2 - j^2$, then

a. A is a skew - symmetric matrix and |A| = 0
b. A is symmetric matrix and |A| is a square
c. A is symmetric matrix and |A| = 0
d. None of these

Solution

Correct option: (d)

Let $A=\begin{bmatrix}0 & -3\\ 3 & 0\end{bmatrix}$ $[\because a_{ij} = i^2 - j^2]$

$|A| = 0-(-9) = 9 \neq 0$

Q24

If $A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$, then $A^T + A = I_2$, if

a. $\theta = n\pi, n \in Z$
b. $\theta = (2n+1)\frac{\pi}{2}, n \in Z$
c. $\theta = 2n\pi + \frac{\pi}{3}, n \in Z$
d. none of these

Solution

Correct option: (c)

If $A = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$A^T + A = I_2$

$\Rightarrow \begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} + \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow \begin{bmatrix}2\cos\theta & 0\\ 0 & 2\cos\theta\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$\Rightarrow 2\cos\theta = 1$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}\ n \in Z$

Q25

If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric and skew – symmetric matrix,

then the symmetric matrix is

a. $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Solution

Correct option: (a)

$A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$

The symmetric matrix is

$\frac{1}{2}(A + A^T)$

$= \frac{1}{2}\left(\begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}\right)$

$= \frac{1}{2}\left(\begin{bmatrix} 4 & 4 & -8 \\ 4 & 6 & 8 \\ -8 & 8 & 4 \end{bmatrix}\right)$

$= \begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$

Q26

Out of the following matrices, choose that matrix which is a scalar matrix:

a. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Solution

Correct option: (a)
A diagonal matrix with all diagonal elements are equal is a scalar matrix.

Q27

The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is

a. 27
b. 18
c. 81
d. 512

Solution

Correct option: (d)

Let us consider a matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

The element a can have two values 0 or 1 in two ways.
Similarly all other elements can also have two values 0 or 1 in two ways each.
So the total number of combinations is $2^9 = 512$.
So total no of matrices will be 512.

Q28

Which of the given values of x and y make the following pairs of matrices equal?

$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}$ and, $\begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

a. $x = -\frac{1}{3}, y = 7$

b. $y = 7, x = -\frac{2}{3}$

c. $x = -\frac{1}{3}, 4 = -\frac{2}{5}$

d. Not possible to find

Solution

Correct option: (d)

$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$

$\Rightarrow 3x + 7 = 0$

$\Rightarrow x = \frac{-7}{3}$

$5 = y - 2$

$\Rightarrow y = 7$

$y + 1 = 8$

$\Rightarrow y = 7$

$2 - 3x = 4$

$\Rightarrow x = \frac{-2}{3}$

We are getting two values of x. So it is not possible to find.

Q29

If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b, are respectively

a. -6, -12, -18
b. -6, 4, 9
c. -6, -4, -9
d. -6, 12, 18

Solution

Correct option: (c)

$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$

$kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$

$\Rightarrow -4k = 24$

$\Rightarrow k = -6$

$2k = 3a$

$\Rightarrow a = -4$

$3k = 2b$

$\Rightarrow b = -9$

Q30

If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B equals

a. I cos θ + J sin θ
b. I sin θ + J cos θ
c. I cos θ - J sin θ
d. - I cos θ + J sin θ

Solution

Correct option: (a)

$I\cos\theta + J\sin\theta$

$= \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$= B$

Q31

The trace of the matrix $A = \begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is

a. 17
b. 25
c. 3
d. 12

Solution

Correct option: (a)

Trace = sum of diagonal elements

$= 1 + 7 + 9 = 17$

Q32

If $A = [a_{ij}]$ is a scalar matrix of order $n \times n$ such that $a_{ii} = k$ for all I, then trace of A is equal to

a. nk
b. n + k
c. $\frac{n}{k}$
d. none of these

Solution

Correct option: (a)

$$\text{Trace} = \sum_{i=1}^{n} a_{ii} = nk$$

Q33

The matrix $A = \begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$ is a

a. square matrix
b. diagonal matrix
c. unit matrix
d. none of these

Solution

Correct option: (a)
No of rows=no of columns.

Q34

The number of possible matrices of order 3×3 with each entry 2 or 0 is

a. 9
b. 27
c. 81
d. none of these

Solution

Correct option: (d)

Let us consider a matrix $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

The element a can have two values 0 or 2 in two ways.
Similarly all other elements can also have two values 0 or 2 in two ways each.
So the total number of combinations is 2^9.
So total no of matrices will be 2^9.

Q35

If $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$, then the value of x and y is

a. x = 3, y = 1
b. x = 2, y = 3
c. x = 2, y = 3
d. x = 3, y = 3

Solution

Correct option: (c)

Given $\begin{bmatrix} 2x+y & 4x \\ 5x-7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y-13 \\ y & x+6 \end{bmatrix}$

Equating the terms, we get

$4x = x + 6$

$\Rightarrow x = 2$

and

$2x + y = 7$

$\Rightarrow y = 3$

Q36

If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3 - 7A$ is equal to

a. A
b. I - A
c. I + A
d. 3A

Solution

Correct option: (a)

$(A-I)^3+(A+I)^3-7A$

$=(A-I+A+I)\left((A-I)^2+(A+I)^2-(A-I)(A+I)\right)-7A$

$=2A\left(2A^2+2I-A^2+I^2\right)-7A$

$=2A\left(A^2+3I\right)-7A$

$=2A(I+3I)-7A$

$=8A-7A$

$=A$

Q37

If A and B are two matrix of order 3 × m and 3 × n respectively and m = n, then the order of 5A - 2B is

a. m × n
b. 3 × 3
c. m × n
d. 3 × n

Solution

Correct option: (d)
In scalar multiplication and in addition or substarction of matrices the order doesn't change.

Q38

If A is a matrix of order m × n and B is a matrix such that AB^T and B^TA are both defined, then the order of matrix B is

a. m × n
b. n × n
c. n × m
d. 3 × n

Solution

Correct option: (a)
A is m×n matrix and AB^T is defined then
number of columns in A=number of rows in B^T = n
B^TA is also defined then
number of columns in B^T=number of rows in A = m
Order of B is m×n

Q39

If A and B are matrices of the same order, then $(AB^T-BA^T)^T$ is a

a. skew-symmetric matrix
b. null matrix
c. unit matrix
d. symmetric matrix

Solution

Correct option: (a)

$(AB^T - BA^T)^T$

$= (AB^T)^T - (BA^T)^T$

$= BA^T - AB^T$

$= -(AB^T - BA^T)$

Q40

If matrix $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = \begin{cases} 1, \text{if } i \neq j \\ 0, \text{if } i = j \end{cases}$, then A^2 is equal to

a. I
b. A
c. O
d. -I

Solution

Correct option: (a)

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

Q41

If $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$, $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$,

then A – B is equal to

a. I
b. 0
c. 2I
d. $\frac{1}{2}I$

Solution

Correct option: (d)

Given $A = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix}$ $B = \frac{1}{\pi}\begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$,

$$A - B = \frac{1}{\pi}\begin{bmatrix} \sin^{-1}(x\pi) + \cos^{-1}(\pi x) & 0 \\ 0 & \cot^{-1}(\pi x) + \tan^{-1}(\pi x) \end{bmatrix}$$

$$= \frac{1}{\pi}\begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2}I$$

Q42

If A and B are square matrices of the same order, then (A + B) (A - B) is equal to

a. $A^2 - B^2$
b. $A^2 - BA - AB - B^2$
c. $A^2 - B^2 + BA - AB$
d. $A^2 - BA + B^2 + AB$

Solution

Correct option: (c)

$(A + B)(A - B)$

$= A^2 - AB + BA - B^2$

Q43

If $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, then

a. Only AB is defined
b. Only BA is defined
c. AB ad BA both are defined
d. AB and BA both are not defined

Solution

Correct option: (c)

Given $A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 23 \\ 13 & -17 \end{bmatrix}$$

So AB is defined as no of columns in A is equal to number of rows in B.

$$BA = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -8 & 13 & 9 \\ 16 & -14 & 10 \\ -18 & 24 & 8 \end{bmatrix}$$

So BA is also defined no of columns in B is equal to number of rows in A.

Q44

The matrix $A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$ is a

a. Diagonal matrix
b. Symmetric matrix
c. Skew-symmetric matrix
d. Scalar matrix

Solution

Correct option: (c)

Given $A = \begin{bmatrix} 0 & -5 & 8 \\ 5 & 0 & 12 \\ -8 & -12 & 0 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & 5 & -8 \\ -5 & 0 & -12 \\ 8 & 12 & 0 \end{bmatrix}$$

$\Rightarrow A = -A^T$

So A is skew-symmetric matrix.

Q45

The matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is

a. Identity matrix
b. Symmetric matrix
c. Skew-symmetric matrix
d. Diagonal matrix

Solution

Correct option: (d)
A matrix is called Diagonal matrix if all the elements, except those in the leading diagonal, are zero.

Exercise 5VSAQ

Q1

If A is an $m \times n$ matrix and B is $n \times p$ matrix does AB exist?
If yes, write its order.

Solution

Given,
Order of $A = m \times n$
Order of $B = n \times p$
Since number of columns of A = n = Number of rows of B
$\Rightarrow$ AB exists
and order of AB = number of rows of A $\times$ Number of columns of $B = m \times p$.

Q2

If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$. Write the orders of AB and BA.

Solution

Order of $A = 2 \times 3$
Order of $B = 3 \times 2$

So,
$A_{2\times 3} \times B_{3\times 2}$ has order = 2×2
$B_{3\times 2} \times A_{2\times 3}$ has order = 3×3

Hence,
Order of $AB = 2 \times 2$
Order of $BA = 3 \times 3$.

Q3

If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, write AB.

Solution

Given,

$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -16+9 \\ -4+6 \end{bmatrix}$$

$$= \begin{bmatrix} -7 \\ 2 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} -7 \\ 2 \end{bmatrix}$$

Q4

If $A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, write AA^T.

Solution

Given,

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Hence,

$$AA^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Q5

Given an example of two non-zero 2×2 matrices A and B such that $AB = O$.

Solution

Let,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq O$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \neq O$$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Q6

If $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$, find $A + A^T$.

Solution

Given,

$$A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$$

$$A + A^T = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 5 \\ 3 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 3+5 \\ 5+3 & 7+7 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$$

Hence,

$$A + A^T = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$$

Q7

If $A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$, write A^2.

Solution

Given,

$$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$A^2 = A \times A$

$$= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$= \begin{bmatrix} i^2+0 & 0+0 \\ 0+0 & 0+i^2 \end{bmatrix}$$

$$= \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \{\text{Since, } i^2 = -1\}$$

Hence,

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Q8

If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x satisfying $0 < x < \frac{\pi}{2}$ when $A + A^T = I$.

Solution

Given,

$$A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow \quad A^T = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$A + A^T = I$

$$\Rightarrow \quad \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \cos x + \cos x & \sin x - \sin x \\ -\sin x + \sin x & \cos x + \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$2\cos x = 1$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} \qquad$ since $0 < x < \frac{\pi}{2}$

So,

$x = \frac{\pi}{3}$.

Q9

If $A=\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}$, find AA^T

Solution

$$A=\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}$$
$$\Rightarrow \quad A^T=\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$$
$$AA^T=\begin{bmatrix}\cos x & -\sin x\\ \sin x & \cos x\end{bmatrix}\begin{bmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{bmatrix}$$
$$=\begin{bmatrix}\cos^2 x+\sin^2 x & \cos x\sin x-\sin x\cos x\\ \cos x\sin x-\sin x\cos x & \sin^2 x+\cos^2 x\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=I$$

So,
$$AA^T=I$$

Q10

If $\begin{bmatrix}1 & 0\\ y & 5\end{bmatrix}+2\begin{bmatrix}x & 0\\ 1 & -2\end{bmatrix}=I$, where I is 2×2 unit matrix. Find x and y.

Solution

Given,
$$\begin{bmatrix}1 & 0\\ y & 5\end{bmatrix}+2\begin{bmatrix}x & 0\\ 1 & -2\end{bmatrix}=I$$
$$\Rightarrow \quad \begin{bmatrix}1 & 0\\ y & 5\end{bmatrix}+\begin{bmatrix}2x & 0\\ 2 & -4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix}1+2x & 0\\ y+2 & 5-4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \quad \begin{bmatrix}1+2x & 0\\ y+2 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Since, corresponding matrices of equal matrices are equal, so

$1+2x=1$

$\Rightarrow \quad x=0$

And $\quad y+2=0$

$\Rightarrow \quad y=-2$

Hence,
$$x=0, y=-2$$

Q11

If $A=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}$ satisfies $A^4=\lambda A$, then write the value of λ.

Solution

Given,

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

and

$$A^2 = KA$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$k = 2$$

Q12

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value of λ.

Solution

Given,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

And

$$A^4 = \lambda A$$

$$\Rightarrow \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right)^2 = \lambda\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow \left(\begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix}\right)^2 = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+4 & 4+4 \\ 4+4 & 4+4 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$$\lambda = 8$$

Q13

If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^2.

Solution

Given,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$A^2 = A \times A$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A^2 = I$

Hence,

$A^2 = I$

$$\text{Thus, we can say that, } A^2 = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = -A = I_3$$

Q14

If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, find A^3.

Solution

Given,

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0-1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0-1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^3 = A$$

Hence. $A^3 = A$

Q15

If $A = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$, find A^4.

Solution

Given,

$$A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$

$$A^2 = A \times A$$
$$= \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}\begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$$
$$= \begin{bmatrix} 9+0 & 0+0 \\ 0+0 & 0+9 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$

$$A^4 = A^2 \times A^2$$
$$= \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}\begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$$
$$= \begin{bmatrix} 81+0 & 0+0 \\ 0+0 & 0+81 \end{bmatrix}$$
$$= \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$$

Hence,

$$A^4 = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$$

Q16

If $\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$, find x.

Solution

Given,

$$\begin{bmatrix} x & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2$$
$$\Rightarrow \begin{bmatrix} 2x + 6 \end{bmatrix} = 2$$
$$\Rightarrow 2x + 6 = 2$$
$$\Rightarrow 2x = 2 - 6$$
$$\Rightarrow x = \frac{-4}{2}$$
$$\Rightarrow x = -2$$

Q17

If $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a 2×2 matrix such that $a_{ij} = i + 2j$, write A.

Solution

Here,

$a_{ij} = i + 2j$

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$= \begin{bmatrix} 1+2(1) & 1+2(2) \\ 2+2(1) & 2+2(2) \end{bmatrix}$

$= \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Hence,

$A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Q18

Write matrix A satisfying $A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.

Solution

Given,

$A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 3-2 & -6-3 \\ -3+1 & 8-4 \end{bmatrix}$

$\Rightarrow \quad A = \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$

Hence,

$A = \begin{bmatrix} 1 & -9 \\ -2 & 4 \end{bmatrix}$

Q19

If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric.

Solution

Given,

$A = [a_{ij}]$

Such that $a_{ij} = i^2 - j^2$

$\Rightarrow \quad a_{ji} = (j)^2 - i^2$

$\Rightarrow \quad a_{ji} = j^2 - i^2$

$\Rightarrow \quad a_{ji} = -(i^2 - j^2)$

$\Rightarrow \quad a_{ji} = -a_{ij}$

We know that, A square matrix $A = [a_{ij}]$ is skew-symmetric if $a_{ji} = -a_{ij}$.

So,

A is a skew symmetric matrix.

Q20

For any square matrix write whether AA^T is symmetric or skew-symmetric.

Solution

$(AA^T)^T = (A^T)^T \times A^T \quad \{\text{since, } (AB)^T = B^T A^T\}$

$\therefore (AA^T)^T = (AA^T) \quad$ ---(i) $\quad \{\text{since, } (A^T)^T = A\}$

We know that, a square matrix A is symmetric if $A^T = A$

So, from equation (i)

(AA^T) is a symmetric matrix.

Q21

If $A = [a_{ij}]$ is a skew-symmetric matrix, then write the value of $\sum_i a_{ii}$.

Solution

Given,

$A = [a_{ij}]$ is skew symmetric

$\Rightarrow \quad a_{ij} = -a_{ji}$

$\Rightarrow \quad a_{ii} = -a_{ii}$

$\Rightarrow \quad a_{ii} + a_{ii} = 0$

$\Rightarrow \quad 2a_{ii} = 0$

$\Rightarrow \quad a_{ii} = 0$

$\sum_i a_{ii} = 0 + 0 + \ldots + 0$ (i times)

$= 0$

So,

$\sum_i a_{ii} = 0$

Q22

If $A = \left[a_{ij}\right]$ is a skew-symmetric matrix, then write the value of $\sum_i \sum_j a_{ij}$.

Solution

Given,

$A = \left[a_{ij}\right]$ is a skew symmetric

$\Rightarrow \quad a_{ij} = -a_{ji}$

$\Rightarrow \quad a_{ii} = 0$

$$\sum_i \sum_j a_{ij} = a_{11} + a_{12} + a_{13} + \ldots + a_{21} + a_{22} + a_{23} + \ldots + a_{31} + a_{32} + a_{33} + \ldots$$
$$= 0 + a_{12} + a_{13} + \ldots - a_{12} + 0 + a_{23} + \ldots - a_{31} - a_{23} + 0 + \ldots$$
$$= 0$$

So,

$$\sum_i \sum_j a_{ij} = 0$$