Exercise 5.3

Q1

Compute the indicated products:

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

Solution

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

$= \begin{bmatrix} (a)(a)+(b)(b) & (a)(-b)+(b)(a) \\ (-b)(a)+(a)(b) & (-b)(-b)+(a)(a) \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & -ab+ab \\ -ab+ab & b^2+a^2 \end{bmatrix}$

$= \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$

Hence,

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix}$$

(ii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix}$

$= \begin{bmatrix} (1)(1)+(-2)(-3) & (1)(2)+(-2)(2) & (1)(3)+(-2)(-1) \\ (2)(1)+(3)(-3) & (2)(2)+(3)(2) & (2)(3)+(3)(-1) \end{bmatrix}$

$= \begin{bmatrix} 1+6 & 2-4 & 3+2 \\ 2-9 & 4+6 & 6-3 \end{bmatrix}$

$= \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$

Hence,

$$\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -3 & 2 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -2 & 5 \\ -7 & 10 & 3 \end{bmatrix}$$

(iii) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} (2)(1)+(3)(0)+(4)(3) & (2)(-3)+(3)(2)+(4)(0) & (2)(5)+(3)(4)+(4)(5) \\ (3)(1)+(4)(0)+(5)(3) & (3)(-3)+(4)(2)+(5)(0) & (3)(5)+(4)(4)+(5)(5) \\ (4)(1)+(5)(0)+(6)(3) & (4)(-3)+(5)(2)+(6)(0) & (4)(5)+(5)(4)+(6)(5) \end{bmatrix}$

$= \begin{bmatrix} 2+0+12 & -6+6+0 & 10+12+20 \\ 3+0+15 & -9+8+0 & 15+16+25 \\ 4+0+18 & -12+10+0 & 20+20+30 \end{bmatrix}$

$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

Hence,

$$\begin{bmatrix} 2 & 3 & 4 \\ 2 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Q2

Show that $AB \neq BA$ in $A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

Solution

$$\text{Given, } A = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$$

$$AB = \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 10-3 & 5-4 \\ 12+21 & 6+28 \end{bmatrix}$$

$$AB = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 10+6 & -2+7 \\ 15+24 & -3+28 \end{bmatrix}$$

$$BA = \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Q3

Show that $AB \neq BA$ in $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Solution

$$\text{Given, } A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+0 & -2+1+0 & -3+0+0 \\ +0+01 & 0-1+1 & 0+0+0 \\ 2+0+4 & 4+3+4 & 6+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+0+6 & 1-2+9 & 0+2+12 \\ 0+0+0 & 0-1+0 & 0+1+0 \\ -1+0+0 & 1-1+0 & 0+1+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad \text{---(ii)}$$

From (i) and (ii), $AB \neq BC$

Q4

Show that $AB \neq BA$ in $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$$BA = \begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+3+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 1+0+0 & 0+0+0 \\ 0+1+0 & 4+0+0 & 0+0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 4 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 0 & 10 & 1 \\ 1 & 0 & 0 \\ 0 & 5 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 0 \\ 1 & 1 & 0 \\ 4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+0+0 & 3+0+0 & 0+0+0 \\ 0+5+4 & 0+5+1 & 0+0+0 \end{bmatrix} \quad \text{----}(ii)$$

$$BA = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 0 \\ 9 & 6 & 0 \end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii), we get

$$AB \neq BA$$

Q5

Compute the products AB and BA whichever exists in $A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Solution

$A = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Since order of A is 2×2 and order of B is 2×3,
So AB is possible but BA is not possible order of AB is 2×3.

$$AB = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(1)+(-2)(2) & (1)(2)+(-2)(3) & (1)(3)+(-2)(1) \\ (2)(1)+(3)(2) & (2)(2)+(3)(3) & (2)(3)+(3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 1-4 & 2-6 & 3-2 \\ 2+6 & 4+9 & 6+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

BA does not exits

Q6

Compute the products AB and BA whichever exists in $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Solution

Here, $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$

Order of $A = 3 \times 2$ and order of $B = 2 \times 3$ So,

AB and BA Both exits and order of $AB = 3 \times 3$ and order of $BA = 2 \times 2$

$$AB = \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (3)(4)+(2)(0) & (3)(5)+(2)(1) & (3)(6)+(2)(2) \\ (-1)(4)+(0)(0) & (-1)(5)+(0)(0) & (-1)(6)+(0)(2) \\ (-1)(4)+(1)(0) & (-1)(5)+(1)(1) & (-1)(6)+(1)(2) \end{bmatrix}$$

$$= \begin{bmatrix} 12+0 & 15+2 & 18+4 \\ -4+0 & -5+0 & -6+0 \\ -4+0 & -5+0 & -6+2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 5 & 6 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ -1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (4)(3)+(5)(-1)+(6)(-1) & (4)(2)+(5)(0)+(6)(1) \\ (0)(3)+(1)(-1)+(2)(-1) & (0)(2)+(1)(0)+(2)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 12-5-6 & 8+0+6 \\ 0-1-2 & 0+0+2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Hence,

$$AB = \begin{bmatrix} 12 & 17 & 22 \\ -4 & -5 & -6 \\ -4 & -4 & -4 \end{bmatrix}, BA = \begin{bmatrix} 1 & 14 \\ -3 & 2 \end{bmatrix}$$

Q7

Compute the products AB and BA whichever exists in $A = \begin{bmatrix} 1 & -1 & 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 2 \end{bmatrix}$

Solution

Here,

$$A=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}, B=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$

Order of $A = 1\times 4$ and order of $B=4\times 1$ So,
AB and BA both exist and order of $AB = 1\times 1$ and order of $BA=4\times 4$, So

$$AB=\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}\begin{bmatrix}0\\1\\3\\2\end{bmatrix}$$
$$=\left[(1)(0)+(-1)(1)+(2)(3)+(3)(2)\right]$$
$$=\left[0-1+6+6\right]$$
$$AB=[11]$$
$$BA=\begin{bmatrix}0\\1\\3\\2\end{bmatrix}\begin{bmatrix}1 & -1 & 2 & 3\end{bmatrix}$$

$$BA=\begin{bmatrix}(0)(1) & (0)(-1) & (0)(2) & (0)(3)\\(1)(1) & (1)(-1) & (1)(2) & (1)(3)\\(3)(1) & (3)(-1) & (3)(2) & (3)(3)\\(2)(1) & (2)(-1) & (3)(2) & (2)(3)\end{bmatrix}$$
$$BA=\begin{bmatrix}0 & 0 & 0 & 0\\1 & -1 & 2 & 3\\3 & -3 & 6 & 9\\2 & -2 & 4 & 6\end{bmatrix}$$

Hence,

$$AB=[11]$$
$$BA=\begin{bmatrix}0 & 0 & 0 & 0\\1 & -1 & 2 & 3\\3 & -3 & 6 & 9\\2 & -2 & 4 & 6\end{bmatrix}$$

Q8

Compute the products AB and BA whichever exists in $[a,b]\begin{bmatrix}c\\d\end{bmatrix}+[a,b,c,d]\begin{bmatrix}a\\b\\c\\d\end{bmatrix}$

Solution

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix}+\begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$=\left[ac+bd\right]+\left[a^2+b^2+c^2+d^2\right]$$

$$=\left[ac+bd=a^2+b^2+c^2+d^2\right]$$

Hence,

$$\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} c \\ d \end{bmatrix}+\begin{bmatrix} a & b & c & d \end{bmatrix}\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$$=\left[ac+bd+a^2+b^2+c^2+d^2\right]$$

Q9

Show that $AB \neq BA$ in $A=\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$

Solution

$$A=\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}, B=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 2 & -4 \end{bmatrix}$$

$$AB=\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}$$

$$=\begin{bmatrix} -2-3+6 & 3+6-9 & -1-3+4 \\ -4+1+6 & 6-2-9 & -2+1+4 \\ -6+0+6 & 9+0-9 & -3+0+4 \end{bmatrix}$$

$$AB=\begin{bmatrix} 1 & 0 & 0 \\ 3 & -5 & 3 \\ 0 & 0 & 1 \end{bmatrix} \quad ---(i)$$

$$BA=\begin{bmatrix} -2 & 3 & -1 \\ -1 & 2 & -1 \\ -6 & 9 & -4 \end{bmatrix}\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & -1 \\ 3 & 0 & -1 \end{bmatrix}$$

$$=\begin{bmatrix} -2+6-3 & -6-3+0 & 2-3+1 \\ -1+4-3 & -3-2+0 & 1-2+1 \\ -6+18-12 & -18-9+0 & 6-9+4 \end{bmatrix}$$

$$BA=\begin{bmatrix} 1 & -9 & 0 \\ 0 & -5 & 0 \\ 0 & -27 & 1 \end{bmatrix} \quad ---(ii)$$

From equation (i) and (ii),

$AB \neq BA$

Q10

Show that $AB \neq BA$ in $A=\begin{bmatrix} 10 & -4 & -1 \\ -11 & 5 & 0 \\ 9 & -5 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & 2 \\ 1 & 3 & 2 \end{bmatrix}$

Solution

$$A=\begin{bmatrix}10 & -4 & -1\\ -11 & 5 & 0\\ 9 & -5 & 1\end{bmatrix}, B=\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\\ 1 & 3 & 2\end{bmatrix}$$

$$AB=\begin{bmatrix}10 & -4 & -1\\ -11 & 5 & 0\\ 9 & -5 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\\ 1 & 3 & 2\end{bmatrix}$$

$$=\begin{bmatrix}10-12-1 & 20-16-3 & 10-8-2\\ -11+15+0 & -22+20+0 & -11+10+0\\ 9-15+1 & 18-20+3 & 9-10+2\end{bmatrix}$$

$$AB=\begin{bmatrix}-3 & 1 & 0\\ 4 & -2 & -1\\ -5 & 1 & 1\end{bmatrix} \quad ---(i)$$

$$BA=\begin{bmatrix}1 & 2 & 1\\ 3 & 4 & 2\\ 1 & 3 & 2\end{bmatrix}\begin{bmatrix}10 & -4 & -1\\ -11 & 5 & 0\\ 9 & -5 & 1\end{bmatrix}$$

$$=\begin{bmatrix}10-22+9 & -4+10-5 & -9+0+1\\ 30-44+10 & -12+20-10 & -3+0+2\\ 10-33+18 & -4+15-10 & -1+0+2\end{bmatrix}$$

$$BA=\begin{bmatrix}-3 & 1 & 0\\ 4 & -2 & -1\\ -5 & 1 & 1\end{bmatrix} \quad ---(ii)$$

From equation (i) and (ii)

$$AB \neq BA$$

Q11

Evaluate $\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$

Solution

$$\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$$

$$=\left(\begin{bmatrix}1+3 & 3-2\\ -1-1 & -4+1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}$$

$$=\begin{bmatrix}4+2 & 12+4 & 20+6\\ -2-6 & -6-12 & -10-18\end{bmatrix}$$

$$=\begin{bmatrix}6 & 16 & 26\\ -8 & -18 & -28\end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix}1 & 3\\ -1 & -4\end{bmatrix}+\begin{bmatrix}3 & -2\\ -1 & 1\end{bmatrix}\right)\begin{bmatrix}1 & 3 & 5\\ 2 & 4 & 6\end{bmatrix}=\begin{bmatrix}6 & 16 & 26\\ -8 & -18 & -28\end{bmatrix}$$

Q12

Evaluate $\begin{bmatrix}1 & 2 & 31\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 2 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}$

Solution

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 2 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}$$

$$=\begin{bmatrix}1+4+0 & 0+0+3 & 2+0+6\end{bmatrix}\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}$$

$$=\begin{bmatrix}5 & 3 & 10\end{bmatrix}\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}$$

$$=\begin{bmatrix}10+12+60\end{bmatrix}$$

$$=\begin{bmatrix}82\end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & 2 & 3\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 2 & 0 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2\\ 4\\ 6\end{bmatrix}=\begin{bmatrix}82\end{bmatrix}$$

Q13

Evaluate $\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\ 2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\ 1 & 0 & 2\end{bmatrix}\right)$

Solution

$$\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\ 2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\ 1 & 0 & 2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 3\end{bmatrix}\left(\begin{bmatrix}1-0 & 0-1 & 2-2\\ 2-1 & 0-0 & 1-2\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 3\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 1 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1-1 & -1+0 & 0+1\\ 0+2 & 0+0 & 0-2\\ 2+3 & -2+0 & 0-3\end{bmatrix}$$

$$=\begin{bmatrix}0 & -1 & 1\\ 2 & 0 & -2\\ 5 & -2 & -3\end{bmatrix}$$

Hence,

$$\begin{bmatrix}1 & -1\\ 0 & 2\\ 2 & 3\end{bmatrix}\left(\begin{bmatrix}1 & 0 & 2\\ 2 & 0 & 1\end{bmatrix}-\begin{bmatrix}0 & 1 & 2\\ 1 & 0 & 2\end{bmatrix}\right)=\begin{bmatrix}0 & -1 & 1\\ 2 & 0 & -2\\ 5 & -2 & -3\end{bmatrix}$$

Q14

If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$,
then show that $A^2 = B^2 = C^2 = I_2$.

Solution

Given, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, C \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = I_2$ ---(i)

$B^2 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B^2 = I_2$ ---(ii)

$C^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$

$C^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C^2 = I_2$ ---(iii)

Hence,

From equation (i), (ii) and (iii),

$A^2 = B^2 = C^2 = I_2$

Q15

If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, find $3A^2 - 2B + I$

Solution

Given, $A=\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}$

$3A^2-2B+I$

$=3\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}\begin{bmatrix}2 & -1\\ 3 & 2\end{bmatrix}-2\begin{bmatrix}0 & 4\\ -1 & 7\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=3\begin{bmatrix}4-3 & -2-2\\ 6+6 & -3+4\end{bmatrix}-\begin{bmatrix}0 & 8\\ -2 & 14\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=3\begin{bmatrix}1 & -4\\ 12 & 1\end{bmatrix}-\begin{bmatrix}0 & 8\\ -2 & 14\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}3 & -12\\ 36 & 3\end{bmatrix}-\begin{bmatrix}0 & 8\\ -2 & 14\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$=\begin{bmatrix}3-0+1 & -12+8+0\\ 36+2+0 & 3-14+1\end{bmatrix}$

$=\begin{bmatrix}4 & -20\\ 38 & -10\end{bmatrix}$

Hence,

$3A^2-2B+I=\begin{bmatrix}4 & -20\\ 38 & -10\end{bmatrix}$

Q16

If $A=\begin{bmatrix}4 & 2\\ -1 & 1\end{bmatrix}$, prove that $(A-2I)(A-3I)=O$

Solution

Given, $A=\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}$

$(A-2I)(A-3I)$

$=\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right)$

$=\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}\right)\left(\begin{bmatrix}4 & 2\\ -1 & -1\end{bmatrix}-\begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix}\right)$

$=\left(\begin{bmatrix}4-2 & 2-0\\ -1-0 & 1-2\end{bmatrix}\right)\left(\begin{bmatrix}4-3 & 2-0\\ -1-0 & 1-3\end{bmatrix}\right)$

$=\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}$

$=\begin{bmatrix}2-2 & 4-4\\ -1+1 & -2+2\end{bmatrix}$

$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

$=0$

Hence,

$(A-2I)(A-3I)=0$

Q17

If $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$, show that $A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $A^3=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$

Solution

Given, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

$$A^2=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}1+0 & 1+1\\ 0+0 & 0+1\end{bmatrix}$$
$$A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$
$$A^3=A^2\cdot A$$
$$=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}1+0 & 1+2\\ 0+0 & 0+1\end{bmatrix}$$
$$A^3=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$$

Hence,

$$A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}, A^3=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix}$$

Q18

If $A=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}$, show that $A^2=O$

Solution

Given, $A=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}$

$$A^2=\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}\begin{bmatrix}ab & b^2\\ -a^2 & -ab\end{bmatrix}$$
$$=\begin{bmatrix}a^2b^2-a^2b^2 & ab^3-ab^3\\ -a^3b+a^3b & -a^2b^2+a^2b^2\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$=0$$

Hence,

$$A^2=0$$

Q19

If $A=\begin{bmatrix}\cos 2\theta & \sin 2\theta\\ -\sin 2\theta & \cos 2\theta\end{bmatrix}$, find A^2

Solution

Given, $A = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$

$$A^2 = A.A$$

$$= \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix} \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2 2\theta - \sin^2 2\theta & \cos 2\theta \sin^2 + \cos 2\theta \sin^2 \theta \\ -\cos 2\theta \sin^2 \theta - \sin^2 \theta \cos^2 \theta & -\sin^2 2\theta + \cos^2 2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 4\theta & 2 \sin^2 \theta \cos^2 \theta \\ -2 \sin^2 \cos 2\theta & \cos 4\theta \end{bmatrix}$$

$\{\text{since } \cos^2 \theta - \sin^2 \theta = \cos 2\theta\}$

$$= \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

$\{\text{since } \sin^2 \theta = 2 \sin\theta \cos\theta\}$

Hence,

$$A^2 = \begin{bmatrix} \cos 4\theta & \sin 4\theta \\ -\sin 4\theta & \cos 4\theta \end{bmatrix}$$

Q20

If $A = \begin{bmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ show that $AB = BA = O_{3\times 3}$

Solution

Given, $A=\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}, B=\begin{bmatrix}-1&3&5\\1&-3&-5\\-1&3&5\end{bmatrix}$

$$AB=\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}\begin{bmatrix}-1&3&5\\1&-3&-5\\-1&3&5\end{bmatrix}$$

$$=\begin{bmatrix}-2-3+5&6+9-15&10+15+25\\1+4-5&-3-12+15&-5-20+25\\-1-3+4&3+9-12&5+15-20\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$AB=O_{3\times3} \quad \text{---(i)}$$

$$BA=\begin{bmatrix}-1&3&5\\1&-3&-5\\-1&3&5\end{bmatrix}\begin{bmatrix}2&-3&-5\\-1&4&5\\1&-3&-4\end{bmatrix}$$

$$=\begin{bmatrix}-2-3+5&3+12-15&5+15-20\\2+3-5&-3-12+15&-5-15+20\\-2-3+5&3+12-15&5+15-20\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$BA=O_{3\times3} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$AB=BA=O_{3\times3}$$

Q21

If $A=\begin{bmatrix}0&c&-b\\-c&0&a\\b&-a&0\end{bmatrix}$ and $B=\begin{bmatrix}a^2&ab&ac\\ab&b^2&bc\\ac&bc&c^2\end{bmatrix}$, show that $AB=BA=O_{3\times3}$.

Solution

Given, $A=\begin{bmatrix}0&c&-b\\-c&o&a\\b&-a&0\end{bmatrix}, B=\begin{bmatrix}a^2&ab&ac\\ab&b^2&bc\\ac&bc&c^2\end{bmatrix}$

$$AB=\begin{bmatrix}0&c&-b\\-c&0&a\\b&-a&0\end{bmatrix}\begin{bmatrix}a^2&ab&ac\\ab&b^2&bc\\ac&bc&c^2\end{bmatrix}$$

$$=\begin{bmatrix}0+abc-abc&0+b^2c-b^2c&0+bc^2-bc^2\\-a^2c+0+a^2c&-abc+0+abc&-ac^2+0+ac^2\\a^2b-a^2b+0&ab^2-ab^2+0&abc-abc+0\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$AB=O_{3\times3} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$AB=BA=O_{3\times3}$$

Q22

If $A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$ and $B=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$, show that $AB=A$ and $BA=B$.

Solution

$$\text{Given, } A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}, B=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$$

$$AB=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$$

$$=\begin{bmatrix}4+3-5 & -4-9+10 & -8-12+15\\ -2-4+5 & 2+12-10 & 4+16-15\\ 2+3-4 & -2-9+18 & -4-12+12\end{bmatrix}$$

$$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$$

$$AB=A$$

$$BA=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$$

$$=\begin{bmatrix}4+2-4 & -6-8+12 & -10-10+16\\ -2-3+4 & 3+12-12 & 5+15-16\\ 2+2-3 & -3-8+9 & -5-10+12\end{bmatrix}$$

$$=\begin{bmatrix}2 & -2 & -4\\ -1 & 3 & 4\\ 1 & -2 & -3\end{bmatrix}$$

$$BA=B$$

Q23

Let $A=\begin{bmatrix}-1 & 1 & -1\\ 3 & -3 & 3\\ 5 & 5 & 5\end{bmatrix}$ and $B=\begin{bmatrix}0 & 4 & 3\\ 1 & -3 & -3\\ -1 & 4 & 4\end{bmatrix}$, compute A^2-B^2.

Solution

Given, $A = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$

$$A^2 = \begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}\begin{bmatrix} -1 & 1 & -1 \\ 3 & -3 & 3 \\ 5 & 5 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -1-3-5 & 1+3-5 \\ -3-9+15 & 3+9+15 & -3-9+15 \\ -5+15+25 & 5-15+25 & -5+15+25 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} \quad ---(i)$$

$$B^2 = \begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}\begin{bmatrix} 0 & 4 & 3 \\ 1 & -3 & -3 \\ -1 & 4 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 0+4-3 & 0-12+12 & 0-12+12 \\ 0-3+3 & 4+9-12 & 3+9-12 \\ 0+4-4 & -4-12+16 & -3-12+16 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ---(ii)$$

Subtracting equation (ii) from equation (i),

$$A^2 - B^2 = \begin{bmatrix} -1 & -9 & -1 \\ 3 & 27 & 3 \\ 35 & 15 & 35 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1-1 & -9-0 & -1-0 \\ 3-0 & 27-1 & 3-0 \\ 35-0 & 15-0 & 35-1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Hence,

$$A^2 - B^2 = \begin{bmatrix} -2 & -9 & -1 \\ 3 & 26 & 3 \\ 35 & 15 & 34 \end{bmatrix}$$

Q24

For the following matrices verify the associativity of matrix multiplication i.e.

$(AB)C = A(BC)$: $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Solution

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix}$ and

$$C = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$(AB)C = \left(\begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \right) \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+0 & 0+4+0 \\ -1+0+0 & +0+0+3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -4 \\ -1 & -3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \qquad ---(i)$$

$$A(BC) = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 \\ -1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1+0 \\ -1-2 \\ 0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1-6+0 \\ -1+0-3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -5 \\ -4 \end{bmatrix} \qquad ---(ii)$$

From equation (i) and (ii) we get,

$$(AB)C = A(BC)$$

Q25

For the following matrices verify the associativity of matrix multiplication i.e.

$(AB)C = A(BC)$: $A = \begin{bmatrix} 4 & 2 & 3 \\ 1 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

Solution

(ii) Given,

$$A=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}, B=\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}, C=\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$(AB)C=\left(\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}\right)\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}4+0+6&-4+2-3&4+4+3\\1+0+4&-1+1-2&1+2+2\\3+0+2&-3+0-1&3+0+1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}10&-5&11\\5&-2&5\\5&-4&4\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\8&0&1\end{bmatrix}$$

$$=\begin{bmatrix}10-15+0&20+0+0&-10+5+11\\5-6+0&10+0+0&-5-2+5\\5-12+0&10+0+0&-5-4+4\end{bmatrix}$$

$$(AB)C=\begin{bmatrix}-5&20&-4\\-1&10&-2\\-7&10&-5\end{bmatrix} \quad \text{---(i)}$$

$$A(BC)=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\left(\begin{bmatrix}1&-1&1\\0&1&2\\2&-1&1\end{bmatrix}\begin{bmatrix}1&2&-1\\3&0&1\\0&0&1\end{bmatrix}\right)$$

$$=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}1-3+0&2+0+0&-1-1+1\\0+3+0&0+0+0&0+1+2\\2-3+0&4+0+0&-2-1+1\end{bmatrix}$$

$$=\begin{bmatrix}4&2&3\\1&1&2\\3&0&1\end{bmatrix}\begin{bmatrix}-2&2&-1\\3&0&3\\-1&4&-2\end{bmatrix}$$

$$=\begin{bmatrix}-8+6-3&8+0+12&-4+6-6\\-2+3-2&2+0+8&-1+3-4\\-6+0-1&6+0+4&-3+0-2\end{bmatrix}$$

$$A(BC)=\begin{bmatrix}-5&20&-4\\-1&10&-2\\-7&10&-5\end{bmatrix} \quad \text{--(ii)}$$

From equation (i) and (ii),

$$(AB)C=A(BC)$$

Q26

For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. $A(B+C)=AB+AC$:

$$A=\begin{bmatrix}1&-1\\0&2\end{bmatrix}, B=\begin{bmatrix}-1&0\\2&1\end{bmatrix} \text{and } C=\begin{bmatrix}0&1\\1&-1\end{bmatrix}$$

Solution

Given, $A=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}, B=\begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix}, C=\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$

$$A(B+C)=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\left(\begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix}+\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}\right)$$

$$=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}-1+0 & 0+1\\ 2+1 & 1-1\end{bmatrix}$$

$$=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}-1 & 1\\ 3 & 0\end{bmatrix}$$

$$=\begin{bmatrix}-1-3 & 1+0\\ 0+6 & 0+0\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}-4 & 1\\ 6 & 0\end{bmatrix} \qquad ---(i)$$

$$AB=AC=\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}-1 & 0\\ 2 & 1\end{bmatrix}+\begin{bmatrix}1 & -1\\ 0 & 2\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & -1\end{bmatrix}$$

$$=\begin{bmatrix}-1-2 & 0-1\\ 0+4 & 0+2\end{bmatrix}+\begin{bmatrix}0+-1 & 1+1\\ 0+2 & 0-2\end{bmatrix}$$

$$=\begin{bmatrix}-3 & -1\\ 4 & 2\end{bmatrix}+\begin{bmatrix}-1 & 2\\ 2 & -2\end{bmatrix}$$

$$=\begin{bmatrix}-3-1 & -1+2\\ 4+2 & 2-2\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}-4 & 1\\ 6 & 0\end{bmatrix} \qquad ---(ii)$$

Using equation (i) and (ii),

$$A(B+C)=AB+AC$$

Q27

For the following matrices verify the distributivity of matrix multiplication over matrix addition i.e. $A(B+C)=AB+AC$:

$$A=\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}, B=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix} \text{ and } C=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

Solution

Given, $A=\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}, B=\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}, C=\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$

$$A(B+C)=\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}\left(\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}+\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}\right)$$

$$=\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}0+1 & 1-1\\ 1+0 & 1+1\end{bmatrix}$$

$$=\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}2-1 & 0+2\\ 1+1 & 0+2\\ -1+2 & 0+4\end{bmatrix}$$

$$A(B+C)=\begin{bmatrix}1 & -2\\ 2 & 2\\ 1 & 4\end{bmatrix} \quad \text{---(i)}$$

$$AB+AC=\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix}+\begin{bmatrix}2 & -1\\ 1 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}0+1 & 2-1\\ 0+1 & 1+1\\ 0+2 & -1+2\end{bmatrix}+\begin{bmatrix}2+0 & -2-1\\ 1+0 & -1+1\\ -1+0 & 1+2\end{bmatrix}$$

$$=\begin{bmatrix}-1 & 1\\ 1 & 2\\ 2 & 1\end{bmatrix}+\begin{bmatrix}2 & -3\\ 1 & 0\\ -1 & 3\end{bmatrix}$$

$$=\begin{bmatrix}-1+2 & 1-3\\ 1+1 & 2+0\\ 2-1 & 1+3\end{bmatrix}$$

$$AB+AC=\begin{bmatrix}1 & -2\\ 2 & 2\\ 1 & 4\end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$A(B+C)=AB+AC$$

Q28

If $A=\begin{bmatrix}1 & 0 & -2\\ 3 & -1 & 0\\ -2 & 1 & 1\end{bmatrix}, B=\begin{bmatrix}0 & 5 & -4\\ -2 & 1 & 3\\ -1 & 0 & 2\end{bmatrix}$ and $C=\begin{bmatrix}1 & 5 & 2\\ -1 & 1 & 0\\ 0 & -1 & 1\end{bmatrix}$, verify that $A(B-C)=AB-AC$.

Solution

Given,

$$A=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}, B=\begin{bmatrix}0&5&-4\\-2&1&3\\-1&0&2\end{bmatrix}$$

$$C=\begin{bmatrix}1&5&2\\-1&1&0\\0&-1&1\end{bmatrix}$$

$$A(B-C)=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\left[\begin{bmatrix}0&5&-4\\-2&1&3\\-1&0&2\end{bmatrix}-\begin{bmatrix}1&5&2\\-1&1&0\\0&-1&1\end{bmatrix}\right]$$

$$=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}0-1&5-5&-4-2\\-2+1&1-1&3-0\\-1-0&0+1&2-1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}-1&0&-6\\-1&0&3\\-1&1&1\end{bmatrix}$$

$$=\begin{bmatrix}-1+0+2&0+0-2&-6+0-2\\-3+1+0&0+0+0&-18-3+0\\2-1-1&0+0+1&12+3+1\end{bmatrix}$$

$$A(B-C)=\begin{bmatrix}1&-2&-8\\-2&0&-21\\0&1&16\end{bmatrix} \quad \text{---(i)}$$

$$AB-AC=\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}0&5&-4\\-2&1&3\\-1&0&2\end{bmatrix}-\begin{bmatrix}1&0&-2\\3&-1&0\\-2&1&1\end{bmatrix}\begin{bmatrix}1&5&2\\-1&1&0\\0&-1&1\end{bmatrix}$$

$$=\begin{bmatrix}0+0+2&5+0+0&-4+0-4\\0+2+0&15-1+0&-12-3+0\\0-2-1&-10+1+0&8+3+2\end{bmatrix}-\begin{bmatrix}1+0+0&5+0+2&2+0-2\\3+1+0&15-1+0&6+0+0\\0-2-1&-10+1+1&-4+0+1\end{bmatrix}$$

$$=\begin{bmatrix}2&5&-8\\2&14&-15\\-3&-9&13\end{bmatrix}-\begin{bmatrix}1&7&0\\4&14&6\\-3&-10&-3\end{bmatrix}$$

$$=\begin{bmatrix}2-1&5-7&-8-0\\2-4&14-14&-14-6\\-3+3&-9+10&13+3\end{bmatrix}$$

$$AB-AC=\begin{bmatrix}1&-2&-8\\-2&0&-21\\0&1&16\end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii),

$$A(B-C)=AB-AC$$

Q29

Compute the elements a_{43} and a_{22} of the matrix:

$$A=\begin{bmatrix}0&1&0\\2&0&2\\0&3&2\\4&0&4\end{bmatrix}\begin{bmatrix}2&-1\\-3&2\\4&3\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

Solution

Given,

$$A=\begin{bmatrix}0&1&0\\2&0&2\\0&3&2\\4&0&4\end{bmatrix}\begin{bmatrix}2&-1\\-3&2\\4&3\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

$$A=\begin{bmatrix}0-3+0&0+2+0\\4+0+8&-2+0+6\\0-9+8&0+6+6\\8+0+16&-4+0+12\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

$$A=\begin{bmatrix}-3&2\\12&4\\-1&12\\24&8\end{bmatrix}\begin{bmatrix}0&1&-1&2&-2\\3&-3&4&-4&0\end{bmatrix}$$

$$A=\begin{bmatrix}0+6&-3-6&3+8&-6-8&6+0\\0+12&12-12&-12+16&24-16&-24+0\\0+36&-1-36&1+48&-2-48&2+0\\0+24&24-24&-24+34&48-32&-48+0\end{bmatrix}$$

$$A=\begin{bmatrix}6&-9&11&-14&6\\12&0&4&8&-24\\36&-37&49&-50&2\\24&0&8&16&-48\end{bmatrix}$$

Here, $a_{43}=8, a_{22}=0$

Q30

If $A=\begin{bmatrix}0&1&0\\0&0&1\\p&q&r\end{bmatrix}$, and I is the identity matrix of order 3, show that $A^3=pI+qA+rA^2$.

Solution

Given,

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+0 & 0+0+0 & 0+1+0 \\ 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}$$

$$= \begin{bmatrix} 0+0+p & 0+0+q & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+0+pq+pr^2 & pr+0+q^2+qr^2 & 0+p+qr+qr+r^2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix} \quad \text{---(i)}$$

$$pI + qA + rA^2$$

$$= p\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + q\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix} + r\begin{bmatrix} 0 & 0 & 1 \\ p & q & r \\ pr & p+qr & q+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} p+0+0 & 0+q+0 & 0+0+r \\ 0+0+pr & p+0+qr & 0+q+r^2 \\ 0+pq+pr^2 & 0+q^2+pr+qr^2 & p+qr+qr+r^2 \end{bmatrix}$$

$$pI + qA + rA^2$$

$$= \begin{bmatrix} p & q & r \\ pr & p+qr & q+r^2 \\ pq+pr^2 & pr+q^2+qr^2 & p+2qr+r^2 \end{bmatrix}$$

From equation (i) and (ii)

$$A^3 = pI + qA + rA^2$$

Q31

If w is a complex cube root of unity, show that

$$\left(\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix} + \begin{bmatrix} w & w^2 & 1 \\ w^2 & 1 & w \\ w & w^2 & 1 \end{bmatrix}\right)\begin{bmatrix} 1 \\ w \\ w^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Solution

Given, W is a complex cube root of unity,

$$\left(\begin{bmatrix}1 & w & w^2\\ w & w^2 & 1\\ w^2 & 1 & w\end{bmatrix}+\begin{bmatrix}w & w^2 & 1\\ w^2 & 1 & w\\ w & w^2 & 1\end{bmatrix}\right)\begin{bmatrix}1\\ w\\ w^2\end{bmatrix}$$

$$=\begin{bmatrix}1+w & w+w^2 & w^2+1\\ w+w^2 & w^2+1 & 1+w\\ w^2+w & 1+w^2 & w+1\end{bmatrix}\begin{bmatrix}1\\ w\\ w^2\end{bmatrix}$$

$$=\begin{bmatrix}-w^2 & -1 & -w\\ -1 & -w & -w^2\\ -1 & -w & -w^2\end{bmatrix}\begin{bmatrix}1\\ w\\ w^2\end{bmatrix} \qquad \begin{Bmatrix}Since\ 1+w+w^2=0\\ and\ w^3=1\end{Bmatrix} \qquad ---(i)$$

$$=\begin{bmatrix}-w^2-w-w^3\\ -1-w^2-w^4\\ -1-w^2-w^2\end{bmatrix}$$

$$=\begin{bmatrix}-w\left(1+w+w^2\right)\\ -1-w^2-w^3w\\ -1-w^2-w^3w\end{bmatrix}$$

$$=\begin{bmatrix}-w.0\\ -1-w^2-w\\ -1-w^2-w\end{bmatrix} \qquad \{using\ reason\ (i)\}$$

$$=\begin{bmatrix}0\\ -\left(1+w+w^2\right)\\ -\left(1+w+w^2\right)\end{bmatrix}$$

$$=\begin{bmatrix}0\\ -(0)\\ -(0)\end{bmatrix}$$

$$=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

Hence,

$$\left(\begin{bmatrix}1 & w & w^2\\ w & w^2 & 1\\ w^2 & 1 & w\end{bmatrix}+\begin{bmatrix}w & w^2 & 1\\ w^2 & 1 & w\\ w & w^2 & 1\end{bmatrix}\right)\begin{bmatrix}1\\ w\\ w^2\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

Q32

If $A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$, show that $A^2=A$

Solution

Given, $A=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$

$A^2 = A.A$

$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$

$=\begin{bmatrix}4+3-5 & -6-12+15 & -10-15+20\\ -2-4+5 & 3+16-15 & 5+20-20\\ 2+3-4 & -3-12+12 & -5-15+16\end{bmatrix}$

$=\begin{bmatrix}2 & -3 & -5\\ -1 & 4 & 5\\ 1 & -3 & -4\end{bmatrix}$

$= A$

Hence,

$A^2 = A$

Q33

If A=$\begin{bmatrix}4 & -1 & -4\\ 3 & 0 & -4\\ 3 & -1 & -3\end{bmatrix}$, show that $A^2 = I_3$

Solution

Given, $A=\begin{bmatrix}4 & -1 & -4\\ 3 & 0 & -4\\ 3 & -1 & -3\end{bmatrix}$

$A^2 = A.A$

$=\begin{bmatrix}4 & -1 & -4\\ 3 & 0 & -4\\ 3 & -1 & -3\end{bmatrix}\begin{bmatrix}4 & -1 & -4\\ 3 & 0 & -4\\ 3 & -1 & -3\end{bmatrix}$

$=\begin{bmatrix}16-3-12 & -4+0+4 & -16+4+12\\ 12+0-12 & -3+0+4 & -12+0+12\\ 12-3-9 & -3+0+3 & -12+4+9\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$

$= I_3$

Hence,

$A^2 = I_3$

Q34

If $\begin{bmatrix}1 & 1 & x\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}=0$, find x.

Solution

Given,

$$\begin{bmatrix}1 & 1 & x\end{bmatrix}\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}1+0+2x & 0+2+x & 2+1+0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}2x+1 & 2+x & 3\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix}2x+1+2+x+3\end{bmatrix} = 0$$

$$\Rightarrow 3x+6 = 0$$

$$\Rightarrow x = -\frac{6}{3}$$

$$\Rightarrow x = -2$$

Q35

If $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$ find x

Solution

Given that $\begin{bmatrix}2 & 3\\5 & 7\end{bmatrix}\begin{bmatrix}1 & -3\\-2 & 4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$

By multiplication of matrices, we have,

$$\begin{bmatrix}2\times1+3\times(-2) & 2\times(-3)+3\times4\\5\times1+7\times(-2) & 5\times(-3)+7\times4\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-4 & 6\\-9 & 13\end{bmatrix} = \begin{bmatrix}-4 & 6\\-9 & x\end{bmatrix}$$

$$\Rightarrow x = 13$$

Q36

If $\begin{bmatrix}x & 4 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 2\\1 & 0 & 2\\0 & 2 & -4\end{bmatrix}\begin{bmatrix}x\\4\\-1\end{bmatrix} = 0$, find x.

Solution

Given,

$$\begin{bmatrix} x & 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4+0 & x+0+2 & 2x+8-4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 2x+4 & x+2 & 2x+4 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ -1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \left[(2x+4)x + 4(x+2) - 1(2x+4)\right] = 0$$

$$\Rightarrow \quad 2x^2 + 4x + 4x + 8 - 2x - 4 = 0$$

$$\Rightarrow \quad 2x + 6x + 4 = 0$$

$$\Rightarrow \quad 2x^2 + 2x + 4x + 4 = 0$$

$$\Rightarrow \quad 2x(x+1) + 4(x+1) = 0$$

$$\Rightarrow \quad (x+1)(2x+4) = 0$$

$$\Rightarrow \quad x+1 = 0 \text{ or } 2x+4 = 0$$

$$\Rightarrow \quad x = -1 \text{ or } x = -2$$

Hence, $x = -1$ or -2

Q37

If $\begin{bmatrix} 1 & -1 & x \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$, find x.

Solution

Given,

$$\begin{bmatrix} 1 & -1 & x \end{bmatrix}\begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} 0-2+x & 1-1+x & -1-3+x \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \begin{bmatrix} x-2 & x & x-4 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = 0$$

$$\Rightarrow \quad \left[0(x-2) + x.1 + 1.(x-4)\right] = 0$$

$$\Rightarrow \quad 0 + x + x - 4 = 0$$

$$\Rightarrow \quad 2x - 4 = 0$$

$$\Rightarrow \quad x = 2$$

Hence,

$x = 2$

Q38

If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then prove that $A^2-A+2I=O$.

Solution

Given, $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}, I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$A^2-A+2I$$
$$=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}-\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}+2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}9-8 & -6+4\\ 12-8 & -8+4\end{bmatrix}-\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}+\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$
$$=\begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}-\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}+\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$
$$=\begin{bmatrix}1-3+2 & -2+2+0\\ 4-4+0 & -4+2+2\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$=0$$

Hence,

$$A^2-A+2I=0$$

Q39

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then find λ so that $A^2=5A+\lambda I$.

Solution

Given, $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}, I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

And

$$A^2=5A+\lambda I$$
$$\Rightarrow \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}=5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+\lambda\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}9-1 & 3+2\\ -3+2 & -1+4\end{bmatrix}=\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}=\begin{bmatrix}15+\lambda & 5\\ -5 & 10+\lambda\end{bmatrix}$$

Since, Corresponding entries of equal matrices are equal, So

$$8=15+\lambda$$
$$\lambda=8-15$$
$$\lambda=-7$$

Q40

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I_2=O$

Solution

Given, $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$$A^2 - 5A + 7I_2$$
$$= \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$$
$$= \begin{bmatrix} 8-15+7 & 5-5+0 \\ -5+5+0 & 3-10+7 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= 0$$

Hence, $A^2 - 5A + 7I_2 = 0$

Q41

If $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$, show that $A^2 - 2A + 3I_2 = O$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}$

$$A^2 - 2A + 3I_2$$
$$= \begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 \\ -1 & 0 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4-3 & 6+0 \\ -2+0 & -3+0 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 6 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 6 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$
$$= \begin{bmatrix} 1-4+3 & 6-6+0 \\ -2+2+0 & -3+0+3 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= 0$$

Hence,

$$A^2 - 2A + 3I_2 = 0$$

Q42

Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^3 - 4A^2 + A = 0$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$A^2 = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix}$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}$

$A^3 = A^2.A$

$= \begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 14+12 & 21+24 \\ 8+7 & 12+14 \end{bmatrix}$

$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix}$

Hence, $A^3 - 4A^2 + A$

$= \begin{bmatrix} 26 & 45 \\ 15 & 26 \end{bmatrix} - 4\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} + \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 26-28+2 & 45-48+3 \\ 15-16+1 & 26-28+2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

So, $A^3 - 4A^2 + A = 0$

Q43

Show that the matrix $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$ is root of the equation $A^2 - 12A - I = O$

Solution

Given, $A = \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}$

$A^2 - 12A - I$

$= \begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - 12\begin{bmatrix} 5 & 3 \\ 12 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 25+36 & 15+21 \\ 60+84 & 36+49 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 61 & 36 \\ 144 & 85 \end{bmatrix} - \begin{bmatrix} 60 & 36 \\ 144 & 84 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 61-60-1 & 36-36-0 \\ 144-144-0 & 85-84-1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

Since $A^2 - 12A - I = 0$

So,

A is a root of the equation $A^2 - 12A - I = 0$

Q44

If $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$, find that $A^2-5A-14I$

Solution

Given, $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}$

$$A^2=5A-14I$$

$$=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}-5\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}-14\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}9+20 & -15-10\\ -12-8 & 20+4\end{bmatrix}-\begin{bmatrix}15 & -25\\ -20 & 10\end{bmatrix}-\begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix}$$

$$=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}-\begin{bmatrix}15 & -25\\ -20 & 10\end{bmatrix}-\begin{bmatrix}14 & 0\\ 0 & 14\end{bmatrix}$$

$$=\begin{bmatrix}29-15-14 & -25+25-0\\ -20+20-0 & 24-10-14\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$$=0$$

So,

$$A^2-5A-14I=0$$

Q45

If $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$, show that $A^2-5A+7I=0$

Use this to find A^4

Solution

It is given that $A=\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$

$$\therefore A^2 = A\cdot A = \begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}$$
$$=\begin{bmatrix}3(3)+1(-1) & 3(1)+1(2)\\ -1(3)+2(-1) & -1(1)+2(2)\end{bmatrix}$$
$$=\begin{bmatrix}9-1 & 3+2\\ -3-2 & -1+4\end{bmatrix}=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}$$

$\therefore$ L.H.S. $= A^2-5A+7I$

$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}+7\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\begin{bmatrix}8 & 5\\ -5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\ -5 & 10\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$
$$=\begin{bmatrix}-7 & 0\\ 0 & -7\end{bmatrix}+\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$$
$$=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

$= O =$ R.H.S.

$\therefore A^2-5A+7I=O$

Since $A^2-5A+7I=O$, we have

$A^2=5A-7I$

Therefore, $A^4=A^2\times A^2=(5A-7I)(5A-7I)$

$\Rightarrow A^4=25A^2-35AI-35IA+49I$

$\Rightarrow A^4=25A^2-70A+49I$

$\Rightarrow A^4=25(5A-7I)-70A+49I$

$\Rightarrow A^4=125A-175I-70A+49I$

$\Rightarrow A^4=55A-126I$

$$\Rightarrow A^4=55\begin{bmatrix}3 & 1\\ -1 & 2\end{bmatrix}-126\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow A^4=\begin{bmatrix}165 & 55\\ -55 & 110\end{bmatrix}-\begin{bmatrix}126 & 0\\ 0 & 126\end{bmatrix}$$
$$\Rightarrow A^4=\begin{bmatrix}165-126 & 55-0\\ -55-0 & 110-126\end{bmatrix}$$
$$\Rightarrow A^4=\begin{bmatrix}39 & 55\\ -55 & -16\end{bmatrix}$$

Q46

If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, find k so that $A^2 = kA - 2I$

Solution

$$A^2 = A\cdot A = \begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$$
$$= \begin{bmatrix}3(3)+(-2)(4) & 3(-2)+(-2)(-2)\\ 4(3)+(-2)(4) & 4(-2)+(-2)(-2)\end{bmatrix} = \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix}$$

Now $A^2 = kA - 2I$

$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix} = k\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix} - 2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix} = \begin{bmatrix}3k & -2k\\ 4k & -2k\end{bmatrix} - \begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}1 & -2\\ 4 & -4\end{bmatrix} = \begin{bmatrix}3k-2 & -2k\\ 4k & -2k-2\end{bmatrix}$$

Comparing the corresponding elements, we have:

$3k - 2 = 1$
$\Rightarrow 3k = 3$
$\Rightarrow k = 1$

Thus, the value of k is 1.

Q47

If $A=\begin{bmatrix}1 & 0\\ -1 & 7\end{bmatrix}$, find k such that $A^2 - 8A + kI = 0$.

Solution

Here,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

And

$$A^2 - 8A + kI = 0$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} - 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} - \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-8+k & 0+0+0 \\ -8+8+0 & 49-56+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -7+k & 0 \\ 0 & -7+k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since,

corresponding entries of equal matrices are equal, so

$-7 + k = 0$

$k = 7$

Q48

If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$, $f(x) = x^2 - 2x - 3$, show that $f(A) = 0$

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \text{ and } f(x) = x^2 - 2x - 3$$

$$f(A) = A^2 - 2A - 3I$$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4-0 \\ 4-4-0 & 5-2-3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

So,

$$f(A) = 0$$

Q49

If $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then find λ μ so that $A^2 = \lambda A + \mu I$

Solution

Given, $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Given,

$$A^2 = \lambda A + \mu I$$

$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} = \lambda \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} + \mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4+3 & 6+6 \\ 2+2 & 3+4 \end{bmatrix} = \begin{bmatrix} 2\lambda & 3\lambda \\ \lambda & 2\lambda \end{bmatrix} + \begin{bmatrix} \mu & 0 \\ 0 & \mu \end{bmatrix}$$

$$\begin{bmatrix} 7 & 12 \\ 4 & 7 \end{bmatrix} = \begin{bmatrix} 2\lambda + \mu & 3\lambda \\ \lambda & 2\lambda + \mu \end{bmatrix}$$

Since corresponding entries of equal matrices are equal, so

$2\lambda + \mu = 7$ ---(i)

$\lambda = 4$ ---(ii)

Put λ from equation (ii) in equation (i),

$2(4) + \mu = 7$

$\mu = 7 - 8$

$\mu = -1$

Hence. $\lambda = 4, \mu = -1$

Q50

Find the value of x for which the matrix product

$$\begin{bmatrix} 2 & 0 & 7 \\ 0 & 1 & 0 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -x & 14x & 7x \\ 0 & 1 & 0 \\ x & -4x & -2x \end{bmatrix}$$

equals an identity matrix.

Solution

Given,

$$\begin{bmatrix}2 & 0 & 7\\0 & 1 & 0\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}-x & 14x & 7x\\0 & 1 & 0\\x & -4x & -2x\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}-2x+0+7x & 28x+0-28x & 14x+0-14x\\0+0+0 & 0+1+0 & 0+0+0\\-x+0+x & 14x-2-4x & 7x+0-2x\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}5x & 0 & 0\\0 & 1 & 0\\0 & 10x-2 & 5x\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$5x = 1$ and $10x - 2 = 0$

$\Rightarrow x = \frac{1}{5}$ and $x = \frac{1}{5}$

Hence, $x = \frac{1}{5}$

Q51

Solve the matrix equation $\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-2 & -3\end{bmatrix}\begin{bmatrix}x\\5\end{bmatrix}=0$

Solution

Here,

$$\begin{bmatrix}x & 1\end{bmatrix}\begin{bmatrix}1 & 0\\-2 & -3\end{bmatrix}\begin{bmatrix}x\\5\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}x-2 & 0-3\end{bmatrix}\begin{bmatrix}x\\5\end{bmatrix}=0$$

$$\Rightarrow \begin{bmatrix}(x-2)x-15\end{bmatrix}=0$$

$$\Rightarrow x^2-2x-15=0$$

$$\Rightarrow x^2-5x+3x-15=0$$

$$\Rightarrow x(x-5)+3(x-5)=0$$

$$\Rightarrow (x-5)(x+3)=0$$

$$\Rightarrow x-5=0 \quad \text{or} \quad x+3=0$$

$$\Rightarrow x=5 \quad \text{or} \quad x=-3$$

So,

$x = 5$ or -3

Q52

Solve the matrix equation $\begin{bmatrix}1 & 2 & 1\end{bmatrix}\begin{bmatrix}1 & 2 & 0\\2 & 0 & 1\\1 & 0 & 2\end{bmatrix}\begin{bmatrix}0\\2\\x\end{bmatrix}=O$?

Solution

We have:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 1+4+1 & 2+0+0 & 0+2+2 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} 6 & 2 & 4 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = O$$

$\Rightarrow \left[6(0)+2(2)+4(x)\right] = O$

$\Rightarrow [4+4x] = [0]$

$\therefore 4 + 4x = 0$

$\Rightarrow x = -1$

Thus, the required value of x is -1.

Q53

Solve the matrix equation $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$

Solution

We have:

$$\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x+0-2 & 0-10+0 & 2x-5-3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$$

$\Rightarrow \left[x(x-2)-40+2x-8\right] = O$

$\Rightarrow \left[x^2-2x-40+2x-8\right] = [0]$

$\Rightarrow \left[x^2-48\right] = [0]$

$\therefore x^2 - 48 = 0$

$\Rightarrow x^2 = 48$

$\Rightarrow x = \pm 4\sqrt{3}$

Q54

If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$, Compute $A^2 - 4A + 3I_3$.

Solution

Given, $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}$

$$A^2 - 4A + 3I_3$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 0 \\ 3 & -4 & 5 \\ 0 & -1 & 3 \end{bmatrix} + 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+6+0 & 2-8+0 & 0+10+0 \\ 3-12+0 & 6+16-5 & 0-20+15 \\ 0-3+0 & 0+4-3 & 0-5+9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -6 & 10 \\ -9 & 17 & -5 \\ -3 & 1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 0 \\ 12 & -16 & 20 \\ 0 & -4 & 12 \end{bmatrix} + \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 7-4+3 & -6-8+0 & 10-0+0 \\ -9-12+0 & 17+16+3 & -5-20+0 \\ -3-0+0 & 1+4+0 & 4-12+3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Hence,

$$A^2 - 4A + 3I_3 = \begin{bmatrix} 6 & -14 & 10 \\ -21 & 36 & -25 \\ -3 & 5 & -5 \end{bmatrix}$$

Q55

If $f(x) = x^2 - 2x$, find $f(A)$ where $A = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 0 & 2 & 3 \end{bmatrix}$

Solution

Given, $A=\begin{bmatrix}0&1&2\\4&5&0\\0&2&3\end{bmatrix}$

And $f(x)=x^2-2x$

$\Rightarrow$ $f(A)=A^2-2A$

$$\Rightarrow f(A)=\begin{bmatrix}0&1&2\\4&5&0\\0&2&3\end{bmatrix}\begin{bmatrix}0&1&2\\4&5&0\\0&2&3\end{bmatrix}-2\begin{bmatrix}0&1&2\\4&5&0\\0&2&3\end{bmatrix}$$

$$\Rightarrow f(A)=\begin{bmatrix}0+4+0&0+5+4&0+0+6\\0+20+0&4+25+0&8+0+0\\0+8+0&0+10+6&0+0+9\end{bmatrix}-\begin{bmatrix}0&2&4\\8&10&0\\0&4&6\end{bmatrix}$$

$$\Rightarrow f(A)=\begin{bmatrix}4&9&6\\20&29&8\\8&16&9\end{bmatrix}-\begin{bmatrix}0&2&4\\8&10&0\\0&4&6\end{bmatrix}$$

$$\Rightarrow f(A)=\begin{bmatrix}4-0&9-2&6-4\\20-8&29-10&8-0\\8-0&16-4&9-6\end{bmatrix}$$

$$\Rightarrow f(A)=\begin{bmatrix}4&7&2\\12&19&8\\8&12&3\end{bmatrix}$$

Q56

If $f(x)=x^3+4x^2-x$, find $f(A)$, where $A=\begin{bmatrix}0&1&2\\2&-3&0\\1&-1&0\end{bmatrix}$

Solution

Given,

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

And $f(x) = x^3 + 4x^2 - x$

$\Rightarrow \quad f(x) = A^3 + 4A^2 - A \qquad$ ---(i)

$A^2 = A \times A$

$$= \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+2+2 & 0-3-2 & 0+0+0 \\ 0-6+0 & 2+9+0 & 4+0+0 \\ 0-2+0 & 1+3+0 & 0+0+0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}$$

$$A^3 = A^2 \times A$$

$$= \begin{bmatrix} 4 & -5 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0-10+0 & 4+15+0 & 8+0+0 \\ 0+22+4 & -6-33-4 & -12+0+0 \\ 0+8+2 & -2-12-2 & -4+0+0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix}$$

Put the value of A, A^2, A^3 in equation (i)

$f(A) = A^3 + 4A^2 - A$

$$= \begin{bmatrix} -10 & 19 & 8 \\ 26 & -43 & -12 \\ 10 & -16 & -4 \end{bmatrix} + 4\begin{bmatrix} 4 & -15 & 0 \\ -6 & 11 & 4 \\ -2 & 4 & 2 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 2 \\ 2 & -3 & 0 \\ 1 & -1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -10+16-0 & 19-20-1 & 8+0-2 \\ 26-24-2 & -43+44+3 & -12+16+0 \\ 10-8-1 & -16+16+1 & -4+8-0 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Hence,

$$f(A) = \begin{bmatrix} 6 & -2 & 6 \\ 0 & 4 & 4 \\ 1 & 1 & 4 \end{bmatrix}$$

Q57

If $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ then show that A is a root of the polynomial
$f(x)=x^3-6x^2+7x+2$

Solution

Given that, $A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}$ and $f(x)=x^3-6x^2+7x+2$

Therefore, $f(A)=A^3-6A^2+7A+2I_3$

First find A^2:

$$A^2=A\times A=\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}\times\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}$$

Now, Let us find A^3:

$$A^3=A^2\times A=\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}\times\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}$$

Thus,

$$f(A)=A^3-6A^2+7A+2I_3$$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-6\begin{bmatrix}5&0&8\\2&4&5\\8&0&13\end{bmatrix}+7\begin{bmatrix}1&0&2\\0&2&1\\2&0&3\end{bmatrix}+2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}21&0&34\\12&8&23\\34&0&55\end{bmatrix}-\begin{bmatrix}30&0&48\\12&24&30\\48&0&78\end{bmatrix}+\begin{bmatrix}7&0&14\\0&14&7\\14&0&21\end{bmatrix}+\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$=\begin{bmatrix}21-30+7+2&0&34-48+14+0\\12-12+0&8-24+14+2&23-30+7+0\\34-48+14+0&0&55-78+21+2\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

Thus, A is a root of the polynomial.

Q58

If $A=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$, then prove that $A^2-4A-5I=0$.

Solution

Given,

$$A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$A^2 - 4A - 5I$

$$= \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+4 & 2+2+4 & 2+4+2 \\ 2+2+4 & 4+1+4 & 4+2+2 \\ 2+4+2 & 4+2+2 & 4+4+1 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 9-4-5 & 8-8-0 & 8-8-0 \\ 8-8-0 & 9-4-5 & 8-8-0 \\ 8-8-0 & 8-8-0 & 9-4-5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= 0$$

Hence,

$$A^2 - 4A - 5I = 0$$

Q59

If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, show that $A^2 - 7A + 10I_3 = O$

Solution

Given,

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$A^2 - 7A + 10I_3$

$$= \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9+2+0 & 6+8+0 & 0+0+0 \\ 3+4+0 & 2+16+0 & 0+0+0 \\ 0+0+0 & 0+0+0 & 0+0+25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$$

$$= \begin{bmatrix} 11-21+10 & 14-14+0 & 0-0+0 \\ 7-7+0 & 18-28+10 & 0-0+0 \\ 0-0+0 & 0-0+0 & 25-35+10 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$= 0$

Hence,

$$A^2 - 7A + 10I_3 = 0$$

Q60

Without using the concept of inverse of a matrix, find the matrix $\begin{bmatrix} x & y \\ z & u \end{bmatrix}$ such that

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Solution

Given,

$$\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 5x - 7z & 5y - 7u \\ -2x + 3z & -2y + 3u \end{bmatrix} = \begin{bmatrix} -16 & -6 \\ 7 & 2 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$5x - 7z = -16$ ---(i)

$-2x + 3z = 7$ ---(ii)

$5y - 7u = -6$ ---(iii)

$-2y + 3u = 2$ ---(iv)

Solving equation (i) and (ii)

$$\begin{array}{r} 10x - 14z = -32 \\ \underline{-10x + 15z = 35} \\ z = 3 \end{array}$$

Put the value of z in equation (i)

$5x - 7(3) = -16$

$\Rightarrow 5x = 16 + 21$

$\Rightarrow 5x = 5$

$\Rightarrow x = 1$

Solving equation (iii) and (iv)

$$\begin{array}{r} 10y - 14u = -12 \\ \underline{-10y + 15u = 10} \\ u = -2 \end{array}$$

Put the value of u in equation (iii)

$5y - 7u = -6$

$\Rightarrow 5y - 7(-2) = -6$

$\Rightarrow 5y + 14 = -6$

$\Rightarrow 5y = -20$

$\Rightarrow y = -4$

So,

$$\begin{bmatrix} x & y \\ z & u \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & -2 \end{bmatrix}$$

Q61

Find the matrix A such that

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A = \begin{bmatrix} 3 & 3 & 5 \\ 1 & 0 & 1 \end{bmatrix}$$

Solution

Given,

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}A=\begin{bmatrix}3 & 3 & 5\\1 & 0 & 1\end{bmatrix}$$

Since, $\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}_{2\times 2}$ $\quad A=\begin{bmatrix}3 & 3 & 5\\1 & 0 & 1\end{bmatrix}_{2\times 3}$

$\Rightarrow$ A is a matrix of order 2×3

So,

$$\text{Let } A=\begin{bmatrix}a & b & c\\d & e & f\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}a & b & c\\d & e & f\end{bmatrix}=\begin{bmatrix}3 & 3 & 5\\1 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a+d & b+e & c+c\\0+d & 0+e & 0+f\end{bmatrix}=\begin{bmatrix}3 & 3 & 5\\1 & 0 & 1\end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$d=1,\ e=0,\ f=1$

And $a+d=3$

$a+1=3$

$a=3-1$

$a=2$

$b+e=3$

$b+0=3$

$b=3$

And $c+f=5$

$c+1=5$

$c=4$

Hence,

$$A=\begin{bmatrix}a & b & c\\d & e & f\end{bmatrix}$$

$$A=\begin{bmatrix}2 & 3 & 4\\1 & 0 & 1\end{bmatrix}$$

Q62

Find the matrix A so that $A\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\2 & 4 & 6\end{bmatrix}$

Solution

It is given that:

$$A\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

The matrix given on the R.H.S. of the equation is a 2×3 matrix and the one given on the L.H.S. of the equation is a 2×3 matrix. Therefore, X has to be a 2×2 matrix.

Now, let $X=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$

Therefore, we have:

$$\begin{bmatrix}a & c\\ b & d\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}a+4c & 2a+5c & 3a+6c\\ b+4d & 2b+5d & 3b+6d\end{bmatrix}=\begin{bmatrix}-7 & -8 & -9\\ 2 & 4 & 6\end{bmatrix}$$

Equating the corresponding elements of the two matrices, we have:

$a+4c=-7,\quad 2a+5c=-8,\quad 3a+6c=-9$

$b+4d=2,\quad 2b+5d=4,\quad 3b+6d=6$

Now, $a+4c=-7\Rightarrow a=-7-4c$

$\therefore 2a+5c=-8\Rightarrow -14-8c+5c=-8$

$\Rightarrow -3c=6$

$\Rightarrow c=-2$

$\therefore a=-7-4(-2)=-7+8=1$

Now, $b+4d=2\Rightarrow b=2-4d$

$\therefore 2b+5d=4\Rightarrow 4-8d+5d=4$

$\Rightarrow -3d=0$

$\Rightarrow d=0$

$\therefore b=2-4(0)=2$

Thus, $a=1,\ b=2,\ c=-2,\ d=0$

Hence, the required matrix X is $\begin{bmatrix}1 & -2\\ 2 & 0\end{bmatrix}$.

Q63

Find the matrix A such that

$$\begin{bmatrix}4\\ 1\\ 3\end{bmatrix}A=\begin{bmatrix}-4 & 8 & 4\\ -1 & 2 & 1\\ -3 & 6 & 3\end{bmatrix}$$

Solution

We know that two matrices B and C are eligible for the product BC only when number of columns of B is equal to number of rows in C. So, from the given definition we can conclude that the order of matrix A is 1×3 i.e. we can assume $A = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$.

Therefore,

$$\begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}_{3\times 1} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}_{1\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3},$$

$$\Rightarrow \begin{bmatrix} 4\times(x_1) & 4\times(x_2) & 4\times(x_3) \\ 1\times(x_1) & 1\times(x_2) & 1\times(x_3) \\ 3\times(x_1) & 3\times(x_2) & 3\times(x_3) \end{bmatrix}_{3\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$$\Rightarrow \begin{bmatrix} 4x_1 & 4x_2 & 4x_3 \\ x_1 & x_2 & x_3 \\ 3x_1 & 3x_2 & 3x_3 \end{bmatrix}_{3\times 3} = \begin{bmatrix} -4 & 8 & 4 \\ -1 & 2 & 1 \\ -3 & 6 & 3 \end{bmatrix}_{3\times 3}$$

$\Rightarrow \quad 4x_1 = -4 \;,\; 4x_2 = 8 \,,\; 4x_3 = 4$

Solving $\quad x_1 = -1 \,,\; x_2 = 2 \,,\; x_3 = 1$

So, matrix $A = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$.

Q64

Find the matrix A such that

$$\begin{bmatrix} 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = A$$

Solution

Using matrix multiplication,

Let, $A_1 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}$, $A_2 = \begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$\text{Now, } A_1.A_2 = \begin{bmatrix} 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} (2\times -1)+(1\times -1)+(3\times 0) & (2\times 0)+(1\times 1)+(3\times 1) & (2\times -1)+(1\times 0)+(3\times 1) \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}$$

$$\text{and } (A_1.A_2)A_3 = \begin{bmatrix} -3 & 4 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} (-3\times 1)+(4\times 0)+(1\times -1) \end{bmatrix}$$

$$(A_1.A_2)A_3 = \begin{bmatrix} -4 \end{bmatrix} = A$$

Therefore matrix $A = \begin{bmatrix} -4 \end{bmatrix}$

Note : The problem can also be solved by calculating (A_2A_3) first then pre multiplying it with A_1 as matrix multiplication is associative but one must not change the order of multiplication.

Q65

Find a 2×2 matrix A such that

$$A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6I_2$$

Solution

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given,

$$A\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+b & -2a+4b \\ c+d & -2c+4d \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

Since, corresponding entries of equal matrices are equal, so

$a + b = 6$ ---(i)

$-2a + 4b = 0$ ---(ii)

$c + d = 0$ ---(iii)

$-2c + 4d = 6$ ---(iv)

Solving equation (i) and (ii)

$$\begin{aligned} 4a + 4b &= 24 \\ -2a + 4b &= 0 \\ (+) \quad (-) & \\ \hline 6a \quad &= 24 \end{aligned}$$

$$\Rightarrow a = \frac{24}{6}$$

$$a = 4$$

Put $a = 4$ in equation (i)

$a + b = 6$

$4 + b = 6$

$b = 6 - 4$

$b = 2$

Solving equation (iii) and (iv)

$$\begin{aligned} 2c + 2d &= 0 \\ \underline{-2c + 4d} &\underline{= 6} \\ 6d &= 6 \\ d &= \frac{6}{6} \\ d &= 1 \end{aligned}$$

Put $d = 1$ in equation (iii)

$c + d = 0$

$c = -1$

Hence,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Q66

If $A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$, find A^{16}.

Solution

Given,

$$A = \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$A^2 = A \times A$$

$$= \begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 4 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= 0$$

$$A^4 = A^2 \times A^2$$

$$= 0 \times 0$$

$$= 0$$

$$A^{16} = A^4 \times A^4$$

$$= 0 \times 0$$

$$= 0$$

So,
A^{16} is null matrix.

Q67

If $A = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $x^2 = -1$,

Then show that $(A + B)^2 = A^2 + B^2$.

Solution

Solving the LHS of the given equation we have ,

$\Rightarrow \quad A + B = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A + B = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}^2 = \begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}\begin{bmatrix} 0 & -x+1 \\ x+1 & 0 \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} (0\times 0)+((-x+1)\times(x+1)) & (0\times(-x+1))+((-x+1)\times 0) \\ ((x+1)\times 0)+(0\times(x+1)) & ((x+1)\times(-x+1))+(0\times 0) \end{bmatrix}$

$(A + B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}.$

Solving the RHS we get,

$\Rightarrow \quad A^2 + B^2 = \begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix}\begin{bmatrix} 0 & -x \\ x & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} (0\times 0)+((-x)\times(x)) & (0\times(-x))+((-x)\times 0) \\ ((x)\times 0)+(0\times(x)) & ((x)\times(-x))+(0\times 0) \end{bmatrix} + \begin{bmatrix} (0\times 0)+(1\times 1) & (0\times 1)+(1\times 0) \\ (1\times 0)+(0\times 1) & (1\times 1)+(0\times 0) \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} -x^2 & 0 \\ 0 & -x^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix}$

Subsituting the value of $x^2 = -1$ in the LHS and RHS above,

$\Rightarrow \quad (A + B)^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$, and

$A^2 + B^2 = \begin{bmatrix} 1-x^2 & 0 \\ 0 & 1-x^2 \end{bmatrix} = \begin{bmatrix} 1+1 & 0 \\ 0 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \quad (A + B)^2 = A^2 + B^2.$

Q68

If $A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$, then verify that $A^2 + A = A(A + I)$,

where I is theidentify matrix.

Solution

Solving the LHS i.e.

$$A^2 + A = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}^2 + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -3 & -6 \\ 4 & 4 & 0 \\ 2 & 2 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Solving the RHS i.e.

$$A(A+I) = \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \left(\begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 & 0 & -3 \\ 2 & 1 & 3 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & -3 \\ 2 & 2 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & -9 \\ 6 & 5 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

So, LHS = RHS verified.

Q69

If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Solution

We have,

$$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

Now, $A^2 = AA = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} (3\times3)+(-5\times-4) & (3\times-5)+(-5\times2) \\ (-4\times3)+(2\times-4) & (-4\times-5)+(2\times2) \end{bmatrix}$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} \text{ and } -14I = \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + \begin{bmatrix} -15 & 25 \\ 20 & -10 \end{bmatrix} + \begin{bmatrix} -14 & 0 \\ 0 & -14 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25+0 \\ -20+20+0 & 24-10+-14 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Now,

$$A^2 - 5A - 14I = 0$$

$$\Rightarrow \quad A^2 = 5A + 14I$$

$$\Rightarrow \quad A^3 = A^2.A = (5A + 14I)A$$

$$\Rightarrow \quad A^3 = A^2.A = 5A^2 + 14A \qquad \left[\text{By using dist. of matrices over matrix addition}\right]$$

$$\Rightarrow \quad A^3 = 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix}$$

$$\Rightarrow \quad A^3 = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$$

Q70

If $p(x) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$, then show that $P(x)\,P(y) = P(x+y) = p(y)P(x)$.

Solution

We have,

$$P(x).\,P(y) = \begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}\begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}$$

$$\Rightarrow P(x).\,P(y) = \begin{bmatrix} \cos x \cos y - \sin x \sin y & \sin y \cos x + \sin x \cos y \\ -\sin x \cos y - \cos x \sin y & -\sin x \sin y + \cos x \cos y \end{bmatrix}$$

$$\Rightarrow P(x).\,P(y) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

Now,

$$P(y).\,P(x) = \begin{bmatrix} \cos y & \sin y \\ -\sin y & \cos y \end{bmatrix}\begin{bmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{bmatrix}$$

$$\Rightarrow P(y).\,P(x) = \begin{bmatrix} \cos y \cos x - \sin y \sin x & \sin x \cos y + \sin y \cos x \\ -\sin y \cos x - \cos y \sin x & -\sin y \sin x + \cos y \cos x \end{bmatrix}$$

$$\Rightarrow P(y).\,P(x) = \begin{bmatrix} \cos(x+y) & \sin(x+y) \\ -\sin(x+y) & \cos(x+y) \end{bmatrix} = P(x+y)$$

$$\therefore P(x).\,P(y) = P(x+y) = P(y).\,P(x)$$

Q71

if $P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ and $Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$, Prove that $PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP$

Solution

We have,

$$P = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}, \ Q = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$\text{So,} \quad PQ = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} x \times a & 0 & 0 \\ 0 & y \times b & 0 \\ 0 & 0 & z \times c \end{bmatrix}$$

$$= \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix}$$

$$\text{and} \quad QP = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}\begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$= \begin{bmatrix} a \times x & 0 & 0 \\ 0 & b \times y & 0 \\ 0 & 0 & c \times z \end{bmatrix}$$

$$= \begin{bmatrix} ax & 0 & 0 \\ 0 & by & 0 \\ 0 & 0 & zc \end{bmatrix}$$

as, $xa = ax$, $yb = by$, $zc = cz$

$$\therefore \ PQ = \begin{bmatrix} xa & 0 & 0 \\ 0 & yb & 0 \\ 0 & 0 & zc \end{bmatrix} = QP.$$

Q72

If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$.

Solution

We have,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

Then ,

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 2\times2+0\times2+1\times1 & 2\times0+0\times1+1\times-1 & 2\times1+0\times3+1\times0 \\ 2\times2+1\times2+3\times1 & 2\times0+0\times1+1\times-1 & 2\times1+1\times3+3\times0 \\ 1\times2+-1\times2+0\times1 & 1\times0+-1\times1+0\times-1 & 1\times1+-1\times3+0\times0 \end{bmatrix},$$

$$-5A = \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix}, \quad 4I = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Hence, $A^2-5A+4I = \begin{bmatrix} 5 & -1 & 5 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} + \begin{bmatrix} -10 & 0 & -5 \\ -10 & -5 & -15 \\ -5 & 5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$A^2-5A+4I = \begin{bmatrix} 5-10+4 & -1+0+0 & 5-5+0 \\ 9-10+0 & -2-5+4 & 5-15+0 \\ 0-5-0 & -1+5+0 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

Now, given is $A^2-5A+4I+X=0$

$\Rightarrow X = -(A^2-5A+4I)$

$$X = -\begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 10 \\ 5 & 4 & 2 \end{bmatrix}$$

Q73

If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for all positive integers n.

Solution

Given,

$$A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

To prove $A^n=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n=1$

$$A^1=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$$

So,

A^n is true for $n=1$

Step 2: Let, A^n be true for $n=k$, then

$$A^k=\begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix} \qquad \text{---(i)}$$

Step 3: We have to show that $A^{k+1}=\begin{bmatrix}1 & k+1\\ 0 & 1\end{bmatrix}$

So,

$$A^{k+1}=A^k\times A$$

$$=\begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \qquad \{\text{using equation (i) and given}\}$$

$$=\begin{bmatrix}1+0 & 1+k\\ 0+0 & 0+1\end{bmatrix}$$

$$A^{k+1}=\begin{bmatrix}1 & 1+k\\ 0 & 1\end{bmatrix}$$

This shows that A^n is true for $n=k+1$ whenever it is true for $n=k$

Hence, by the principle of mathematical induction A^n is true for all positive integer.

Q74

If $A=\begin{bmatrix}a & b\\ 0 & 1\end{bmatrix}$, prove that $A^n=\begin{bmatrix}a^n & \dfrac{b\left(a^n-1\right)}{a-1}\\ 0 & 1\end{bmatrix}$ for every positive integer n.

Solution

Given,

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

To prove $A^n = \begin{bmatrix} a^n & \frac{b\left(a^n - 1\right)}{a-1} \\ 0 & 1 \end{bmatrix}$ we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} a^1 & \frac{b\left(a^1 - 1\right)}{a-1} \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n is true for $n = k$, so,

$$A^k = \begin{bmatrix} a^k & \frac{b\left(ak - 1\right)}{a-1} \\ 0 & 1 \end{bmatrix} \qquad \text{---(i)}$$

Step 3: We have to show that

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b\left(a^k - 1\right)}{a-1} \\ 0 & 1 \end{bmatrix}$$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} a^k & \frac{b\left(a^k - 1\right)}{a-1} \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix} \quad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} a^{k+1} + 0 & a^k b + \frac{b\left(a^k - 1\right)}{a-1} \\ 0 + 0 & 0 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & \frac{a^{k+1}b - a^k b + a^k b - b}{a-1} \\ 0 & 1 \end{bmatrix}$$

$$A^{k+1} = \begin{bmatrix} a^{k+1} & \frac{b\left(a^{k+1} - 1\right)}{a-1} \\ 0 & 1 \end{bmatrix}$$

So,

A^n is true for $n = k + 1$ whenever it is true $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer n.

Q75

If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, then prove by principle of mathematical induction that

$A^n = \begin{bmatrix} \cos n\theta & i\sin n\theta \\ i\sin n\theta & \cos n\theta \end{bmatrix}$ for all $n \in N$.

Solution

Given,

$$A=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$$

To show that,

$$A^n=\begin{bmatrix}\cos n\theta & i\sin n\theta\\ i\sin n\theta & \cos n\theta\end{bmatrix}\text{ for all } n\in N.$$

Put $n=1$

$$A^1=\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$$

So,

A^n is true for $n=1$

Let, A^n is true for $n=k$, so

$$A^k=\begin{bmatrix}\cos k\theta & i\sin k\theta\\ i\sin k\theta & \cos k\theta\end{bmatrix}\qquad ---(i)$$

Now, we have to show that,

$$A^{k+1}=\begin{bmatrix}\cos(k+1)\theta & i\sin(k+1)\theta\\ i\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$$

Now, $A^{k+1}=A^k\times A$

$$=\begin{bmatrix}\cos k\theta & i\sin k\theta\\ i\sin k\theta & \cos k\theta\end{bmatrix}\begin{bmatrix}\cos\theta & i\sin\theta\\ i\sin\theta & \cos\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos k\theta\cos\theta+i^2\sin k\theta\sin\theta & i^2\cos k\theta\sin\theta+i\sin k\theta\cos\theta\\ i\sin k\theta\cos\theta+i\cos k\theta\sin\theta & i^2\sin k\theta\sin\theta+\cos\theta\cos k\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos k\theta\cos\theta-\sin k\theta\sin\theta & i(\cos k\theta\sin\theta+\sin k\theta\cos\theta)\\ i(\sin k\theta\cos\theta+\cos k\theta\sin\theta) & \cos k\theta\cos\theta-\sin k\theta\sin\theta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(k+1)\theta & i\sin(k+1)\theta\\ i\sin(k+1)\theta & \cos(k+1)\theta\end{bmatrix}$$

So, A^n is true for $n=k+1$ whenever it is true for $n=k$.

Hence, By principle of mathematical induction A^n is true for all positive integer.

Q76

If $A=\begin{bmatrix}\cos\alpha+\sin\alpha & \sqrt{2}\sin\alpha\\ -\sqrt{2}\sin\alpha & \cos\alpha-\sin\alpha\end{bmatrix}$, prove that

$$A^n=\begin{bmatrix}\cos n\alpha+\sin n\alpha & \sqrt{2}\sin n\alpha\\ -\sqrt{2}\sin n\alpha & \cos\alpha-\sin n\alpha\end{bmatrix}\text{ for all } n\in N.$$

Solution

Given,

$$A=\begin{bmatrix}\cos\alpha+\sin\alpha & \sqrt{2}\sin\alpha\\ -\sqrt{2}\sin\alpha & \cos\alpha-\sin\alpha\end{bmatrix}$$

To prove P(n): $A^n=\begin{bmatrix}\cos n\alpha+\sin n\alpha & \sqrt{2}\sin n\alpha\\ -\sqrt{2}\sin n\alpha & \cos n\alpha-\sin n\alpha\end{bmatrix}$ we use mathematical induction.

Step 1: To show P(1) is true.

A^n is true for $n=1$

Step 2: Let, $P(k)$ be true, so

$$A^k=\begin{bmatrix}\cos k\alpha+\sin k\alpha & \sqrt{2}\sin k\alpha\\ -\sqrt{2}\sin k\alpha & \cos k\alpha-\sin k\alpha\end{bmatrix} \quad \text{---(i)}$$

Step 3: Let, $P(k)$ is true.

Now, we have to show that

$$A^{k+1}=\begin{bmatrix}\cos(k+1)\alpha+\sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha\\ -\sqrt{2}\sin(k1)\alpha & \cos(k+1)\alpha-\sin(k+1)\alpha\end{bmatrix}$$

Now,

$$A^{k+1}=A^k\times A$$

$$=\begin{bmatrix}\cos k\alpha+\sin k\alpha & \sqrt{2}\sin k\alpha\\ -\sqrt{2}\sin k\alpha & \cos k\alpha-\sin k\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha+\sin\alpha & \sqrt{2}\sin\alpha\\ -\sqrt{2}\sin\alpha & \cos\alpha-\sin\alpha\end{bmatrix}$$

$$=\begin{bmatrix}(\cos k\alpha+\sin k\alpha)(\cos\alpha+\sin\alpha)-2\sin\alpha\sin k\alpha & (\cos k\alpha+\sin k\alpha)\sqrt{2}\sin\alpha+\sqrt{2}\sin k\alpha(\cos\alpha-\sin\alpha)\\ (\cos\alpha+\sin\alpha)\left(-\sqrt{2}\sin k\alpha\right)-\sqrt{2}\sin\alpha(\cos k\alpha-\sin k\alpha) & -2\sin k\alpha\sin\alpha+(\cos k\alpha-\sin k\alpha)(\cos\alpha-\sin\alpha)\end{bmatrix}$$

$$=\begin{bmatrix}\cos k\alpha\cos\alpha+\sin k\alpha\cos\alpha+\cos k\alpha\sin\alpha+\sin\alpha\sin k\alpha-2\sin\alpha\sin k\alpha & \sqrt{2}\cos k\alpha\sin\alpha+\sqrt{2}\sin\alpha\sin k\alpha+\sqrt{2}\sin k\alpha\cos\alpha-\sqrt{2}\sin k\alpha\sin\alpha\\ -\sqrt{2}\cos\alpha\sin\alpha-\sqrt{2}\sin\alpha\sin k\alpha-\sqrt{2}\sin\alpha\cos k\alpha+\sqrt{2}\sin\alpha\sin k\alpha & -2\sin k\alpha\sin\alpha+\cos k\alpha\cos\alpha-\cos\alpha\sin k\alpha-\sin\alpha\cos k\alpha+\sin\alpha\sin k\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha\cos k\alpha+\sin\alpha\sin k\alpha & \\ \sin\alpha\cos k\alpha+\sin k\alpha\cos\alpha & \sqrt{2}(\sin k\alpha\cos\alpha+\cos k\alpha\sin\alpha)\\ -\sqrt{2}(\sin k\alpha\cos\alpha+\cos k\alpha\sin\alpha) & \cos k\alpha\cos\alpha-\sin k\alpha\sin\alpha-(\sin k\alpha\cos\alpha+\sin\alpha\cos k\alpha)\end{bmatrix}$$

$$=\begin{bmatrix}\cos(k+1)\alpha+\sin(k+1)\alpha & \sqrt{2}\sin(k+1)\alpha\\ -\sqrt{2}\sin(k+1)\alpha & \cos(k+1)\alpha-\sin(k+1)\alpha\end{bmatrix}$$

So, $P(k+1)$ is true whenever $P(k)$ is true.

Hence, by principle of mathematical induction $P(n)$ is true for all positive integer.

Q77

If $A=\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}$, then use the principle of mathematical inductin to show that

$A^n=\begin{bmatrix}1 & n & n(n+1)/2\\ 0 & 1 & n\\ 0 & 0 & 1\end{bmatrix}$ for every positive integer n.

Solution

Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

To prove, $A^n = \begin{bmatrix} 1 & n & \frac{n(n+1)}{2} \\ 0 & 1 & n \\ 0 & 0 & 1 \end{bmatrix}$, we will use the principle of mathematical induction.

Step 1: Put $n = 1$

$$A^1 = \begin{bmatrix} 1 & 1 & \frac{1(1+1)}{2} \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

So, A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: We will prove that A^n be true for $n = k+1$

Now,

$$A^{k+1} = A^k \times A$$

$$= \begin{bmatrix} 1 & k & \frac{k(k+1)}{2} \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \{\text{using equation (i) and given}\}$$

$$= \begin{bmatrix} 1+0+0 & 1+k+0 & 1+k+\frac{k(k+1)}{2} \\ 0+0+0 & 0+1+0 & 0+1+k \\ 0+0+0 & 0+0+0 & 0+0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & (k+1) & \frac{(k+1)(k+2)}{2} \\ 0 & 1 & (k+1) \\ 0 & 0 & 1 \end{bmatrix}$$

Hence, A^n is true for $n = k+1$ whenever it is true for $n = k$.

So, by principle of mathematical induction A^n is true for all positive integer n.

Q78

If B, C are n rowed square matrices and if $A = B + C$, $BC = CB$, $C^2 = 0$, then show that for every $n \in N$, $A^{n+1} = B^n\left(B + (n+1)C\right)$.

Solution

We will prove P(n): $A^{n+1} = B^n\left[B + (n+1)C\right]$ is true for all natural numbers using mathematical induction.

Given,

$$A = B + C,\ BC = CB,\ C^2 = 0$$
$$A = B + C$$

Squaring both the sides, so

$$A^2 = (B + C)^2$$
$$\Rightarrow \quad A^2 = (B + C)(B + C)$$
$$\Rightarrow \quad A^2 = B \times B + BC + CB + C \times C \qquad \{\text{using distributive property}\}$$
$$\Rightarrow \quad A^2 = B^2 + BC + BC + C^2 \qquad \{\text{using } BC = CB \text{ given}\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC + 0 \qquad \{\text{since, given } C^2 = 0\}$$
$$\Rightarrow \quad A^2 = B^2 + 2BC \qquad \text{---(1)}$$
$$A^2 = B(B + 2C)$$

Now, consider

$$P(n):\ A^{n+1} = B^n\left[B + (n+1)C\right]$$

Step 1: Tp prove P(1) is true, put $n = 1$

$$A^{1+1} = B^1\left[B + (1+1)C\right]$$
$$A^2 = B\left[B + 2C\right]$$
$$A^2 = B^2 + 2BC$$

From equation (i), P(1) is true.

Step 2: Suppose P(k) is true.

$$\therefore A^{k+1} = B^k\left[B + (k+1)C\right] \qquad \text{---(2)}$$

Step 3: Now, we have to show that P(k+1) is true.

That is we need to prove that,

$$A^{k+2} = B^{k+1}\left[B + (k+2)C\right]$$

Now,

$$A^{k+2} = A^k \times A^2$$
$$= B^{(k-1)}\left[B + kC\right] \times \left[B(B + 2C)\right]$$
$$= B^k\left[B + kC\right] \times \left[B + 2C\right]$$
$$= B^k\left[B \times B + B \times 2C + kC \times B + 2kC^2\right]$$
$$= B^k\left[B^2 + 2BC + kBC + 2k \times 0\right] \qquad \{\text{since } BC = CB,\ C^2 = 0\}$$
$$= B^k\left[B^2 + BC(2 + k)\right]$$
$$= B^k \times B\left[B + (k+2)C\right]$$
$$= B^{k+1}\left[B + (k+2)C\right]$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$

Therefore by principle of mathematical induction $P(n)$ is true for all natural number.

Q79

If $A = \text{diag}(a,b,c)$, show that $A^n = \text{diag}(a^n, b^n, c^n)$ for all positive integer n.

Solution

Given,

$$A = \text{diag}(a,b,c)$$

Show that,

$$A^n = \text{diag}(a^n, b^n, c^n)$$

Step 1: Put $n = 1$

$$A^1 = \text{diag}(a^1, b^1, c^1)$$

$$A = \text{diag}(a,b,c)$$

So,

A^n is true for $n = 1$

Step 2: Let, A^n be true for $n = k$, so,

$$A^k = \text{diag}(a^k, b^k, c^k) \quad \text{---(i)}$$

Step 3: Now, we have to show that,

$$A^{k+1} = \text{diag}(a^{k+1}, b^{k+1}, c^{k+1})$$

Now,

$$A^{k+1} = A^k \times k3$$

$$= \text{diag}(a^k, b^k, c^k) \times \text{diag}(a,b,c) \quad \{\text{using equation (i) and given}\}$$

$$A^{k+1} = \begin{bmatrix} a^k & 0 & 0 \\ 0 & b^k & 0 \\ 0 & 0 & c^k \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} a^k \times a + 0 + 0 & 0 + 0 + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + b^k \times b + 0 & 0 + 0 + 0 \\ 0 + 0 + 0 & 0 + 0 + 0 & 0 + 0 + c^k \times c \end{bmatrix}$$

$$= \begin{bmatrix} a^{k+1} & 0 & 0 \\ 0 & b^{k+1} & 0 \\ 0 & 0 & c^{k+1} \end{bmatrix}$$

$$A^{k+1} = \text{diag}(a^{k+1}, b^{k+1}, c^{k+1})$$

So, $P(n)$ is true for $n = k + 1$ whenever $P(n)$ is true for $n = k$.

Hence, by principle of mathematical induction A^n is true for all positive integer.

Q80

A matrix X has a+b rows a+2 columns the matrix Y has b+1 rows and a+3 columns. Both matrices XY and XY exist. Find a and b.
Can you say XY and YX are of the same type? Are they equal.

Solution

Given,

order of matrix $X = (a+b)\times(a+2)$

order of matrix $Y = (b+1)\times(a+3)$

Given, $X_{(a+b)\times(a+2)} \cdot Y_{(b+1)\times(a+3)}$ exist.

$\Rightarrow \quad a+2 = b+1$

$\Rightarrow \quad a-b = -1 \qquad ---(i)$

And

$Y_{(b+1)\times(a+3)} \cdot X_{(a+b)\times(a+2)}$ exists.

$\Rightarrow \quad a+3 = a+b$

$\Rightarrow \quad b = 3$

Put $b = 3$ in equation(i),

$a-b = -1$

$a-3 = -1$

$a = 3-1$

$a = 2$

So, $a = 2, b = 3$

So,

$Order\ of\ X = (a+b)\times(a+2)$

$= (2+3)\times(2+2)$

$= 5\times 4$

$Order\ of\ Y = (b+1)\times(a+3)$

$= (3+1)\times(2+3)$

$= 4\times 5$

$Order\ of\ X_{5\times 4} \cdot Y_{4\times 5} = 5\times 5$

$Order\ of\ X_{4\times 5} \cdot Y_{5\times 4} = 4\times 4$

So, *order* of XY and YX are not same and they are not equal but both are square matrices.

Q81

Give an example of matrices: A and B such that $AB \neq BA$

Solution

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix} \quad \text{---(i)}$$

$$BA = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

From equation(i) and (ii)

$AB \neq BA$

when $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b \\ 0 & 0 \end{bmatrix}$

Q82

Given an example of matrices A and B such that $AB = O$ but $A \neq 0$, $B \neq 0$.

Solution

Let, $A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$

$$AB = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Hence,

$AB = 0$

When,

$A = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

$B = \begin{bmatrix} 0 & 0 \\ 0 & b \end{bmatrix} \neq 0$

Q83

Give an example of matrices A and B such that $AB = O$ but $BA \neq O$.

Solution

Let, $A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}\begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$AB = 0$$
$$BA = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0+0 & ab+0 \\ 0+0 & 0+0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & ab \\ 0 & 0 \end{bmatrix}$$
$$BA \neq 0$$

Hence,

for $AB = 0$ and $BA \neq 0$ we have,

$A = \begin{bmatrix} 0 & a \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} b & 0 \\ 0 & 0 \end{bmatrix}$

Q84

Given an example of matrices A, B and C such that $AB = AC$ but $B \neq C$, $A \neq 0$.

Solution

Let, $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Here,

$A \neq 0$, $B \neq C$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix} = \begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

LHS = RHS

So,

for $A \neq 0$, $BC \neq 0$ but $AB = AC$

We have,

$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

Q85

Let A and B be square matrices of the same order. Does $(A+B)^2 = A^2+2AB+B^2$ hold? If not, why?

Solution

Given,

A and B are square matices of same order

$$(A+B)^2 = (A+B)(A+B)$$
$$= A(A+B)+B(A+B) \quad \{\text{using distributive property}\}$$
$$= A\times A + AB + BA + B^2$$
$$= A^2 + AB + BA + B^2$$

But,

$(A+B)^2 = A^2+2AB+B^2$ is possible only when $AB = BA$

Here, we can not say that $AB = BA$

So,

$(A+B)^2 = A^2+2AB+B^2$ does not hold.

Q86

If A and B are square matrices of the same order, explain, why in general

(i) $(A+B)^2 \neq A^2+2AB+B^2$ (ii) $(A-B)^2 \neq A^2-2AB+B^2$

(iii) $(A+B)(A-B) \neq A^2-B^2$.

Solution

Given, A and B are square matrices of same order.

(i) $(A+B)^2 = (A+B)(A+B)$

$= A(A+B)+B(A+B)$ {using distributive property}

$= A \times A + AB + BA + B \times B$

$= A^2 + AB + BA + B^2$

$\neq A^2 + 2AB + B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$

So, $(A+B)^2 \neq A^2 + 2AB + B^2$

(ii) $(A-B)^2 = (A-B)(A-B)$

$= A(A-B) - B(A-B)$ {using distributive property} .

$= A \times A - AB - BA + B \times B$

$= A^2 - AB - BA + B^2$

$\neq A^2 - 2AB + B^2$

Since, in general matrix multiplication is not commutative $(AB \neq BA)$, so

So, $(A-B)^2 \neq A^2 - 2AB + B^2$

(iii) $(A+B)(A-B) = A(A-B) + B(A-B)$ {using distributive property}

$= A \times A - AB + BA - B \times B$

$= A^2 - AB + BA - B^2$

$\neq A^2 - B^2$

Since, in general matix multiplication is not commutative $(AB \neq BA)$,

So, $(A+B)(A-B) \neq A^2 - B^2$

Q87

Let A and B be square matrices of the order 3 × 3.
Is $(AB)^2 = A^2 B^2$? Give reasons.

Solution

The given equality is true only when we choose A and B to be a square matrix in such a way that AB = BA else the result is not true in general.

Example: Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Here $AB = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times 0+0\times 2+0\times 0 & 1\times 1+0\times 1+0\times 0 & 1\times 0+0\times 0+0\times 1 \\ 1\times 0+1\times 2+0\times 0 & 1\times 1+1\times 1+0\times 0 & 1\times 0+1\times 0+0\times 1 \\ 0\times 0+0\times 2+1\times 0 & 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $BA = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0\times 1+1\times 1+0\times 0 & 0\times 0+1\times 1+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 2\times 1+1\times 1+0\times 0 & 2\times 0+1\times 1+0\times 0 & 2\times 0+1\times 0+0\times 1 \\ 0\times 1+0\times 1+1\times 0 & 0\times 0+0\times 1+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$AB \neq BA$

Now, $(AB)^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0\times 0+1\times 1+0\times 0 & 0\times 1+1\times 2+0\times 0 & 0\times 0+1\times 0+0\times 1 \\ 1\times 0+2\times 1+0\times 0 & 1\times 1+1\times 2+0\times 0 & 1\times 0+2\times 0+0\times 1 \\ 0\times 0+0\times 1+1\times 0 & 0\times 1+0\times 2+1\times 0 & 0\times 0+0\times 0+1\times 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^2=\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times1+0\times0 & 1\times0+0\times1+0\times0 & 1\times0+0\times0+0\times1\\1\times1+1\times1+0\times0 & 1\times0+1\times1+0\times0 & 1\times0+1\times0+0\times1\\0\times1+0\times1+1\times0 & 0\times0+0\times1+1\times0 & 0\times0+0\times0+1\times1\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\2&1&0\\0&0&1\end{bmatrix}$$

$$B^2=\begin{bmatrix}0&1&0\\2&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}0&1&0\\2&1&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}0\times0+1\times2+0\times0 & 0\times1+1\times1+0\times0 & 0\times0+1\times0+0\times1\\2\times0+1\times2+0\times0 & 2\times1+1\times1+0\times0 & 2\times0+1\times0+0\times1\\0\times0+0\times2+1\times0 & 0\times1+0\times1+1\times0 & 0\times0+0\times0+1\times1\end{bmatrix}$$

$$=\begin{bmatrix}1&1&0\\1&2&0\\0&0&1\end{bmatrix}$$

$$A^2B^2=\begin{bmatrix}1&0&0\\2&1&0\\0&0&1\end{bmatrix}\begin{bmatrix}1&1&0\\1&2&0\\0&0&1\end{bmatrix}$$

$$=\begin{bmatrix}1\times1+0\times1+0\times0 & 1\times1+0\times2+0\times0 & 1\times0+0\times0+0\times1\\2\times1+1\times1+0\times0 & 2\times1+1\times2+0\times0 & 2\times0+1\times0+0\times1\\0\times1+0\times1+1\times0 & 0\times1+0\times2+1\times0 & 0\times0+0\times0+1\times1\end{bmatrix}$$

$$=\begin{bmatrix}1&1&0\\3&4&0\\0&0&1\end{bmatrix}$$

We can see that if we have A and B two square matrices with $AB \neq BA$ then $(AB)^2 \neq A^2B^2$

Q88

If A and B be square matrices of the same order such that $AB = BA$, then show that $(A + B)^2 = A^2 + 2AB + B^2$.

Solution

Given,

A and B two square matrices of same order such that

AB = BA.

To prove : $(A+B)^2 = A^2 + 2AB + B^2$

Now, solving LHS gives,

$$(A+B)^2 = (A+B)(A+B)$$

$$= A(A+B) + B(A+B) \quad \begin{bmatrix}\text{by dist. of matrix multiplication} \\ \text{over addition}\end{bmatrix}$$

$$= A^2 + AB + BA + B^2 \quad \begin{bmatrix}\text{by dist. of matrix multiplication} \\ \text{over addition}\end{bmatrix}$$

$$= A^2 + 2AB + B^2 \quad [\text{As, } AB = BA]$$

$$= RHS$$

Hence proved.

Q89

Let $A = \begin{bmatrix}1 & 1 & 1 \\ 3 & 3 & 3\end{bmatrix}, B = \begin{bmatrix}3 & 1 \\ 5 & 2 \\ -2 & 4\end{bmatrix}$ and $C = \begin{bmatrix}4 & 2 \\ -3 & 5 \\ 5 & 0\end{bmatrix}$

Verify that $AB = AC$ though $B \neq C$, $A \neq O$.

Solution

Given, $A = \begin{bmatrix}1 & 1 & 1 \\ 3 & 3 & 3\end{bmatrix}, B = \begin{bmatrix}3 & 1 \\ 5 & 2 \\ -2 & 4\end{bmatrix}, C = \begin{bmatrix}4 & 2 \\ -3 & 5 \\ 5 & 0\end{bmatrix}$

$$AB = \begin{bmatrix}1 & 1 & 1 \\ 3 & 3 & 3\end{bmatrix}\begin{bmatrix}3 & 1 \\ 5 & 2 \\ -2 & 4\end{bmatrix}$$

$$= \begin{bmatrix}3+5-2 & 1+2+4 \\ 9+15-6 & 3+6+12\end{bmatrix}$$

$$AB = \begin{bmatrix}6 & 7 \\ 18 & 21\end{bmatrix} \quad \text{---(i)}$$

$$AC = \begin{bmatrix}1 & 1 & 1 \\ 3 & 3 & 3\end{bmatrix}\begin{bmatrix}4 & 2 \\ -3 & 5 \\ 5 & 0\end{bmatrix}$$

$$= \begin{bmatrix}4-3+5 & 2+5+0 \\ 12-9+15 & 6+15+0\end{bmatrix}$$

$$AC = \begin{bmatrix}6 & 7 \\ 18 & 21\end{bmatrix} \quad \text{---(ii)}$$

From equation (i) and (ii)

$AB = AC$

Q90

Three shopkeepers A, B and C go to a store to buy stationary. A purchases 12 dozen notebooks, 5 dozen pens and 6 dozen pencils. B purchases 10 dozen notebooks, 6 dozen pens and 7 dozen pencils. C purchases 11 dozen notebooks, 13 dozen pens and 8 dozen pencils. A notebook costs 40 paise, a pen costs Rs. 1.25 and a pencil costs 35 paise. Use matrix multiplication to calculate each individual's bill.

Solution

The number of items purchased by A, B and C are represented in matrix form as,

$$X = \begin{array}{c} \\ A \\ B \\ C \end{array} \begin{array}{c} \begin{array}{ccc} \text{Notebook} & \text{Pens} & \text{Pencils} \end{array} \\ \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \end{array}$$

Now, matrix formed by the cost of each items is given by,

$$Y = \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix} \begin{array}{l} \text{Note book} \\ \text{Pen} \\ \text{Pencil} \end{array}$$

Individual bill can be calculated by

$$XY = \begin{bmatrix} 144 & 60 & 72 \\ 120 & 72 & 84 \\ 132 & 156 & 96 \end{bmatrix} \begin{bmatrix} 0.40 \\ 1.25 \\ 0.35 \end{bmatrix}$$

$$XY = \begin{bmatrix} 57.60 + 75.00 + 25.20 \\ 48.00 + 90.00 + 29.40 \\ 52.80 + 195.00 + 33.60 \end{bmatrix}$$

$$XY = \begin{bmatrix} 157.80 \\ 167.40 \\ 281.40 \end{bmatrix}$$

So,

Bill of A = Rs 157.80

Bill of B = Rs 167.40

Bill of C = Rs 281.40

Q91

The cooperative stores of a particular school has 10 dozen physics books, 8 dozen chemistry books and 5 dozen mathematics books. Their selling prices are Rs. 8.30, Rs. 3.45 and Rs. 4.50 each respectively. Find the total amount the store will receive from selling all the items.

Solution

Matrix representation of stock of various types of book in the store is given by,

$$X = \begin{array}{ccc} \text{Physics} & \text{Chemistry} & \text{Mathematics} \\ \end{array}$$
$$X = \begin{bmatrix} 120 & 96 & 60 \end{bmatrix}$$

Matrix representation of sellin price (Rs.) of each book is given by

$$Y = \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix} \begin{array}{l} \text{Physics} \\ \text{Chemistry} \\ \text{Mathematics} \end{array}$$

So, totaol amount recieved by the store from sellin all the items is given by,

$$XY = \begin{bmatrix} 120 & 96 & 60 \end{bmatrix} \begin{bmatrix} 8.30 \\ 3.45 \\ 4.50 \end{bmatrix}$$
$$= \left[(120)(8.30) + (96)(3.45) + (60)(4.50)\right]$$
$$= [996 + 331.20 + 270]$$
$$= [1597.20]$$

Required amount = Rs 1597.20

Q92

In a legislative assembly election, a political group hired a public relations firm to promote its candidates in three ways: telephone, house calls and letters. The cost per contact (in paise) is given matrix A as

$$A = \begin{array}{c} \text{cost per contact} \\ \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \end{array} \begin{bmatrix} \text{Telephone} \\ \text{House call} \\ \text{Letter} \end{bmatrix}$$

The number of contacts of each type made in two cities X and Y is given in matrix B as

$$B = \begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{Housecall} & \text{Letter} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \rightarrow X \\ \rightarrow Y \end{array}$$

Solution

Given,

The cost per contact (in paise) is given by

$$A = \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix} \begin{bmatrix} \text{Telephone} \\ \text{Housecall} \\ \text{Letter} \end{bmatrix}$$

The number of contact of each type made in two cities X and y is given by.

$$\begin{array}{cccc} & \text{Telephone} & \text{Housecall} & \text{Letter} \end{array}$$

$$B = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix}$$

Total amount spent by the group in the two cities X and y can be given by

$$BA = \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$$

$$= \begin{bmatrix} 40000 + 50000 + 250000 \\ 120000 + 100000 + 500000 \end{bmatrix}$$

$$= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix} \begin{matrix} X \\ Y \end{matrix}$$

Hence,

Amount spend on X = Rs 3400

Amount spend on Y = Rs 7200

Q93

A trust fund has Rs 30,000 that must be invested in two different types of bonds. The first bond pays 5% interest per year, and the second bond pays 7% interest per year. Using matrix multiplication, determine how to divide Rs 30,000 among the two types of bonds. If the trust fund must obtain an annual total interest of:

(a) Rs 1,800 (b) Rs 2,000

Solution

(a) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.
It is given that the first bond pays 5% interest per year and the second bond pays 7% interest per year.
Therefore, in order to obtain an annual total interest of Rs 1800, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 1800 \qquad \left[\text{S.I. for 1 year} = \frac{\text{Principal} \times \text{Rate}}{100}\right]$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 1800$$
$$\Rightarrow 5x + 210000 - 7x = 180000$$
$$\Rightarrow 210000 - 2x = 180000$$
$$\Rightarrow 2x = 210000 - 180000$$
$$\Rightarrow 2x = 30000$$
$$\Rightarrow x = 15000$$

Thus, in order to obtain an annual total interest of Rs 1800, the trust fund should invest Rs 15000 in the first bond and the remaining Rs 15000 in the second bond.
(b) Let Rs x be invested in the first bond. Then, the sum of money invested in the second bond will be Rs $(30000-x)$.
Therefore, in order to obtain an annual total interest of Rs 2000, we have:

$$\begin{bmatrix} x & (30000-x) \end{bmatrix}\begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix} = 2000$$

$$\Rightarrow \frac{5x}{100} + \frac{7(30000-x)}{100} = 2000$$
$$\Rightarrow 5x + 210000 - 7x = 200000$$
$$\Rightarrow 210000 - 2x = 200000$$
$$\Rightarrow 2x = 210000 - 200000$$
$$\Rightarrow 2x = 10000$$
$$\Rightarrow x = 5000$$

Thus, in order to obtain an annual total interest of Rs 2000, the trust fund should invest Rs 5000 in the first bond and the remaining Rs 25000 in the second bond

Q94

To promote making of toilets for women, an organization tried to generate awareness through (i) house calls (ii) letters and (iii) announcements. The cost for each mode per attempt is given below:

i. Rs. 50
ii. Rs. 20
iii. Rs. 40

The number of attempts made in three villages X, Y, and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
z	500	400	150

Find the total cost incurred by the organization for three villages separately, using matrices.

Solution

The cost for each mode per attempt is represented by 3 x 1 matrix:

$$A = \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

The number of attempts made in the three villages X, Y, and Z are represented by a 3 x 3 matrix:

$$B = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix}$$

The total cost incurred by the prganization for the three villages seperately is given by matrix multiplication

$$BA = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix}$$

$$BA = \begin{bmatrix} 400 \times 50 + 300 \times 20 + 100 \times 40 \\ 300 \times 50 + 250 \times 20 + 75 \times 40 \\ 500 \times 50 + 400 \times 20 + 150 \times 40 \end{bmatrix}$$

$$= \begin{bmatrix} 30{,}000 \\ 23{,}000 \\ 39{,}000 \end{bmatrix}$$

Note: The answer given in the book is incorrect.

Q95

There are 2 families A and B. There are 4 men, 6 women and 2 children in family A, and 2 men, 2 women and 4 children in family B. The recommend daily amount of calories is 2400 for men, 1900 for women, 1800 for children and 45 grams of proteins for men, 55 grams for women and 33 grams for children. Represent the above information using matrix. Using matrix multiplication, Calculate the total requirement of calories and proteins for each of the families. What awareness can you create among people about the planned diet from this question?

Solution

Let F be the family matrix and R be the requirement matrix. Then,

$$F = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Men} & \text{Women} & \text{Children} \end{array} \\ \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ \text{Men} \\ \text{Women} \\ \text{Children} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Portein} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

The requirement of calories and protein of each of the two families is given by the product matrix FR, as matrix F has number of columns equal to number of rows of R thus ,

$$FR = \begin{bmatrix} 4 & 6 & 2 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$FR = \begin{bmatrix} 4 \times 2400 + 6 \times 1900 + 2 \times 1800 & 4 \times 45 + 6 \times 55 + 2 \times 33 \\ 2 \times 2400 + 2 \times 1900 + 4 \times 1800 & 2 \times 45 + 2 \times 55 + 4 \times 33 \end{bmatrix}$$

$$FR = \begin{array}{c} \\ \text{Family A} \\ \text{Family B} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Protein} \end{array} \\ \begin{bmatrix} 24600 & 576 \\ 15800 & 332 \end{bmatrix} \end{array}$$

we can say that balanced diet having the required amount of calories and protein must be taken by each of the family.

Q96

In a parliament election, a political party hired a public relations firm to promote its candidates in three ways - telephone, house calls and letters. The cost per contact (in paisa) is given in matrix A as

$$A = \begin{bmatrix} 140 \\ 200 \\ 150 \end{bmatrix} \begin{array}{l} \text{Telephone} \\ \text{House calls} \\ \text{Letters} \end{array}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{array}{c} \begin{array}{ccc} \text{Telephone} & \text{House calls} & \text{Letters} \end{array} \\ \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{City X} \\ \text{City Y} \end{array}$$

Find the total amount spent by the party in the two cities.
What should one consider before casting his/her vote - party's promotional activity or their social activities?

Solution

The cost per contact (in paisa) is given in matrix A as

$$A=\begin{array}{c}\begin{bmatrix}140\\200\\150\end{bmatrix}\end{array}\begin{array}{l}\text{Telephone}\\\text{House calls}\\\text{Letters}\end{array}$$

The number of contacts of each type made in two cities X and Y is given in the matrix B as

$$B = \begin{array}{c} \\ \text{City X} \\ \text{City Y}\end{array}\begin{array}{c}\begin{array}{ccc}\text{Telephone} & \text{House calls} & \text{Letters}\end{array}\\ \begin{bmatrix}1000 & 500 & 5000\\3000 & 1000 & 10000\end{bmatrix}\end{array}$$

The total amout of money spent by party in each of the city for the election is given by the matrix multiplication :

$$BA = \begin{bmatrix}1000 & 500 & 5000\\3000 & 1000 & 10000\end{bmatrix}\begin{bmatrix}140\\200\\150\end{bmatrix}$$

$$=\begin{bmatrix}1000\times140+500\times200+5000\times150\\3000\times140+1000\times200+10000\times150\end{bmatrix}$$

$$=\begin{array}{c}\text{City X}\\\text{City Y}\end{array}\begin{bmatrix}990000\\2120000\end{bmatrix}$$

The total amout of money spent by party in each of the city for the election in rupees is given by

$$=\left(\frac{1}{100}\right)\begin{array}{c}\text{City X}\\\text{City Y}\end{array}\begin{bmatrix}990000\\2120000\end{bmatrix}$$

$$=\begin{array}{c}\text{City X}\\\text{City Y}\end{array}\begin{bmatrix}9900\\21200\end{bmatrix}$$

One sould consider social activities before casting his/her vote to the party.