Ch 5 – Algebra of Matrices

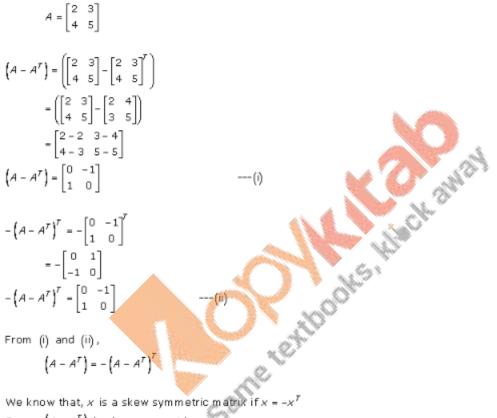
Exercise 5.5

Q1

If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, prove that $A - A^{T}$ is a skew-symmetric matrix.

Solution

Given,



 $(A - A^{T})$ is skew symmetric. So,

Q2

If
$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$
, show that $A - A^{T}$ is a skew-symmetric matrix.

Given, $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ $A - A^{T} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}^{T}$ $= \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$ $= \begin{bmatrix} 3 - 3 & -4 - 1 \\ 1 + 4 & -1 + 1 \end{bmatrix}$ $A - A^T = \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ --- (i) $-\left(A-A^{T}\right)^{T}=-\begin{bmatrix}0&-5\\5&0\end{bmatrix}^{T}$ --[0 5 -5 0] $-\left(A-A^{T}\right)^{T}=\begin{bmatrix}0&-5\\5&0\end{bmatrix}$ ----(ii)

From equation (i) and (ii),

$$\left(A - A^{T}\right) = -\left(A - A^{T}\right)^{t}$$

We know that, x is skewsymmetric matrx if x = -xSo, $(A - A^T)$ is skewsymmetric matrix.

Q3

If the matrix $A = \begin{bmatrix} 5 & 2 \\ y & z & -3 \end{bmatrix}$ is a symmetric matrix, find x, y, z and t.

Solution

Given,

$$A = \begin{bmatrix} 5 & 2 & x \\ y & z & -3 \\ 4 & t & -7 \end{bmatrix}$$
 is a symmetric matrix.

We know that $A = [aij]_{m \times n}$ is a symmetric matrix if aij = aji

So, x = a₁₃ = a₃₁ = 4 $y = a_{21} = a_{12} = 2$

 $z = a_{22} = a_{22} = z$ t = a32 = a32 = -3

Hence,

x = 4, y = 2, t = -3 and z can have any value.

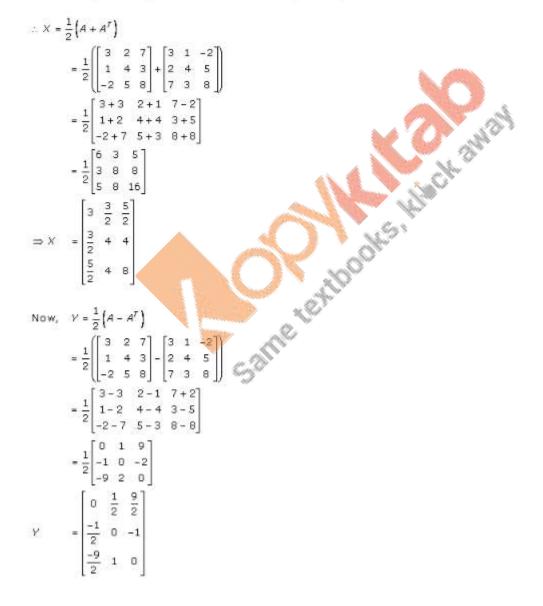
Ch 5 – Algebra of Matrices

Let
$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix}$$
. Find matrix X and Y such that $X + Y = A$, where X is a symmetric and Y is a skew-symmetric matrix.

Solution

Given,

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 1 & 4 & 3 \\ -2 & 5 & 8 \end{bmatrix} \qquad \qquad \Rightarrow \qquad A^{T} = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 5 \\ 7 & 3 & 8 \end{bmatrix}$$



x is a symmetric matrix

Now,

$$x^{7} = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & \frac{4}{4} \\ \frac{5}{2} & \frac{4}{8} \end{bmatrix}^{7} = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{2} \\ \frac{3}{2} & \frac{4}{4} \\ \frac{5}{2} & \frac{4}{8} \end{bmatrix}^{7} = X$$

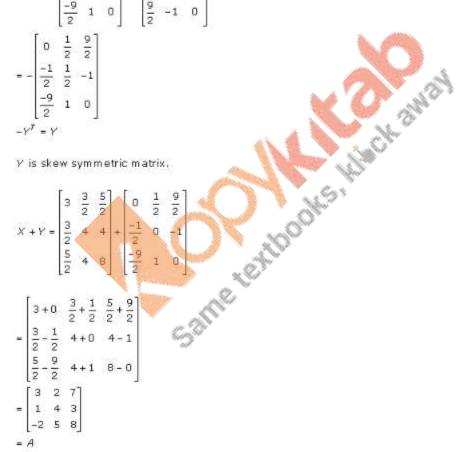
 \Rightarrow

 \Rightarrow

Now,

$$-\gamma^{T} = -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}^{T} = \begin{bmatrix} 0 & \frac{-1}{2} & \frac{-9}{2} \\ \frac{1}{2} & 0 & 1 \\ \frac{9}{2} & -1 & 0 \end{bmatrix}$$
$$= -\begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & \frac{1}{2} & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$
$$-\gamma^{T} = \gamma$$

Y is skew symmetric matrix.



Hence,

$$X = \begin{bmatrix} 3 & \frac{3}{2} & \frac{5}{4} \\ \frac{3}{2} & 4 & 4 \\ \frac{5}{2} & 4 & 8 \end{bmatrix}, Y = \begin{bmatrix} 0 & \frac{1}{2} & \frac{9}{2} \\ \frac{-1}{2} & 0 & -1 \\ \frac{-9}{2} & 1 & 0 \end{bmatrix}$$

Ch 5 – Algebra of Matrices

Express the matrix $A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Solution

Given,

2

$$A = \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} \qquad \Rightarrow \qquad A^{T} = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix}$$

Let
$$X = \frac{1}{2} \left[\begin{pmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{pmatrix} + \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix} \right]$$

$$= \frac{1}{2} \begin{bmatrix} 4 + 4 & 2 + 3 & -1 + 1 \\ 3 + 2 & 5 + 5 & 7 - 2 \\ 1 - 1 & -2 + 7 & 1 + 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 8 & 5 & 0 \\ 5 & 10 & 5 \\ 0 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}$$

$$X^{T} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^{T} = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix}^{T} = x$$

$$X \text{ is symmetric matrix}$$

Now,

$$Y = \frac{1}{2} \left[\begin{pmatrix} 4 - A^{T} \end{pmatrix} \\ = \frac{1}{2} \begin{bmatrix} 4 - 3 & 1 \\ 2 & 5 & -2 \\ 1 - 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 3 & 1 \\ 2 & 5 & -2 \\ -1 & 7 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 4 - 4 & 2 - 3 & -1 - 1 \\ 2 & 3 - 2 & 5 - 5 & 7 + 2 \\ 1 + 1 & -2 - 7 & 1 - 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & 9 \\ 2 & -9 & 0 \end{bmatrix}$$

$$\therefore Y = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}$$

$$\Rightarrow -Y^{T} = -\begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix}^{T} = \begin{bmatrix} 0 & -\frac{1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & -\frac{9}{2} & 0 \end{bmatrix} = Y$$

$$\Rightarrow Y \text{ is a skew symmetric matrix}.$$

Now,

$$X + Y = \begin{bmatrix} 4 & \frac{5}{2} & 0 \\ \frac{5}{2} & 5 & \frac{5}{2} \\ 0 & \frac{5}{2} & 1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-1}{2} & -1 \\ \frac{1}{2} & 0 & \frac{9}{2} \\ 1 & \frac{-9}{2} & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 4+0 & \frac{5}{2} - \frac{1}{2} & 0-1 \\ \frac{5}{2} + \frac{1}{2} & 5+0 & \frac{5}{2} + \frac{9}{2} \\ 0+1 & \frac{5}{2} - \frac{9}{2} & 1+0 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 2 & -1 \\ 3 & 5 & 7 \\ 1 & -2 & 1 \end{bmatrix} = A$$

Q6

Define a symmetric matrix. Prove that for $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$, $A + A^T$ is a symmetric matrix where A^T is the transpose of A. Solution A square matrix A is called a symmetric matrix, if $A^T = A$ Here, $A = \begin{bmatrix} 2 & 4 \end{bmatrix}$

Solution

$$A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2 + 2 & 4 + 5 \\ 5 + 4 & 6 + 6 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

$$A + A^{T} = \begin{bmatrix} 4 & 9 \\ 9 & 12 \end{bmatrix}$$

From equation (i) and (ii),

$$\left(A + A^{T}\right)^{T} = \left(A + A^{T}\right)$$

So,

 $(A + A^T)$ is a symmetric matrix.

Ch 5 – Algebra of Matrices

Express the matrix $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Solution

Here,

 \Rightarrow

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} \implies A^{T} = \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}$$

Let,
$$X = \frac{1}{2} \left(A + A^{T} \right) = \frac{1}{2} \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3 + 3 & -4 + 1 \\ 1 - 4 & -1 - 1 \end{bmatrix} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}$$

Now, $X^{T} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^{T} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^{T} = \begin{bmatrix} 3 & \frac{-3}{2} \\ \frac{-3}{2} & -1 \end{bmatrix}^{T}$
 $\Rightarrow X \text{ is symmetric matrix}$

Let
$$Y = \frac{1}{2} \left(A - A^{T} \right) = \frac{1}{2} \left(\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix} \right) = \frac{1}{2} \begin{bmatrix} 3 - 3 & -4 - 1 \\ 1 + 4 & -1 + 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}$$

Now, $-Y^{T} = -\begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix}^{T} = \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = Y$
 $\Rightarrow Y \text{ is skew symmetric}$
Now, $X + Y = \begin{bmatrix} 3 & \frac{-3}{2} \\ -\frac{3}{2} & -1 \end{bmatrix} + \begin{bmatrix} 0 & \frac{-5}{2} \\ \frac{5}{2} & 0 \end{bmatrix} = \begin{bmatrix} 3 + 0 & -\frac{3}{2} - \frac{5}{2} \\ -\frac{3}{2} + \frac{5}{2} & -1 + 0 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = A$
Q8

Express the following matrix as the sum of a symmetric and skey-

symmetric matrix and verify your result:

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

Let,

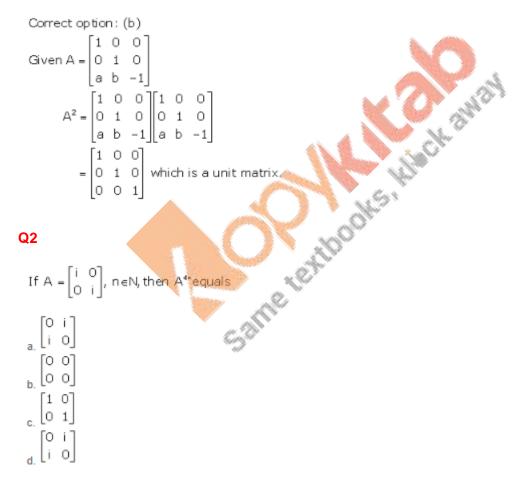
Ch 5 – Algebra of Matrices

Exercise MCQ

Q1

If
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ a & b & -1 \end{bmatrix}$$
, then A^{2} is equal to
a. a null matrix
b. a unit matrix
c. -A
d A

Solution



Correct option: (c)
Given A =
$$\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} = i \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

A⁴ = i⁴ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
= $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
A⁴ⁿ = $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Q3

If A and B are two matrices such that AB = A and BA = B, then B² is equal to

- a. B b. A
- c. 1 d. 0

Solution

```
Correct option: (a)
Given AB=A and BA=B, then
\Rightarrow BAB = B<sup>2</sup>
\Rightarrow BA = B^2
\Rightarrow B = B^2
```

Q4

```
If AB = A and BA = B, where A and B are square matrices, then
```

```
a. B^2 = B and A^2 = A
b. B^2 \neq B and A^2 = A
c. A^2 \neq A, B^2 = B
d. A^2 \neq A, B^2 \neq B
```

Solution

Correct option: (a) Given AB=A and BA=B, then \Rightarrow BAB = B² and ABA=A² \Rightarrow BA = B² and AB=A² \Rightarrow B = B² and A=A²

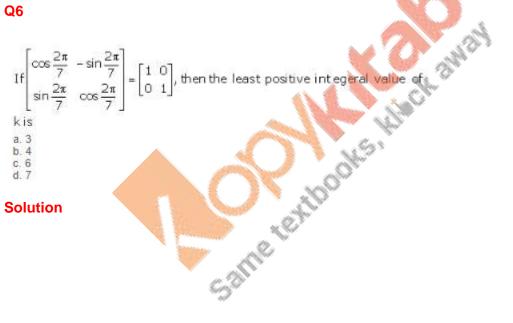
Ch 5 – Algebra of Matrices

If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ is equal to

a. 2 AB b. 2 BA c. A + B d. AB

Solution

Correct option: (c) Given AB=A and BA=B, then \Rightarrow BAB = B² and ABA=A² \Rightarrow BA = B² and AB=A² \Rightarrow B = B² and A=A² $\Rightarrow A^2 + B^2 = A + B$



Correct option: (d)
Let
$$A = \begin{bmatrix} \cos \frac{2\pi}{7} & \sin \frac{-2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix}^{-1} \begin{bmatrix} \operatorname{Re} e^{\frac{2\pi}{7}} & \operatorname{Im} e^{\frac{-2\pi}{7}} \\ \operatorname{Im} e^{\frac{2\pi}{7}} & \operatorname{Re} e^{\frac{2\pi}{7}} \end{bmatrix}^{-1} \\ A^{2} = \begin{bmatrix} \cos \frac{2\pi}{7} & \sin \frac{-2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix} \begin{bmatrix} \cos \frac{2\pi}{7} & \sin \frac{-2\pi}{7} \\ \sin \frac{2\pi}{7} & \cos \frac{2\pi}{7} \end{bmatrix}^{-1} \\ = \begin{bmatrix} \cos^{2} \frac{2\pi}{7} - \sin^{2} \frac{2\pi}{7} & -2\cos \frac{2\pi}{7} \sin \frac{2\pi}{7} \\ 2\cos \frac{2\pi}{7} \sin \frac{2\pi}{7} & \cos^{2} \frac{2\pi}{7} - \sin^{2} \frac{2\pi}{7} \end{bmatrix}^{-1} \\ = \begin{bmatrix} \cos \frac{4\pi}{7} & -\sin \frac{4\pi}{7} \\ \cos \frac{4\pi}{7} & -\sin \frac{4\pi}{7} \end{bmatrix}^{-1} \\ = \begin{bmatrix} \operatorname{Re} e^{\frac{\pi}{7}} & \operatorname{Im} e^{\frac{-4\pi}{7}} \\ \operatorname{Im} e^{\frac{\pi}{7}} & \operatorname{Re} e^{\frac{\pi}{7}} \end{bmatrix}^{-1} \\ \operatorname{Re} e^{\frac{\pi}{7}} & \operatorname{Im} e^{\frac{-4\pi}{7}} \\ \operatorname{Im} e^{\frac{\pi}{7}} & \operatorname{Re} e^{\frac{\pi}{7}} \end{bmatrix}^{-1} \\ \operatorname{Now} \\ \begin{bmatrix} \operatorname{Re} e^{\frac{\pi}{7}} & \operatorname{Im} e^{-\frac{\pi}{7}} \\ \operatorname{Im} e^{\frac{\pi}{7}} & \operatorname{Re} e^{\frac{\pi}{7}} \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ \Rightarrow \operatorname{Re} e^{\frac{\pi}{7}} = 1 \\ \Rightarrow \cos \frac{2\pi}{7} = 1 \\ \Rightarrow \cos \frac{2\pi}{7} = 1 \\ \Rightarrow \cos \frac{2\pi}{7} = 2\pi \text{ for } n \in \mathbb{Z} \\ \Rightarrow k = 7n \\ \text{So positive integral values of k can be 0, 7, 14 etc} \end{bmatrix}$$

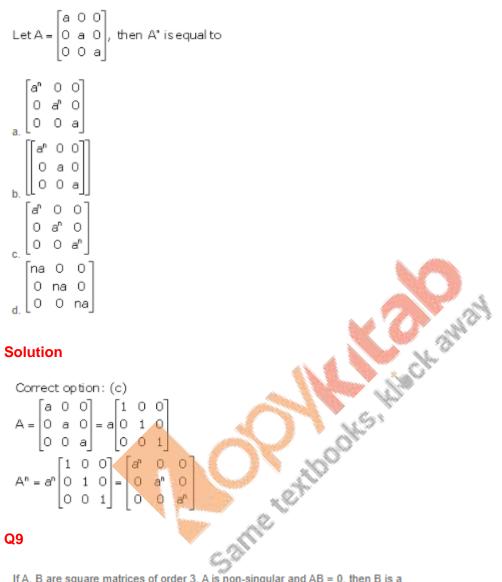
Q7

If the matrix AB is zero , then

a. It is not necessary that either A = 0 or B = 0 b. A = 0 or B = 0 c. A = O and B = 0 d. All the above statements are wrong

Solution

Correct option: (a) Let $A = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$ $AB = \begin{bmatrix} 0 & 3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $A \neq 0, B \neq 0$ but AB = 0



If A, B are square matrices of order 3, A is non-singular and AB = 0, then B is a

a. Null matrix

b. Singular matrix

c. Unit matrix

d. Non-singular matrix

Solution

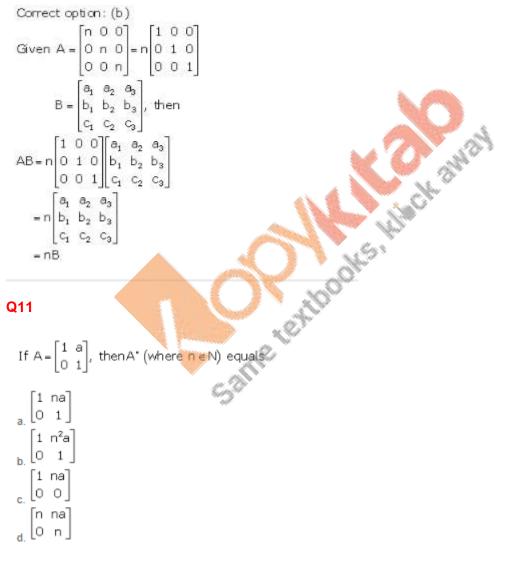
Correct option: (a)

A is non singular that means determinant of A is non zero. $AB=0 \Rightarrow B=0$

Ch 5 – Algebra of Matrices

If A =
$$\begin{bmatrix} n & 0 & 0 \\ 0 & n & 0 \\ 0 & 0 & n \end{bmatrix}$$
 and B = $\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$, then AB is equal to
a. B
b. nB
c. Bⁿ
d. A + B

Solution



Correct option: (a) Given $A = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $A^{2} = \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix}$ $A^3 = A^2A$ $\begin{bmatrix} 1 & 2a \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ $=\begin{bmatrix} 1 & 3a \\ 0 & 1 \end{bmatrix}$ On genaralising we get $A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$

Q12

$$A^{H} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$
Q12
If $A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $AB = I_3$, then $x + y$ equals to the interval of the set of

Solution

```
Given A = \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} and B = \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
AB = I<sub>3</sub>
\Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}
    \Rightarrow \begin{bmatrix} 1 & 2 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
    \Rightarrow \begin{bmatrix} 1 & 0 & \times + \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}
    \Rightarrow x + y = 0
```

Ch 5 – Algebra of Matrices

If
$$A\begin{bmatrix} 1 & -1\\ 2 & -1 \end{bmatrix}$$
, $B = \begin{bmatrix} a & 1\\ b & -1 \end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$,
then values of a and b are

a. a = 4, b = 1 b. a = 1, b = 4 c. a = 0, b = 4 d. a = 2, b = 4

Solution

Correct answer: (b)
Given
$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$$

 $(A + B)^2 = A^2 + B^2$,
 $\Rightarrow A^2 + B^2 + AB + BA = A^2 + B^2$
 $\Rightarrow AB + BA = 0$
 $\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} + \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} a - b & 2 \\ 2a - b & 3 \end{bmatrix} + \begin{bmatrix} a + 2 & -a - 1 \\ b - 2 & -b + 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\Rightarrow \begin{bmatrix} 2a - b + 2 & 2 - a - 1 \\ 2a - 2 & 4 - b \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
 $\Rightarrow a = 1 \text{ and } b = 4$
Q14
If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then
 $a \cdot 1 + \alpha^2 + \beta\gamma = 0$
 $b \cdot 1 - \alpha^2 + \beta\gamma = 0$
 $c \cdot 1 - \alpha^2 - \beta\gamma = 0$

Q14

If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then a. $1 + \alpha^2 + \beta \gamma = 0$

b. 1 - α^2 + $\beta\gamma$ = 0 c. $1 - \alpha^2 - \beta \gamma = 0$ d. $1 + \alpha^2 - \beta \gamma = 0$

Solution

Correct option: (c) Given $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ and $A^2 = I$, then $A^2 = I$ $\Rightarrow \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} \alpha^2 + \beta \gamma & 0 \\ 0 & \alpha^2 + \beta \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\Rightarrow \alpha^2 + \beta \gamma = 1$ $\Rightarrow 1 - \alpha^2 - \beta y = 0$

Ch 5 – Algebra of Matrices

If S = [s_{ii}] is a scalar matrix such that s_{ii} = k and A is a square matrix of the same order, then AS = SA = ?

a. A^k b. k + A c. kA d. kS

Solution

```
Correct option: (c)
   k O O
              [1 0 0]
S= 0 k 0 = k 0 1 0
0 0 k 0 0 1
      [1 0 0]
SA=k 0 1 0 A=kA
      001
```

Q16

If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ - 7A is equal to

a. A b. I - A c. I d. 3A

Solution

Correct option: (c) $(I + A)^{3}$ = 1³ + A³ + 31²A + 31A² = I + A²A + 3A + 3A² = I + A² + 3A + 3A = I + A + 6A = I + 7A (I + A)3 - 7A - I

Q17

If a matrix A is both symmetric and skew-symmetric, then

a. A is a diagonal matrix

b. A is a zero matrix

c. A is a scalar matrix

d. A is a square matrix

Ch 5 – Algebra of Matrices

Correct option: (b) A is symmetric $\Rightarrow a_{ii} = a_{ii} \rightarrow (1)$ A is skew-symmetric $\Rightarrow a_i = -a_i \rightarrow (2)$ and $a_i = -a_i$ $\Rightarrow a_{i} = 0$ means the diagonal entries are zero. From (1) and (2) we can write a; = a; = 0 which means all the off diagonal entries are zero. So A is a null matrix.

Q18

The matrix
$$\begin{bmatrix} 0 & 5 & -7 \\ -5 & 0 & 11 \\ 7 & -11 & 0 \end{bmatrix}$$
 is

a. A skew-symmetric matrix b. A symmetric matrix c. A diagonal matrix d. An upper triangular matrix

Solution

is skew symmetric because $a_1 + a_2$ Correct option: (a) Ō 5 -7 The matrix -5 0 11 7 -11 0 for i,j=1,2,3.

Q19

If A is a square matrix, then AA is a

a. Skew-symmetric matrix

- b. Symmetric matrix
- c. Diagonal matrix
- d. None of these

Solution

Correct option: (d)
Let
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

 $AA = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

Ch 5 – Algebra of Matrices

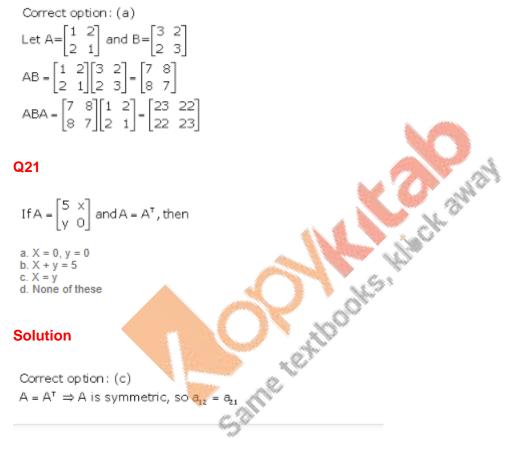
If A and B are symmetric matrices, then ABA is

a. Symmetric matrix

b. Skew-symmetric matrix

- c. Diagonal matrix
- Scalar matrix

Solution



Q22

If A is 3 × 4 matrix and B is a matrix such that A^TB and BA^T are both defined. Then, B is of the type

a. 3 × 4 b. 3 × 3 c. 4 × 4 d. 4 × 3

Solution

Correct option: (a) A is 3×4 matrix so A^T is 4×3 matrix A^TB is defined, so no of columns in A^T =no of rows in B=3 BA^T is defined, so no of columns in B=no of rows in A^T =4 So B is 3×4 matrix.

If A = $[a_{ij}]$ is a square matrix of even order such that a_{ij} = i^2 - $j^2,$ then

a. A is a skew - symmetric matrix and |A| = 0 b. A is symmetric matrix and |A| is a square c. A is symmetric matrix and |A| = 0 d. None of these

Solution

Correct option: (d) Let $A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} \because a_j = i^2 - j^2 \end{bmatrix}$ |A| = 0 - (-9) = 9 ≠ 0

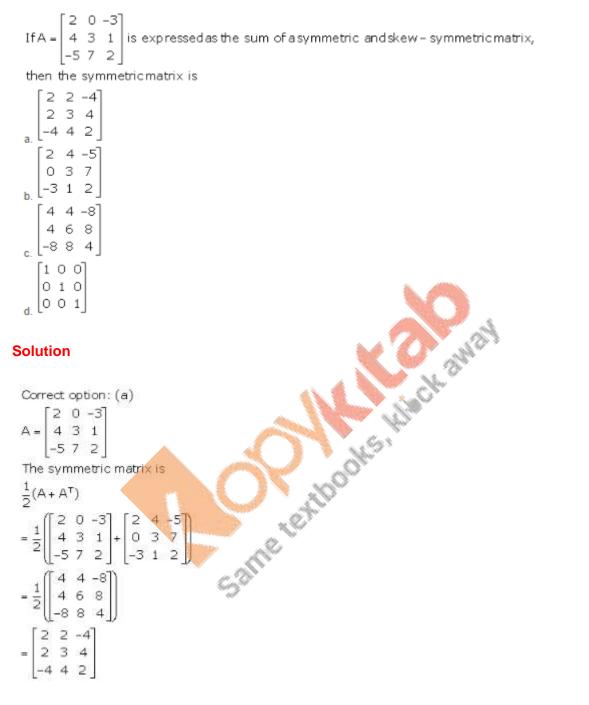
Q24

If
$$A = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$
, then $A^{T} + A = I_{z}$
 $a. \theta = n\pi, n \in Z$
 $b. \theta = (2n + 1)\frac{\pi}{2}, n \in Z$
 $c. \theta = 2n\pi + \frac{\pi}{3}, n \in Z$
 $d. none of these$

Solution

```
r
Correct option: (c)
IfA = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}
A^T + A = I_2
\Rightarrow \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} + \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\Rightarrow \begin{bmatrix} 2\cos\theta & 0\\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix}
\Rightarrow 2\cos\theta = 1
\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3} n \in \mathbb{Z}
```

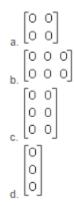
Ch 5 – Algebra of Matrices





Ch 5 – Algebra of Matrices

Out of the following matrices, choose that matrix which is a scalar matrix:



Solution

The number of all possible matrices of order 3×3 with each entry 0 or 1 is the transmitted of the second of t Correct option: (a)

Q27

Solution

The element a can have two values 0 or 1 in two ways. Similarly all other elements can also have two values 0 or 1 in two ways each.

So the total number of combinations is 29 = 512. So total no of matrices will be 512.

Which of the given values of x and y make the following pairs of matrices equal? $\begin{bmatrix} 3x + 7 & 5\\ y + 1 & 2 - 3x \end{bmatrix}$ and, $\begin{bmatrix} 0 & y - 2\\ 8 & 4 \end{bmatrix}$ $x = -\frac{1}{3}, y = 7$ $y = 7, x = -\frac{2}{3}$ $x = -\frac{1}{3}, 4 = -\frac{2}{5}$ C.

d. Not possible to find

Solution

Correct option: (d)

$$\begin{bmatrix} 3x + 7 & 5 \\ y + 1 & 2 - 3x \end{bmatrix} = \begin{bmatrix} 0 & y - 2 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow 3x + 7 = 0$$

$$\Rightarrow x = \frac{-7}{3}$$

$$5 = y - 2$$

$$\Rightarrow y = 7$$

$$y + 1 = 8$$

$$\Rightarrow y = 7$$

$$2 - 3x = 4$$

$$\Rightarrow x = \frac{-2}{3}$$
We are getting two values of x. So it is not possible to find.

Q29 If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$ and $kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b, are respectively. ofk, a, b, are respectively

a. -6, -12, -18 b. -6, 4, 9 c. -6, -4, -9 d. -6, 12, 18

Correct option: (c)

$$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}$$

$$kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 2k \\ 2b & 24 \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$

$$\Rightarrow -4k = 2k$$

$$\Rightarrow -4k = 24$$

$$\Rightarrow k = -6$$

$$2k = 3a$$

$$\Rightarrow a = -4$$

$$3k = 2b$$

$$\Rightarrow b = -9$$

Q30

If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$, then B equals a. I cos θ + J sin θ b. I sin θ + J cos θ c. I cos 0 - J sin 0 d. - I cos θ + J sin θ

Solution

```
Correct option: (a)
Icos 0+ Jsin 0
  cos 0
                  0
                       sine
         0
      cos θ +
    0
                -sine
                        0
   cos 0 sin 0
  -sin0 cos0
= B
```

Q31

The trace of the matrix A =
$$\begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$$
 is

a. 17 b. 25 c. 3

d. 12

```
Correct option: (a)
Trace - sum of diagonal elements
      = 1 + 7 + 9 = 17
```

Ch 5 – Algebra of Matrices

Q32

If A = [a_{ii}] is a scalar matrix of order n × n such that a_{ii} = k for all I, then trace of A is equal to

a. nk b. n + k n c. K d. none of these

Solution

Correct option: (a)

Q33

The matrix A =
$$\begin{bmatrix} 0 & 0 & 4 \\ 0 & 4 & 0 \\ 4 & 0 & 0 \end{bmatrix}$$
 is a

a. square matrix

b. diagonal matrix

c. unit matrix

d. none of these

Solution

Correct option: (a) No of rows=no of columns.

Q34

The number of possible matrices of order 3 × 3 with each entry 2 or 0 is

a. 9 b. 27 c. 81

d. none of these

Ch 5 – Algebra of Matrices

Correct option: (d)

a b c Let us consider a matrix d e f ghi

The element a can have two values 0 or 2 in two ways. Similarly all other elements can also have two values 0 or 2 in two ways each.

So the total number of combinations is 29. So total no of matrices will be 29.

Q35

 $If\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 13 \\ y & x + 6 \end{bmatrix}$, then the value of x and y is a. x = 3, y = 1 b. x = 2, y = 3 c. x = 2, y = 3 d. x = 3, y = 3

Solution

Correct option: (c) Given $\begin{bmatrix} 2x + y & 4x \\ 5x - 7 & 4x \end{bmatrix} = \begin{bmatrix} 7 & 7y - 13 \\ y & x + 6 \end{bmatrix}$ Equating the terms, we get 4x = x + 6⇒×=2 and 2x + y = 7⇒y = 3

Q36

If A is a square matrix such that $A^2 = I$, then $(A - I)^3 + (A + I)^3$ - 7A is equal to

a. A b. I - A c. I + A d. 3A

Ch 5 – Algebra of Matrices

CK away

Correct option: (a) $(A - I)^{3} + (A + I)^{3} - 7A$ $= (A - I + A + I)((A - I)^{2} + (A + I)^{2} - (A - I)(A + I)) - 7A$ = 2A(2A² + 2I - A² + I²) - 7A = 2A(A² + 3I) - 7A = 2A(1+31)-7A = 8A - 7A - A

Q37

If A and B are two matrix of order 3 x m and 3 x n respectively and m = n, then the order of 5A - 2B is

a. m × n b. 3 × 3 c. m × n d. 3 × n

Solution

```
Correct option: (d)
In scalar multiplication and in addition or substanction of matrices
the order doesn't change.
                                                     OK.S.
```

Q38

If A is a matrix of order m × n and B is a matrix such that ABT and BTA are both defined, then the order of matrix B is

a. m × n b. n × n c. n × m d. 3 × n

Solution

Correct option: (a)

A is m x n matrix and AB^T is defined then number of columns in A=number of rows in B^T = n B^TA is also defined then number of columns in B^T=number of rows in A = m Order of B is m x n

Q39

If A and B are matrices of the same order, then $(AB^{T}-BA^{T})^{T}$ is a

a. skew-symmetric matrix b. null matrix

c. unit matrix

d. symmetric matrix

Ch 5 – Algebra of Matrices

Solution

Correct option: (a) $(AB^T - BA^T)^T$ $= (AB^T)^T - (BA^T)^T$ $= BA^T - AB^T$ $= -(AB^{T} - BA^{T})$

Q40

If matrix
$$A = \begin{bmatrix} a_{i} \end{bmatrix}_{2\times 2}$$
, where $a_{ij} = \begin{cases} 1, if i \neq j \\ 0, if i = j \end{cases}$, then A^{2} is equal to
a.1
b.A
c.O
d.-1
Solution
Correct option: (a)
 $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
 $A^{2} - \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
 $-\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - I$
Q41
If $A = \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(\pi) & \tan^{-1}(\frac{x}{\pi}) \\ \sin^{-1}(\frac{x}{\pi}) & \cot^{-1}(\pi x) \end{bmatrix}$, $B = \frac{1}{\pi} \begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}(\frac{x}{\pi}) \\ \sin^{-1}(\frac{x}{\pi}) & -\tan^{-1}(\pi x) \end{bmatrix}$,
then $A = B$ is equal to

a. I b. 0 c. 21 d. 21

Ch 5 – Algebra of Matrices

Correct option: (d)

Given
$$A = \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(x\pi) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & \cot^{-1}(\pi x) \end{bmatrix} B = \frac{1}{\pi} \begin{bmatrix} -\cos^{-1}(\pi x) & \tan^{-1}\left(\frac{x}{\pi}\right) \\ \sin^{-1}\left(\frac{x}{\pi}\right) & -\tan^{-1}(\pi x) \end{bmatrix}$$

 $A - B = \frac{1}{\pi} \begin{bmatrix} \sin^{-1}(x\pi) + \cos^{-1}(\pi x) & 0 \\ 0 & \cot^{-1}(\pi x) + \tan^{-1}(\pi x) \end{bmatrix}$
 $= \frac{1}{\pi} \begin{bmatrix} \frac{\pi}{2} & 0 \\ 0 & \frac{\pi}{2} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \frac{1}{2} I$

Q42

If A and B are square matrices of the same order, then (A + B) (A - B) is equal to

a. A² - B² b. A² - BA - AB - B² c. A² - B² + BA - AB d. A^2 - BA + B² + AB

Solution

Correct option: (c) (A+B)(A-B) $= A^2 - AB + BA - B^2$

Q43

If
$$A = \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$$
 and $B = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$, the

a. Only AB is defined

b. Only BA is defined

c. AB ad BA both are defined

d. AB and BA both are not defined

Correct option: (c)

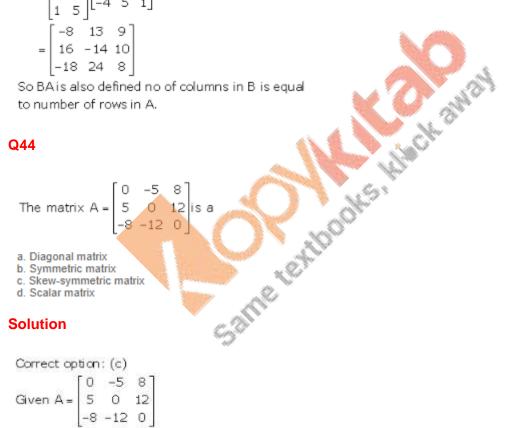
Given A =
$$\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$$
 B = $\begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$
AB = $\begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix}$
= $\begin{bmatrix} 3 & 23 \\ 13 & -17 \end{bmatrix}$

So AB is defined as no of columns in A is equal to number of rows in B.

$$BA = \begin{bmatrix} 2 & 3 \\ 4 & -2 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & -1 & 3 \\ -4 & 5 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} -8 & 13 & 9 \\ 16 & -14 & 10 \\ -18 & 24 & 8 \end{bmatrix}$$

So BA is also defined no of columns in B is equal to number of rows in A.

Q44



[0 5 -8] $A^{T} = -5 \ 0 \ -12$ 8 12 0 $\Rightarrow A = -A^T$

So A is skew-symmetric matrix.

Ch 5 – Algebra of Matrices

The matrix A =
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$
 is

a. Identity matrix b. Symmetric matrix

c. Skew-symmetric matrix

d. Diagonal matrix

Solution

Correct option: (d)

A matrix is called Diagonal matrix if all the elements, except those in the leading diagonal, are zero.



Ch 5 – Algebra of Matrices

Exercise 5VSAQ

Q1

If A is an $m \times n$ matrix and B is $n \times p$ matrix does AB exist? If yes, write its order.

Solution

Given.

```
Order of A = m \times n
       Order of B = n \times p
Since number of columns of A = n = Number of rows of B
⇒
       AB exists
and order of AB = number of rows of A × Number of columns of B = m×p.
```

Q2

If
$$A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$$
 and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$. Write the orders of AB and BA .

Solution

```
Order of A = 2 \times 3
Order of B = 3 \times 2
```

So,

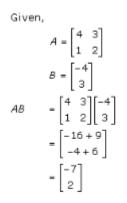
```
A2x3 × B3x2 has order = 2 × 2
B_{3\times 2} \times A_{2\times 3} has order = 3 \times 3
```

Hence,

```
Order of AB = 2 \times 2
Order of BA = 3 \times 3.
```

Q3

If
$$A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$$
 and $B = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, write AB .



Hence,

$$AB = \begin{bmatrix} -7\\2 \end{bmatrix}$$

Q4

If
$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$
, write AA^{T} .

[1]

Solution

Given,

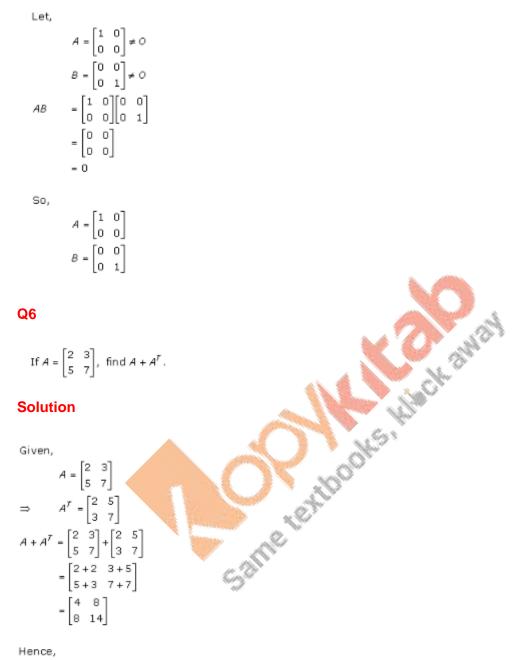
 $A = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ $\Rightarrow A^{T} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ $AA^{T} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ $= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$

Hence,

$$AA^{T} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$$

Q5

Given an example of two non-zero 2 ×2 matrices A and B such that AB = 0.



$$A + A^T = \begin{bmatrix} 4 & 8 \\ 8 & 14 \end{bmatrix}$$

Q7

If
$$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$
, write A^2

Given,

$$A = \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$A^{2} = A \times A$$

$$= \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix} \begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$$

$$= \begin{bmatrix} i^{2} + 0 & 0 + 0 \\ 0 + 0 & 0 + i^{2} \end{bmatrix}$$

$$= \begin{bmatrix} i^{2} & 0 \\ 0 & i^{2} \end{bmatrix}$$

$$A^{2} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$
{Since, $i^{2} = -1$ }

Hence,

$$A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Q8

Solution

If $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, find x satisfying $0 < x < \frac{\pi}{2}$ when $A + A^{T} = I$ Solution Given, $A = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$ $\Rightarrow A^{T} = \begin{bmatrix} \cos x & \sin x \\ \sin x & \cos x \end{bmatrix}$ $A = \begin{bmatrix} \cos x & \sin x \\ \sin x & \cos x \end{bmatrix}$ $A = \begin{bmatrix} \cos x & \sin x \\ \sin x & \cos x \end{bmatrix}$ $\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix} + \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = $\begin{bmatrix} \cos x + \cos x & \sin x - \sin x \\ -\sin x + \sin x & \cos x + \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ⇒ $\begin{bmatrix} 2\cos x & 0 \\ 0 & 2\cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ⇒

Since, corresponding entries of equal matrices are equal, so

 $2\cos x = 1$ $\cos x = \frac{1}{2}$ $X = \frac{\pi}{3}$ since $0 < x < \frac{\pi}{2}$

So,

$$x = \frac{\pi}{3}$$
.

Ch 5 – Algebra of Matrices

```
If A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}, find AA^T
```

Solution

$$A = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

$$\Rightarrow \quad A^{T} = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$AA^{T} = \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix} \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos^{2} x + \sin^{2} x & \cos x \sin x - \sin x \cos x \\ \cos x \sin x - \sin x \cos x & \sin^{2} x + \cos^{2} x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= I$$

$$AA^T = I$$

Q10

Solution

```
If \begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I, where I is 2 \times 2 unit matrix. Find x and y.

Folution

Iven,

\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I

\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I

\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I

\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + 2 \begin{bmatrix} x & 0 \\ 1 & -2 \end{bmatrix} = I

\begin{bmatrix} 1 & 0 \\ y & 5 \end{bmatrix} + \begin{bmatrix} 2x & 0 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}

\begin{bmatrix} 1 + 2x & 0 \\ y + 2 & 5 - 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
                                                             \begin{bmatrix} 1+2x & 0 \\ y+2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
  ⇒
```

Since, corresponding matrices of equal matrices are equal, so

1 + 2x = 1x = 0 ⇒ And y + 2 = 0y = -2⇒

Hence,

x = 0, y = -2

If
$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$
 satisfies $A^4 = \lambda A$, then write the value of λ .

Solution

Given,

 $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ and

	$A^2 = KA$
⇒	$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = k \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$
⇒	$\begin{bmatrix} 1+1 & -1-1 \\ -1-1 & 1+1 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$
⇒	$\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$

Since, corresponding entries of equal matrices are equal, so k = 2

Q12

If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = \lambda A$, then write the value of λ .

Solution

Given,

And

 $A^4 = \lambda A$

 $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

n	$\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^2 = \lambda \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
⇒	$\left(\begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix} \right)^2 = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$
⇒	$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$
n	$\begin{bmatrix} 4+4 & 4+4 \\ 4+4 & 4+4 \end{bmatrix} = \begin{bmatrix} \hat{x} & \hat{x} \\ \hat{x} & \hat{x} \end{bmatrix}$
⇒	$\begin{bmatrix} 8 & 8 \\ 8 & 8 \end{bmatrix} = \begin{bmatrix} \lambda & \lambda \\ \lambda & \lambda \end{bmatrix}$

Since, corresponding entries of equal matrices are equal, so

 $\lambda = 0$

Q13

 $\mathrm{If}\, A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \ \mathrm{find}\, A^2.$

```
Given,
        A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
 A^2 = A \times A
          [-1 0 0][-1 0 0]
        - 0 -1 0 0 -1 0
          0 0 -1
                      0 0 -1
          [1+0+0 0+0+0 0+0+0]
        = 0+0+0 0+1+0 0+0+0
          0+0+0 0+0+0 0+1+0
          [1 0 0]
        = 0 1 0
          0 0 1
                               A^2 = I
 Hence,
        A^{2} = I
 Thus, we can say that, A^2 = -\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} = -A = I_3
Q14
      [-1 0 0]
If A = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}, find A^3.
      0 0 -1
Solution
```

Given,
$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
$A = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$
$A^{2} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
$A^2 = 0 - 1 0 0 - 1 0$
$\begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \end{bmatrix}$
= 0 + 0 + 0 0 + 1 + 0 0 + 0 + 0
$\begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \end{bmatrix}$
$ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $
- 0 1 0
$A^3 = A^2 \times A$
[1 0 0][-1 0 0]
- 0 1 0 0 -1 0
$ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} $
[-1+0+0 0+0+0 0+0+0]
= 0+0+0 0-1+0 0+0+0
0+0+0 0+0+0 0+0-1
[-1 0 0]
$ \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} $
0 0 -1
A ³ = A
Hence, A3 = A
Q15
If $A = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$, find A^4 .
$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $= \begin{bmatrix} -1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0-1+0 & 0+0+0 \\ 0+0+0 & 0+0+0 \end{bmatrix}$ $= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ $A^{3} = A$ Hence. $A^{3} = A$ Q15 If $A = \begin{bmatrix} -3 & 0 \\ 0 & 3 \end{bmatrix}$, find A^{4} . Solution Solution
Calution
Solution

Given, $A = \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$ $A^2 = A \times A$ $= \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -3 & 0 \\ 0 & -3 \end{bmatrix}$ $= \begin{bmatrix} 9+0 & 0+0\\ 0+0 & 0+9 \end{bmatrix}$ - [9 0] 0 9] $= A^2 \times A^2$ A^4 $= \begin{bmatrix} 81+0 & 0+0\\ 0+0 & 0+81 \end{bmatrix}$ $= \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$ Hence, $A^4 = \begin{bmatrix} 81 & 0 \\ 0 & 81 \end{bmatrix}$ Q16 If $\begin{bmatrix} x & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = 2$, find x. **Solution** Given, $\begin{bmatrix} x & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix} = 2$ [2x + 6] = 2⇒ 2x + 6 = 2⇒ ⇒ 2x = 2 - 6 $x = \frac{-4}{2}$ ⇒ x = -2⇒

Q17

If $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a 2×2 matrix such that $a_{ij} = i + 2j$, write A.

Here,

 $a_{ii} = i + 2j$ $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ $= \begin{bmatrix} 1+2(1) & 1+2(2) \\ 2+2(1) & 2+2(2) \end{bmatrix}$ $= \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

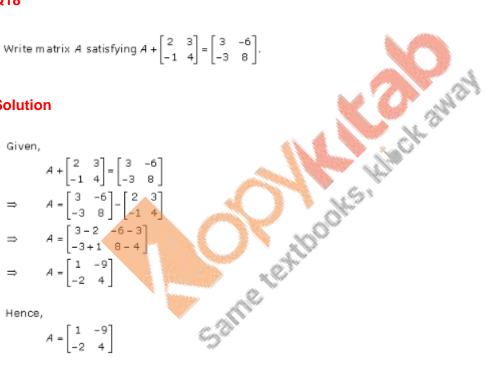
Hence,

 $A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Q18

Write matrix A satisfying $A + \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -3 & 8 \end{bmatrix}$.

Solution



Q19

If $A = [a_{ij}]$ is a square matrix such that $a_{ij} = i^2 - j^2$, then write whether A is symmetric or skew-symmetric.

Ch 5 – Algebra of Matrices

awan

ce, (AB)^r

Given,

$$A = \begin{bmatrix} a_{ij} \end{bmatrix}$$

Such that $a_{ij} = i^2 - j^2$ \rightarrow $= (i)^2 - i^2$

$$\Rightarrow \quad a_j = (j) - j^2$$

$$\Rightarrow \quad a_j = j^2 - j^2$$

$$\Rightarrow \quad a_j = -(j^2 - j^2)$$

$$\Rightarrow \quad \partial_{ji} = -\partial_{ij}$$

We know that, A square matrix $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is skew-symmetric if $a_{ji} = -a_{ij}$.

So,

A is a skew symmetric matrix.

Q20

For any square matrix write whether AA^{T} is symmetric or skew-symmetric

(i)

Solution

 $(AA^T)^T = (A^T)^T \times A^T$

$$\left(AA^{T}\right)^{T} = \left(AA^{T}\right)$$

We know that, a square matrix A is symmetric if $A^{T} = A$ So, from equation (i) (AA^{T}) is a symmetric matrix, Q21

If $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a skew-symmetric matrix, then write the value of $\sum a_{ij}$.

```
Given,
            A = \begin{bmatrix} a_{ij} \end{bmatrix} is skew symmetric
            \partial i_j = -\partial_j
⇒
            a;; = -a;;
⇒
⇒
            a_{ij} + a_{jj} = 0
⇒
            2a_{ii} = 0
⇒
            \partial_{ii} = 0
\sum a_{ii} = 0 + 0 + ... + 0 (i times)
                 - 0
So,
            \sum_{j} a_{ij} = 0
```

Ch 5 – Algebra of Matrices

Q22

If $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a skew-symmetric matrix, then write the value of $\sum_{j} \sum_{j} a_{ij}$.

Solution

Given, $A = \begin{bmatrix} a_{ij} \end{bmatrix}$ is a skew symmetric $\Rightarrow \quad a_{ij} = -a_{ij}$ $\Rightarrow \quad a_{ij} = 0$ $\sum_{i} \sum_{j} a_{ij} = a_{11} + a_{12} + a_{13} + \dots + a_{21} + a_{22} + a_{23} + \dots + a_{31} + a_{32} + a_{33} + \dots$ $= 0 + a_{12} + a_{13} + \dots - a_{12} + 0 + a_{23} + \dots - a_{31} - a_{23} + 0 + \dots$ = 0So, $\sum_{i} \sum_{j} a_{ij} = 0$