# RD Sharma Solutions

# Class 12 Maths

# Chapter 9

# Ex 9.1

We have to check the continuity of function at $x = 0$.

L.H.L $= \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{-h}{|-h|} = \lim_{h \to 0} \frac{-h}{h} = -1$

R.H.L $= \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h}{|h|} = 1$

Thus, LHL $\neq$ R.H.L
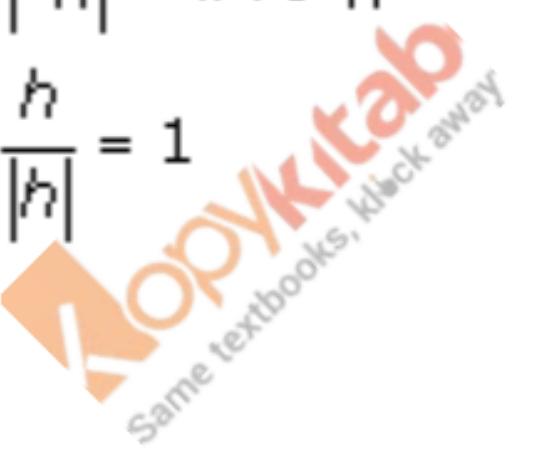
So, the given function in discontinuous and the discontinuity is of first kind.

**Continuity Ex 9.1 Q2**

We have, to check the continuity at $x = 3$.

L.H.L $= \lim\limits_{x \to 3^-} f(x) = \lim\limits_{h \to 0} f(3-h) = \lim\limits_{h \to 0} \dfrac{(3-h)^2 - (3-h) - 6}{(3-h) - 3} = \lim\limits_{h \to 0} \dfrac{h^2 - 5h}{-h} = \lim\limits_{h \to 0} -h + 5 = 5$

R.H.L $= \lim\limits_{x \to 3^+} f(x) = \lim\limits_{h \to 0} f(3+h) = \lim\limits_{h \to 0} \dfrac{(3+h)^2 - (3+h) - 6}{(3+h) - 3} = \lim\limits_{h \to 0} \dfrac{h^2 + 5h}{h} = \lim\limits_{h \to 0} h + 5 = 5$

$f(3) = 5$

Thus, we have, LHL $=$ RHL $= f(3) = 5$

So, The function is continuous at $x = 3$

**Continuity Ex 9.1 Q3**

We have, to check the continuity of the function at $x = 3$.

LHL $= \lim\limits_{x \to 3^-} f(x) = \lim\limits_{h \to 0} f(3-h) = \lim\limits_{h \to 0} \dfrac{(3-h)^2 - 9}{(3-h) - 3} = \lim\limits_{h \to 0} \dfrac{h^2 - 6h}{-h} = \lim\limits_{h \to 0} -h + 6 = 6$

RHL $= \lim\limits_{x \to 3^+} f(x) = \lim\limits_{h \to 0} f(3+h) = \lim\limits_{h \to 0} \dfrac{(3+h)^2 - 9}{(3+h) - 3} = \lim\limits_{h \to 0} \dfrac{h^2 + 6h}{h} = \lim\limits_{h \to 0} h + 6 = 6$

$f(3) = 6$

Thus, we have, LHL $=$ RHL $= f(3) = 6$

So, the given function is continuous at $x = 3$.

**Continuity Ex 9.1 Q4**

We want, to check the continuity of the function at $x = 1$.

LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} \dfrac{(1-h)^2 - 1}{(1-h) - 1} = \lim\limits_{h \to 0} \dfrac{h^2 - 2h}{-h} = \lim\limits_{h \to 0} -h + 2 = 2$

RHL $= \lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) = \lim\limits_{h \to 0} \dfrac{(1+h)^2 - 1}{(1+h) - 1} = \lim\limits_{h \to 0} \dfrac{h^2 + 2h}{h} = \lim\limits_{h \to 0} h + 2 = 2$

$f(1) = 2$

we find that LHL $=$ RHL $= f(1) = 2$

Hence, $f(x)$ is continuous at $x = 1$.

**Continuity Ex 9.1 Q5**

We have, to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin 3(-h)}{-h} = \lim_{h \to 0} \frac{\sin 3h}{-h} = 3$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{\sin 3h}{h} = 3$$

$$f(0) = 1$$

$\text{LHL} = \text{RHL} \neq f(0)$

$\Rightarrow$ Function is discontinuous at $x = 0$. It is removable discontinuty.

**Continuity Ex 9.1 Q6**

We have, to check the continuity of the function at $x = 0$.

$$\text{L.H.L} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} e^{1/-h} = e^{-\infty} = 0$$

$$\text{R.H.L} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} e^{1/h} = e^{\infty} = \infty$$

So, $\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 0$. This is discontinuty of I$^{st}$ kind.

**Continuity Ex 9.1 Q7**

We want, to check the continuity of the given function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{1-\cos(-h)}{(-h)^2}$$

$$= \lim_{h \to 0} \frac{1-\cos h}{h^2} \left[\because \cos(-\theta) = \cos\theta\right]$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{h^2} \left[\because 1-\cos\theta = 2\sin^2 \frac{\theta}{2}\right]$$

$$= \lim_{h \to 0} 2 \cdot \left(\frac{\sin \frac{h}{2}}{h}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{1-\cos h}{h^2} = \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}}{h^2} = \lim_{h \to 0} 2\left(\frac{\sin^2 \frac{h}{2}}{h}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

$$f(0) = 1$$

$\text{LHL} = \text{RHL} \neq f(0)$

Hence, the function is discontinuous at $x = 0$

This is removable discontinuty.

**Continuity Ex 9.1 Q8**

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{-h - |-h|}{2} = \lim_{h \to 0} \frac{-h - h}{2} = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{h - (|h|)}{2} = 0$$

$$f(0) = 2$$

Thus, LHL = RHL $\neq f(0)$

Hence, The function is discontinuous at $x = 0$

This is removable discontinuty.

**Continuity Ex 9.1 Q9**

We want, to check the continuty of the function at $x = a$.

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a - h) = \lim_{h \to 0} \frac{|a - h - a|}{a - h - a} = \lim_{h \to 0} \frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a + h) = \lim_{h \to 0} \frac{|a + h - a|}{a + h - a} = \lim_{h \to 0} \frac{h}{h} = 1$$

Thus, LHL $\neq$ RHL

Hence, function is discontinuous at $x = a$. And the discontinuty is of first kind.

**Continuity Ex 9.1 Q10(i)**

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} |-h| \cos\left(\frac{1}{-h}\right) = \lim_{h \to 0} h \cos\left(\frac{1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} |h| \cos\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, LHL = RHL = $f(0) = 0$

Hence, function is continuous at $x = 0$.

**Continuity Ex 9.1 Q10(ii)**

We want, to check the continuity at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} (-h)^2 \sin\left(\frac{1}{-h}\right) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} h^2 \sin\left(\frac{1}{h}\right) = 0$$

$$f(0) = 0$$

Thus, LHL = RHL = $f(0) = 0$

Hence, the function is continuous at $x = 0$.

**Continuity Ex 9.1 Q10(iii)**

We want, to check the continuity of the function at $x = a$.

$$\text{LHL} = \lim_{x \to a^-} f(x) = \lim_{h \to 0} f(a - h) = \lim_{h \to 0} (a - h - a) \sin\left(\frac{1}{a - h - a}\right) = \lim_{h \to 0} -h \sin\left(\frac{-1}{h}\right) = 0$$

$$\text{RHL} = \lim_{x \to a^+} f(x) = \lim_{h \to 0} f(a + h) = \lim_{h \to 0} (a + h - a) \sin\left(\frac{1}{a + h - a}\right) = \lim_{h \to 0} h \sin\left(\frac{1}{h}\right) = 0$$

$$f(a) = 0$$

Thus, LHL $\neq$ RHL = $f(a) = 0$

Hence, the function is continuous at $x = a$.

**Continuity Ex 9.1 Q10(iv)**

We want, to check the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{e^{-h} - 1}{\log(1 + 2(-h))} = \lim_{h \to 0} \frac{e^{-h} - 1}{\log(1 - 2h)} = DNE$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{e^{h} - 1}{\log(1 + 2h)} = DNE$$

Thus, Both LHL and RHL do not exist

∴ Function is discontinuous and the discontinutiy is of II$^{nd}$ kind.

**Continuity Ex 9.1 Q10(v)**

We want, to check the continuity at $x = 1$

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{1-(1-h)^n}{1-(1-h)} = \lim_{h \to 0} \frac{1-\left[1-nh+\dfrac{n(n-1)}{2!}h^2+\ldots\right]}{h}$$

$$= \lim_{h \to 0} n - \frac{n(n-1)}{2!}h + \ldots$$

$$= n$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{1-(1+h)^n}{1-(1+h)} = \lim_{h \to 0} \frac{1-\left[1+nh+\dfrac{n(n-1)}{2!}h^2+\ldots\right]}{-h}$$

$$= \lim_{h \to 0} n + \frac{n(n-1)}{2!}h + \ldots$$

$$= n$$

$f(1) = n - 1$

Thus, LHL = RHL $\neq f(1)$

Hence, function is discontinuous at $x = 1$

This is removable discontinuity.

**Continuity Ex 9.1 Q10(vi)**

We want, to check the continuity at $x = 1$.

$$\text{LHL} = \lim_{x \to 1^-} f(x) = \lim_{h \to 0} f(1-h) = \lim_{h \to 0} \frac{\left|(1-h)^2-1\right|}{(1-h)-1} = \lim_{h \to 0} \frac{\left|h^2-2h\right|}{-h} = \lim_{h \to 0} -(h-2) = 2$$

$$\text{RHL} = \lim_{x \to 1^+} f(x) = \lim_{h \to 0} f(1+h) = \lim_{h \to 0} \frac{\left|(1+h)^2-1\right|}{1+h-1} = \lim_{h \to 0} \frac{h^2+2h}{h} = 2$$

$f(1) = 2$

$\therefore$     LHL = RHL $= f(1) = 2$

Hence, function is continuous.

**Continuity Ex 9.1 Q10(vii)**

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{2(|-h|)+(-h)^2}{-h} = \lim_{h \to 0} \frac{2h+h^2}{-h} = -2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2\times|h|+h^2}{h} = 2$$

Thus, LHL $\neq$ RHL

Function is not continuous at $x = 0$

This is discontinuity of $\text{I}^{\text{st}}$ kind.

**Continuity Ex 9.1 Q11**

We want to check the continuity at $x = 1$.

$\text{LHL} = \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} 1 + (1-h)^2 = \lim\limits_{h \to 0} 1 + 1 - 2h + h^2 = 2$

$\text{RHL} = \lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) = \lim\limits_{h \to 0} 2 - (1+h) = 1$

$\text{LHL} \neq \text{RHL}$

Hence, the function is discontinuous at $x = 1$

This is discontinuity of $\text{I}^{\text{st}}$ kind.

**Continuity Ex 9.1 Q12**

We want to check the continuity at $x = 0$.

$\text{LHL} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\sin(3 \times (-h))}{\tan(2 \times (-h))} = \lim\limits_{h \to 0} \dfrac{-\sin 3h}{-\tan 2h} = \lim\limits_{h \to 0} \dfrac{\dfrac{\sin 3h}{3h} \times 3h}{\dfrac{\tan 2h}{2h} \times 2h} = \dfrac{3}{2}$

$\text{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} \dfrac{\log(1+3h)}{e^{2h} - 1} = \lim\limits_{h \to 0} \dfrac{\dfrac{\log(1+3h)}{3h} \times 3h}{\dfrac{e^{2h} - 1}{2h} \times 2h} = \dfrac{3}{2}$

$f(0) = \dfrac{3}{2}$

Thus, $\text{LHL} = \text{RHL} = f(0) = \dfrac{3}{2}$

Hence, the function is continuous at $x = 0$

**Continuity Ex 9.1 Q13**

We want to check the continuity of the function at $x = 0$.

$\text{LHL} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} 2(-h) - |-h| = \lim\limits_{h \to 0} -2h - h = 0$

$\text{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} 2h - |h| = 0$

$f(0) = 0$

Thus, $\text{LHL} = \text{RHL} = f(0) = 0$

Hence, the function is continuous at $x = 0$

**Continuity Ex 9.1 Q14**

We want to check the continuty of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} 3(-h) - 2 = \lim_{h \to 0} -3h - 2 = -2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h + 1 = 1 = 0$$

LHL $\neq$ RHL

So, the function is discontinuous
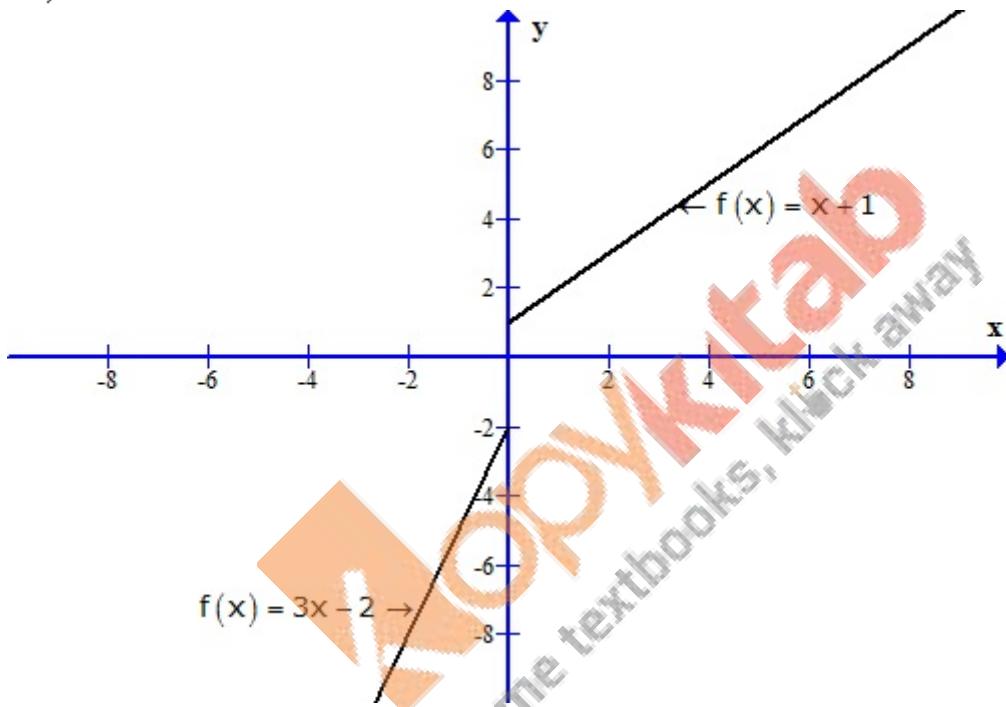


**Continuity Ex 9.1 Q15**

We want to discuss the continuity of the function at $x = 0$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} -(-h) = 0$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h = 0$$

$$f(0) = 1$$

Thus, LHL = RHL $\neq f(0)$

Hence, the function is discontinuous at $x = 0$. And this is removable discontinuity.

**Continuity Ex 9.1 Q16**

We want to discuss the continuity of the function at $x = \frac{1}{2}$.

LHL $= \lim\limits_{x \to \frac{1}{2}^-} f(x) = \lim\limits_{h \to 0} f\left(\frac{1}{2} - h\right) = \lim\limits_{h \to 0} \frac{1}{2} - h = \frac{1}{2}$

RHL $= \lim\limits_{x \to \left(\frac{1}{2}\right)^+} = \lim\limits_{h \to 0} f\left(\frac{1}{2} + h\right) = \lim\limits_{h \to 0} 1 - \left(\frac{1}{2} + h\right) = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \frac{1}{2}$

Thus, LHL = RHL $= f\left(\frac{1}{2}\right) = \frac{1}{2}$

Hence, the function is continuous at $x = \frac{1}{2}$.

**Continuity Ex 9.1 Q17**

We want to check the continuity of the function at $x = 0$.

LHL $= \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0 - h) = \lim\limits_{h \to 0} 2(-h) - 1 = -1$

RHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h) = \lim\limits_{h \to 0} 2h + 1 = 1$

Thus, LHL $\neq$ RHL

Hence, the function is discontinuous at $x = 0$. This is discontinuity of I$^{st}$ kind.

**Continuity Ex 9.1 Q18**

We have given that the function is continuous at $x = 1$

LHL = RHL $= f(1)$ ..... (1)

Now, LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} \frac{(1-h)^2 - 1}{(1-h) - 1} = \lim\limits_{h \to 0} \frac{h^2 - 2h}{-h} = 2$

$f(1) = k$

From (1), LHL $= f(1)$

$\therefore 2 = k$

**Continuity Ex 9.1 Q19**

We have that the function is continuous at $x = 1$

$\therefore$ LHL = RHL $= f(1)$ .... (1)

Now,

LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} \frac{(1-h)^2 - 3(1-h) + 2}{(1-h) - 1} = \lim\limits_{h \to 0} \frac{h^2 + h}{-h} = \lim\limits_{h \to 0} -h - 1 = -1$

$f(1) = k$

From (1), we get,

$k = -1$

**Continuity Ex 9.1 Q20**

We know that a function is continuous at 0 if

LHL = RHL = $f(0)$ ....(1)

Now,

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\sin 5(-h)}{3(-h)} = \lim\limits_{h \to 0} \dfrac{-\sin 5h}{-3h} = \lim\limits_{h \to 0} \dfrac{\sin 5h}{5h} \times \dfrac{5h}{3h} = \dfrac{5}{3}$

$f(0) = k$

Thus, from (1),

$k = \dfrac{5}{3}$

**Continuity Ex 9.1 Q21**

The given function is $f(x) = \begin{cases} kx^2, & \text{if } x \le 2 \\ 3, & \text{if } x > 2 \end{cases}$

The given function $f$ is continuous at $x = 2$, if $f$ is defined at $x = 2$ and if the value of $f$ at $x = 2$ equals the limit of $f$ at $x = 2$

It is evident that $f$ is defined at $x = 2$ and $f(2) = k(2)^2 = 4k$

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$

$\Rightarrow \lim\limits_{x \to 2^-} (kx^2) = \lim\limits_{x \to 2^+} (3) = 4k$

$\Rightarrow k \times 2^2 = 3 = 4k$

$\Rightarrow 4k = 3 = 4k$

$\Rightarrow 4k = 3$

$\Rightarrow k = \dfrac{3}{4}$

Therefore, the required value of $k$ is $\dfrac{3}{4}$.

**Continuity Ex 9.1 Q22**

We have given that the function is continuous at $x = 0$

So, LHL = RHL = $f(0)$ .....(1)

Now,

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\sin 2(-h)}{5(-h)} = \lim\limits_{h \to 0} \dfrac{-\sin 2h}{-5h} = \lim\limits_{h \to 0} \dfrac{\sin 2h}{2h} \times \dfrac{2h}{5h} = \dfrac{2}{5}$

$f(0) = k$

Using (1), $k = \dfrac{2}{5}$

**Continuity Ex 9.1 Q23**

We have given that the function is continuous at $x = 2$

$\text{LHL} = \text{RHL} = f(2) \ldots \ldots (1)$

Now,

$\text{LHL} = \lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2-h) = \lim\limits_{h \to 0} a(2-h) + 5 = 2a + 5$

$f(2) = 2a + 5$

$\text{RHL} = \lim\limits_{x \to 2^+} f(x) = \lim\limits_{h \to 0} f(2+h) = \lim\limits_{h \to 0} 2 + h - 1 = 1$

$\therefore$ Using $(1)$,

$2a + 5 = 1 \Rightarrow a = -2$

**Continuity Ex 9.1 Q24**

We have, at $x = 0$

$\text{LHL} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{-h}{|-h| + 2(-h)^2} = \lim\limits_{h \to 0} \dfrac{-h}{h + 2h^2} = \lim\limits_{h \to 0} \dfrac{-1}{1 + 2h} = -1$

$f(0) = k$

$\text{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} \dfrac{h}{|h| + 2h^2} = \lim\limits_{h \to 0} \dfrac{1}{1 + 2h} = 1$

Since, $\text{LHL} \neq \text{RHL}$, function will remain discontinuous at $x = 0$, regardless the choice of $k$.

**Continuity Ex 9.1 Q25**

*Since $f(x)$ is continuous at $x = \dfrac{\pi}{2}$, L.H.Limit = R.H.Limit.*

$\Rightarrow \lim\limits_{x \to \frac{\pi}{2}^-} f(x) = \lim\limits_{x \to \frac{\pi}{2}^+} f(x) = \lim\limits_{x \to \frac{\pi}{2}} f(x) = f\left(\dfrac{\pi}{2}\right)$

$\Rightarrow \lim\limits_{x \to \frac{\pi}{2}^-} \dfrac{k\cos x}{\pi - 2x} = 3$

$\Rightarrow k \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin\left(\dfrac{\pi}{2} - x\right)}{2\left(\dfrac{\pi}{2} - x\right)} = 3$

$\Rightarrow \dfrac{k}{2} \lim\limits_{x \to \frac{\pi}{2}} \dfrac{\sin\left(\dfrac{\pi}{2} - x\right)}{\left(\dfrac{\pi}{2} - x\right)} = 3$

$\Rightarrow \dfrac{k}{2} = 3$

$\Rightarrow k = 6$

**Continuity Ex 9.1 Q26**

We have given that the function is continuous at $x = 0$

$$\text{LHL} = \text{RHL} = f(0) \ldots (1)$$

$$f(0) = c$$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin(a+1)(-h) + \sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin(ah+h) - \sinh}{-h}$$

$$= \lim_{h \to 0} \frac{\sin(a+h)h}{h} + \lim_{h \to 0} \frac{\sinh}{h}$$

$$= a + 1 + 1 = a + 2$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}}$$

$$= \lim_{h \to 0} \frac{\sqrt{h+bh^2} - \sqrt{h}}{bh^{\frac{3}{2}}} \times \frac{\sqrt{h+bh^2} + \sqrt{h}}{\sqrt{h+bh^2} + \sqrt{h}}$$

$$= \lim_{h \to 0} \frac{h+bh^2 - h}{bh^{\frac{3}{2}}\left(\sqrt{h+bh^2} + \sqrt{h}\right)} = \lim_{h \to 0} \frac{bh^2}{bh^2\left(\sqrt{1+bh} + 1\right)} = \frac{1}{2}$$

$\therefore$ from $(1)$,

$$a + 2 = \frac{1}{2} \Rightarrow a = \frac{-3}{2}$$

$$c = \frac{1}{2} \quad \text{and}$$

$$b \in R - \{0\}$$

Hence, $a = \dfrac{-3}{2}$, $b \in R - \{0\}$, $c = \dfrac{1}{2}$

**Continuity Ex 9.1 Q27**

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$.....(1)

$f(0) = \dfrac{1}{2}$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{1-\cos k(-h)}{-h\sin(-h)} = \lim\limits_{h \to 0} \dfrac{1-\cos kh}{+h\sinh}$

$= \lim\limits_{h \to 0} \dfrac{2\sin^2 \dfrac{kh}{2}}{h.\,2\sin \dfrac{h}{2}.\cos \dfrac{h}{2}}$

$= \lim\limits_{h \to 0} \left(\dfrac{\sin \dfrac{kh}{2}}{\dfrac{kh}{2}}\right)^2 \times \dfrac{\dfrac{k^2h^2}{4}}{\dfrac{\sin \dfrac{h}{2}}{\dfrac{h}{2}} \times \dfrac{h}{2}} \cdot \dfrac{1}{h}$

$= \lim\limits_{h \to 0} \left(\dfrac{\sin \dfrac{kh}{2}}{\dfrac{kh}{2}}\right)^2 \cdot \dfrac{\dfrac{k^2}{4}}{\dfrac{\sin \dfrac{h}{2}}{\dfrac{h}{2}} \cdot \dfrac{1}{2}}$

$= \dfrac{k^2}{2}$

$\therefore$ Using (1) we get,

$\dfrac{k^2}{2} = \dfrac{1}{2} \Rightarrow k = \pm 1$

**Continuity Ex 9.1 Q28**

We have given that the function is continuous at $x = 4$

$\therefore$ LHL = RHL = $f(4)$ .... (1)

$f(4) = a + b$ ..... (A)

LHL = $\lim\limits_{x \to 4^-} f(x) = \lim\limits_{h \to 0} f(4-h) = \lim\limits_{h \to 0} \dfrac{(4-h)-4}{|(4-h)-4|} + a = \lim\limits_{h \to 0} \dfrac{-h}{h} + a = a - 1$ .... (B)

RHL = $\lim\limits_{x \to 4^+} f(x) = \lim\limits_{h \to 0} f(4+h) = \lim\limits_{h \to 0} \dfrac{(4+h)-4}{|(4+h)-4|} + b = \lim\limits_{h \to 0} \dfrac{h}{h} + b = b + 1$ .... (C)

$\therefore$ from (1)

$a - 1 = b + 1 \Rightarrow a - b = 2$ .....(D)

from (A) and (B)

$a + b = a - 1 \Rightarrow b = -1$

from (A) and (C)

$a + b = b + 1 \Rightarrow a = 1$

Thus, $a = 1$ and $b = -1$

**Continuity Ex 9.1 Q29**

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$ ..... (1)

$f(0) = k$

LHL = $\lim\limits_{x \to 0} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\sin 2(0-h)}{-h} = \lim\limits_{h \to 0} \dfrac{-\sin 2h}{-h} = 2$

$\therefore$ using (1), we get $k = 2$

**Continuity Ex 9.1 Q30**

We know that a function is continuous at $x = 0$ if,

LHL = RHL = $f(0)$ .... (1)

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\log\left(1 - \dfrac{h}{a}\right) - \log\left(1 + \dfrac{h}{b}\right)}{(-h)} = \lim\limits_{h \to 0} \dfrac{\log\left(1 + \left(\dfrac{-h}{a}\right)\right)}{\left(\dfrac{-h}{a}\right) \times a} + \dfrac{\log\left(1 + \dfrac{h}{b}\right)}{h}$

$= \dfrac{1}{a} + \dfrac{1}{b} = \dfrac{a+b}{ab}$

from (1),

$f(0) = \dfrac{a+b}{ab}$

**Continuity Ex 9.1 Q31**

We are given that the function is continuous at $x = 2$

$\therefore$ LHL = RHL = $f(2)$     .....(1)

Now,

$f(2) = k$     ....(A)

$$\text{LHL} = \lim_{x \to 2} f(x) = \lim_{h \to 0} f(2-h) = \lim_{h \to 0} \frac{2^{(2-h)+2} - 16}{4^{(2-h)} - 16} = \lim_{h \to 0} \frac{2^{4-h} - 16}{4^{2-h} - 16}$$

$$= \lim_{h \to 0} \frac{2^4 . 2^{-h} - 16}{4^2 . 4^{-h} - 16}$$

$$= \lim_{h \to 0} \frac{16 . 2^{-h} - 16}{16 . 4^{-h} - 16}$$

$$= \lim_{h \to 0} \frac{16\left(2^{-h} - 1\right)}{16\left(4^{-h} - 1\right)}$$

$$= \lim_{h \to 0} \frac{2^{-h} - 1}{\left(2^{-h}\right) - 1^2} \quad \left[\because 2^{-2h} = \left(2^{-h}\right)^2 = 4^{-h}\right]$$

$$= \lim_{h \to 0} \frac{2^{-h} - 1}{\left(2^{-h} - 1\right)\left(2^{-h} + 1\right)} = \frac{1}{2} \quad ......(B)$$

$\therefore$ Using (1) from (A) & (B)

$$k = \frac{1}{2}$$

**Continuity Ex 9.1 Q33**

We know that a function is said to be continuous at $x = \pi$ if

LHL = RHL = value of the function at $x = \pi$.....(1)

$$\text{LHL} = \lim_{x \to \pi} f(x) = \lim_{h \to 0} f(\pi - h) = \lim_{h \to 0} \frac{1 - \cos 7(\pi - h - \pi)}{5\left((\pi - h) - \pi\right)^2} = \lim_{h \to 0} \frac{1 - \cos 7h}{5h^2}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{7}{2}h}{5h^2}$$

$$= \lim_{h \to 0} \frac{2}{5}\left(\frac{\sin \frac{7}{2}h}{\frac{7}{2}h}\right)^2 \times \left(\frac{7}{2}\right)^2$$

$$= \frac{2}{5} \times \frac{49}{4} = \frac{49}{10} \quad ....(B)$$

Thus, using (1) we get,

$$f(\pi) = \frac{49}{10}$$

**Continuity Ex 9.1 Q34**

It is given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0) \ldots (1)$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{2(-h) + 3\sin(-h)}{3(-h) + 2\sin(-h)} = \lim\limits_{h \to 0} \dfrac{-2h - 3\sin h}{-3h - 2\sin h}$

$$= \lim\limits_{h \to 0} \dfrac{\dfrac{2h + 3\sin h}{h}}{\dfrac{3h + 2\sin h}{h}}$$

$$= \lim\limits_{h \to 0} \dfrac{2 + 3\dfrac{\sin h}{h}}{3 + 2\dfrac{\sin h}{h}} = \dfrac{2+3}{3+2} = 1 \qquad \left[ \because \lim\limits_{\theta \to 0} \dfrac{\sin \theta}{\theta} = 1 \right]$$

Using (1) we get,

$f(0) = 1$

**Continuity Ex 9.1 Q35**

It is given that the function is continuous at $x = 0$.

LHL = RHL = $f(0) \ldots (1)$

$f(0) = k \ldots (A)$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{1 - \cos 4(-h)}{8(-h)^2} = \lim\limits_{h \to 0} \dfrac{1 - \cos 4h}{8h^2} = \lim\limits_{h \to 0} \dfrac{2\sin^2 2h}{8h^2} = \lim\limits_{h \to 0} \left( \dfrac{\sin 2h}{2h} \right)^2 = 1$

Thus, using (1) we get,

$k = 1$

**Continuity Ex 9.1 Q36**

The given function will be continuous at $x = 0$ if

$LHL = RHL = f(0)\ldots(1)$

$f(0) = 8\ldots(A)$

$LHL = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \dfrac{1 - \cos 2k\,(-h)}{(-h)^2} = \lim_{h \to 0} \dfrac{1 - \cos 2kh}{h^2} = \lim_{h \to 0} \dfrac{2\sin^2 kh}{h^2}$

$$= \lim_{h \to 0} 2\left(\dfrac{\sin kh}{kh}\right)^2 . k^2$$

$$= 2k^2$$

Thus, using (1) we get,

$2k^2 = 8 \implies k^2 = 4 \implies k = \pm 2$

Hence, $k = \pm 2$

Let $x - 1 = y$

$\implies x = y + 1$

Thus,

$$\lim_{x \to 1}(x-1)\tan\dfrac{\pi x}{2} = \lim_{y \to 0} y\tan\dfrac{\pi(y+1)}{2}$$

$$= \lim_{y \to 0} y\tan\left(\dfrac{\pi y}{2} + \dfrac{\pi}{2}\right)$$

$$= -\lim_{y \to 0} y\cot\dfrac{\pi y}{2}$$

$$= -\lim_{y \to 0} y\dfrac{\cos\dfrac{\pi y}{2}}{\sin\dfrac{\pi y}{2}}$$

$$= -\lim_{y \to 0} y\dfrac{\cos\dfrac{\pi y}{2}}{\dfrac{\left(\sin\dfrac{\pi y}{2}\right)\dfrac{\pi}{2}}{\dfrac{\pi}{2}}}$$

$$= -\lim_{y \to 0} \dfrac{\cos\dfrac{\pi y}{2}}{\dfrac{\left(\sin\dfrac{\pi y}{2}\right)\dfrac{\pi}{2}}{\dfrac{\pi y}{2}}}$$

$$= -\lim_{y \to 0} \dfrac{2}{\pi}\dfrac{\cos\dfrac{\pi y}{2}}{\dfrac{\left(\sin\dfrac{\pi y}{2}\right)}{\dfrac{\pi y}{2}}}$$

$$= -\dfrac{2}{\pi}\lim_{y \to 0}\cos\dfrac{\pi y}{2}$$

$$= -\dfrac{2}{\pi}$$

*Since the function is continuous, L.H.Limit = R.H.Limit*

$$Thus, k = -\frac{2}{\pi}$$

Since the function is continuous at every point, therefore
$$LHL = RHL = f(0)$$
Now
$$f(0) = \cos 0$$
$$= 1$$
Again
$$LHL = \lim_{x \to 0} k(x^2 - 2x)$$
$$= \lim_{h \to 0^+} k(h^2 - 2h)$$
$$= 0$$
Therefore there is no value of $k$
Since the function is continuous at every point, therefore
$$LHL = RHL = f(\pi)$$
Now
$$f(\pi) = k\pi + 1$$
Again
$$RHL = \lim_{x \to \pi^+} \cos x$$
$$= \lim_{h \to 0^+} \cos(\pi - h)$$
$$= -\lim_{h \to 0^+} \cos h$$
$$= -1$$
Therefore we can write
$$k\pi + 1 = -1$$
$$k = -\frac{2}{\pi}$$

We are given that function is continuous at $x = 5$.
$$\therefore LHL = RHL = f(5)....(1)$$

$$f(5) = 5k + 1$$

$$LHL = \lim_{x \to 5^+} f(x) = \lim_{h \to 0} f(5+h) = \lim_{h \to 0} 3(5+h) - 5 = 10$$

Thus, using (1), we get,
$$5k + 1 = 10$$
$$5k = 9$$
$$k = \frac{9}{5}$$

We know that the function will be continuous at $x = 5$. if

LHL = RHL = $f(5)$ ..... (1)

$f(5) = k$

LHL = $\lim\limits_{x \to 5^-} f(x) = \lim\limits_{h \to 0} f(5-h) = \lim\limits_{h \to 0} \dfrac{(5-h)^2 - 25}{(5-h) - 5} = \lim\limits_{h \to 0} \dfrac{h^2 - 10h}{-h} = \lim\limits_{h \to 0} -h + 10 = 10$

Thus, using (1), we get,

$k = 10$

We know that a function will be continuous at $x = 1$. if

LHL = RHL = $f(1)$ .... (1)

$f(1) = k.1^2 = k$

LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} 4 = 4$

Thus, using (1), we get,

$k = 4$

We know that a function will be continuous at $x = 0$. if

LHL = RHL = $f(0)$ ..... (1)

$f(0) = k(0+2) = 2k$

LHL = $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} 3(h) + 1 = 1$

Thus, using (1), we get,

$2k = 1$

$k = \dfrac{1}{2}$

**Continuity Ex 9.1 Q37**

It is given that the function is continuous at $x = 3$ and at $x = 5$

$\therefore$ LHL = RHL = $f(3)$ ..... (1) and

LHL = RHL = $f(5)$ ..... (2)

Now,

$f(3) = 1$

RHL = $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{h \to 0} f(3 + h) = \lim\limits_{h \to 0} a(3 + h) + b = 3a + b$

Thus, using (1), we get,

$3a + b = 1$ .... (3)

$f(5) = 7$

LHL = $\lim\limits_{x \to 5^-} f(x) = \lim\limits_{h \to 0} f(5 - h) = \lim\limits_{h \to 0} a(5 - h) + b = 5a + b$

Thus, using (2), we get

$5a + b = 7$ ..... (4)

Now, solving (3) and (4) we get,

$a = 3$ and $b = -8$

**Continuity Ex 9.1 Q38**

We want to discuss the continuity of the function at $x = 1$

We need to prove that

LHL = RHL = $f(1)$

$f(1) = \dfrac{1^2}{2} = \dfrac{1}{2}$

LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1 - h) = \lim\limits_{h \to 0} \dfrac{(1 - h)^2}{2} = \dfrac{1}{2}$

RHL = $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1 + h) = \lim\limits_{h \to 0} 2(1 + h)^2 - 3(1 + h) + \dfrac{3}{2} = 2 - 3 + \dfrac{3}{2} = \dfrac{1}{2}$

Thus, LHL = RHL = $f(1) = \dfrac{1}{2}$

Hence, function is continuous at $x = 1$

**Continuity Ex 9.1 Q39**

We want to discuss the continuity at $x = 0$ and $x = 1$

Now,

$f(0) = 1$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} |-h| + |-h-1| = 1$.

RHL = $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} |h| + |h-1| = 1$

$\because$ LHL = RHL = $f(0) = 1$, function is continuous at $x = 0$.

For $x = 1$.

$f(1) = 1$

LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} |1-h| + |1-h-1| = 1$

RHL = $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) = \lim\limits_{h \to 0} |1+h| + |1+h-1| = 1$

$\because$ LHL = RHL = $f(1) = 1$ function is continuous at $x = 1$.

For $x = -1$

$f(-1) = |-1-1| + |-1+1| = 2$

LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(-1-h) = \lim\limits_{h \to 0} |-1-h-1| + |-1-h+1| = 2$

RHL = $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(-1+h) = \lim\limits_{h \to 0} |-1+h-1| + |-1+h+1| = 2$

Thus, LHL = RHL = $f(-1) = 2$

Hence, function is continuous at $x = -1$

For $x = 1$

$f(1) = |1-1| + |1+1| = 2$

LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} |1-h-1| + |1-h+1| = 2$

RHL = $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) = \lim\limits_{h \to 0} |1+h-1| + |1+h+1| = 2$

Thus, LHL = RHL = $f(1) = 2$

Hence, function is continuous at $x = 1$

**Continuity Ex 9.1 Q40**

*Since $f(x)$ is continuous at $x = 0$, L.H.Limit = R.H.Limit.*

*Thus, we have*

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$$

$$\Rightarrow \lim_{x \to 0^-} a\sin\frac{\pi}{2}(x+1) = \lim_{x \to 0^+} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a \times 1 = \lim_{x \to 0} \frac{\tan x - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\dfrac{\sin x}{\cos x} - \sin x}{x^3}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\dfrac{\sin x}{x}\left(\dfrac{1}{\cos x} - 1\right)}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\dfrac{\sin x}{x}\left(\dfrac{1-\cos x}{\cos x}\right)}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin x}{x} \times \lim_{x \to 0} \frac{1}{\cos x} \times \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$\Rightarrow a = 1 \times 1 \times \lim_{x \to 0} \frac{1-\cos x}{x^2}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{1-\cos x}{x^2} \times \frac{1+\cos x}{1+\cos x}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{1-\cos^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin^2 x}{x^2(1+\cos x)}$$

$$\Rightarrow a = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \lim_{x \to 0} \frac{1}{1+\cos x}$$

$$\Rightarrow a = 1 \times \lim_{x \to 0} \frac{1}{1+\cos x}$$

$$\Rightarrow a = 1 \times \frac{1}{1+1}$$

$$\Rightarrow a = \frac{1}{2}$$

**Continuity Ex 9.1 Q41**

It is given that function is continuous at $x = 0$. then,

LHL = RHL = $f(0)$ .... (1)

Now,

$f(0) = 2 \cdot 0 + k = k$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} -2(-h)^2 + k = k$

RHL = $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} 2(h^2) + k = k$

Thus, the function will be continuous for any $k \in R$.

**Continuity Ex 9.1 Q42**

The given function $f$ is $f(x) = \begin{cases} \lambda(x^2 - 2x), & \text{if } x \leq 0 \\ 4x + 1, & \text{if } x > 0 \end{cases}$

If $f$ is continuous at $x = 0$, then

$\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = f(0)$

$\Rightarrow \lim\limits_{x \to 0^-} \lambda(x^2 - 2x) = \lim\limits_{x \to 0^+} (4x + 1) = \lambda(0^2 - 2 \times 0)$

$\Rightarrow \lambda(0^2 - 2 \times 0) = 4 \times 0 + 1 = 0$

$\Rightarrow 0 = 1 = 0$, which is not possible

Therefore, there is no value of $\lambda$ for which $f$ is continuous at $x = 0$

At $x = 1$,

$f(1) = 4x + 1 = 4 \times 1 + 1 = 5$

$\lim\limits_{x \to 1} (4x + 1) = 4 \times 1 + 1 = 5$

$\therefore \lim\limits_{x \to 1} f(x) = f(1)$

Therefore, for any values of $\lambda$, $f$ is continuous at $x = 1$

### Continuity Ex 9.1 Q43

The function will be continuous at $x = 2$
if $LHL = RHL = f(2)$ ....(1)

Now,

$f(2) = k$

$LHL = \lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2 - h) = \lim\limits_{h \to 0} 2(2 - h) + 1 = 5.$

Thus, using (1) we get,

$k = 5$
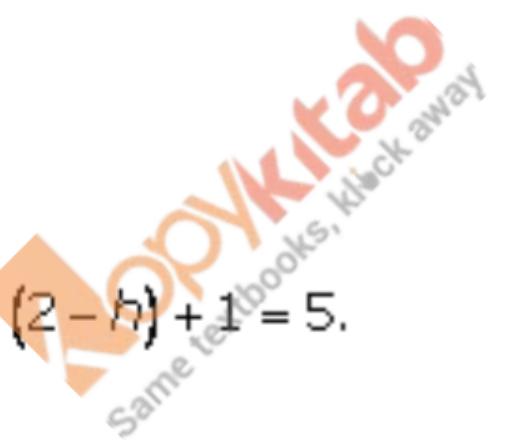
**Continuity Ex 9.1 Q44**

It is given that the function is continuous at $x = \frac{\pi}{2}$

$\therefore$ LHL = RHL = $f\left(\frac{\pi}{2}\right)$ .....(1)

Now,

$f\left(\frac{\pi}{2}\right) = a$

$\text{LHL} = \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right) = \lim_{h \to 0} \frac{1 - \sin^3\left(\frac{\pi}{2} - h\right)}{3\cos^2\left(\frac{\pi}{2} - h\right)} = \lim_{h \to 0} \frac{1 - \cos^3 h}{3\sin^2 h}$

$$= \lim_{h \to 0} \frac{(1 - \cos h)(1 + \cos^2 h + \cos h)}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{2\sin^2 \frac{h}{2}(1 + \cos^2 h + \cos h)}{3\sin^2 h}$$

$$= \lim_{h \to 0} \frac{2\left(\dfrac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \dfrac{h^2}{4}(1 + \cos^2 h + \cos h)}{3\left(\dfrac{\sin h}{h}\right)^2 \cdot h^2}$$

$$= \lim_{h \to 0} \frac{2 \cdot \frac{1}{4}(1 + \cos^2 h + \cos h)}{3} = \frac{1}{2}$$

$\text{RHL} = \lim_{x \to \frac{\pi}{2}} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right) = \lim_{h \to 0} \frac{b\left(1 - \sin\left(\frac{\pi}{2} + h\right)\right)}{\left(\pi - 2\left(\frac{\pi}{2} + h\right)\right)^2} = \lim_{h \to 0} \frac{b(1 - \cos h)}{(\pi - \pi - 2h)^2}$

$$= \lim_{h \to 0} \frac{b \cdot 2\sin^2 \frac{h}{2}}{(2h)^2}$$

$$= \lim_{h \to 0} \frac{b}{2}\left(\frac{\sin \frac{h}{2}}{\frac{h}{2}}\right)^2 \times \frac{1}{4}$$

$$= \lim_{h \to 0} \frac{b}{8} = \frac{b}{8}$$

Thus, using (1) we get,

$a = \dfrac{1}{2}$

And

$\dfrac{b}{8} = \dfrac{1}{2} \Rightarrow b = 4$

Thus, $a = \dfrac{1}{2}$ and $b = 4$

**Continuity Ex 9.1 Q45**

It is given that the function is continuous at $x = 0$, then

LHL = RHL = $f(0)$ ..... (1)

Now,

$f(0) = k$

RHL = $\lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \dfrac{h}{|h|} = 1$ .... (B)

Thus, using (1) we get,

$k = 1$

**Continuity Ex 9.1 Q46**

Since the function is continuous at $x = 3$, therefore

$$LHL = RHL = f(3)$$

Now

$$RHL = \lim_{x \to 3^+} f(x)$$

$$= \lim_{h \to 0} f(3+h)$$

$$= \lim_{h \to 0} b(3+h) + 3$$

$$= \lim_{h \to 0} 3b + 3h + 3$$

$$= 3b + 3$$

Again

$$f(3) = a(3) + 1$$

$$= 3a + 1$$

Thus we can write

$$f(3) = RHL$$

$$3a + 1 = 3b + 3$$

$$3a - 3b = 2$$
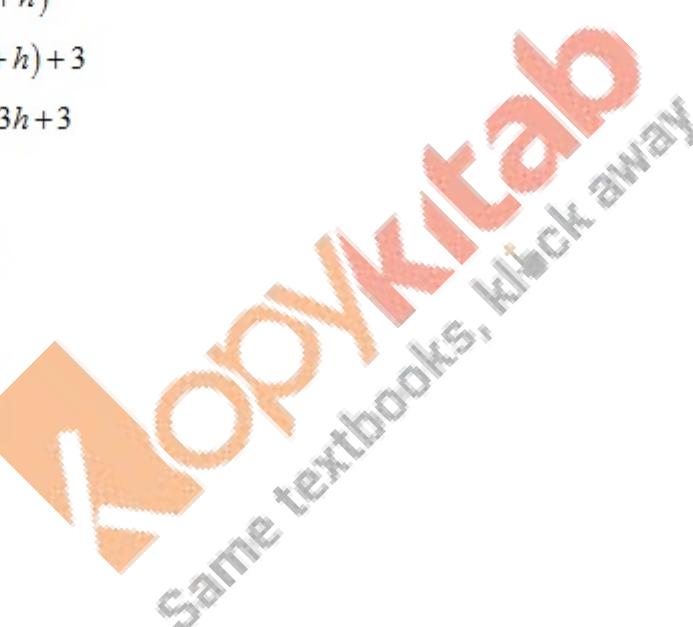
# RD Sharma Solutions
# Class 12 Maths
# Chapter 9
# Ex 9.2
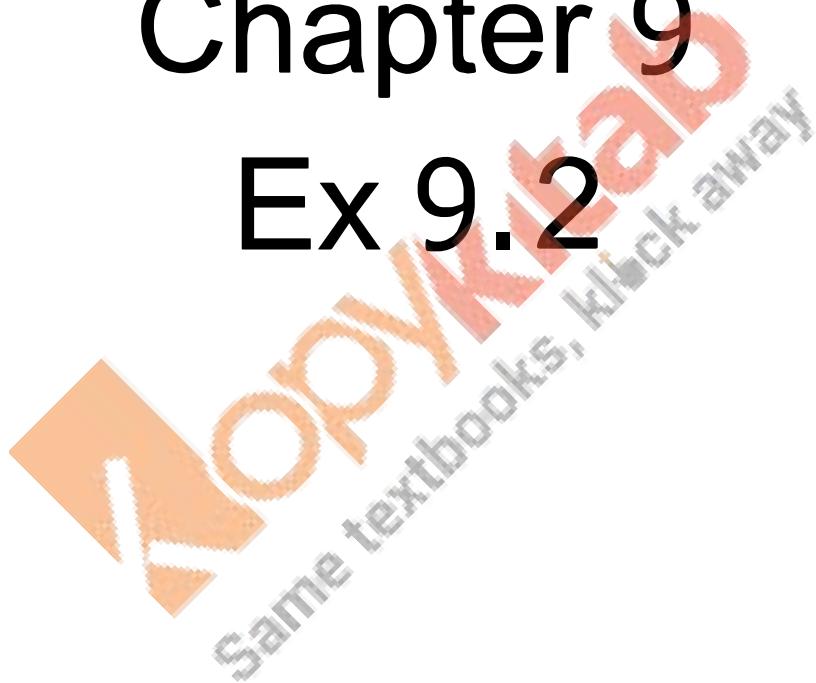
## Chapter 9 Continuity Ex 9.2 Q2

When $x \neq 0$,

$$f(x) = \frac{x}{|x|} = \begin{cases} \dfrac{-x}{x} = -1 & ; \ x < 0 \\[2mm] \dfrac{x}{|x|} = 1 & ; \ x > 0 \end{cases}$$

So, $f(x)$ is a constant function when $x \neq 0$

hence, is continuous for all $x < 0$ and $x > 0$

Now,

Consider the point $x = 0$.

$\text{LHL} = \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{-h}{|-h|} = -1$

$\text{RHL} = \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} \dfrac{h}{|h|} = 1$

So, $\text{LHL} \neq \text{RHL}$

Hence, function is discontinuous at $x = 0$

**Chapter 9 Continuity Ex 9.2 Q3(i)**

When $x \neq 1$

$f(x) = x^3 - x^2 + 2x - 2$ is a polynomial, so is continuous for $x < 1$ and $x > 1$

Now, consider the point $x = 1$

LHL $= \lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1-h) = \lim\limits_{h \to 0} (1-h)^3 - (1-h)^2 + 2(1-h) - 2 = 1 - 1 + 2 - 2 = 0$

RHL $= \lim\limits_{x \to 1^+} f(x) = \lim\limits_{h \to 0} f(1+h) = \lim\limits_{h \to 0} (1+h)^3 - (1+h)^2 + 2(1+h) - 2 = 1 - 1 + 2 - 2 = 0$

$f(1) = 4$

LHL = RHL $\neq f(1)$

Thus, function is not discontinuous at $x = 1$

**Chapter 9 Continuity Ex 9.2 Q3(ii)**

When $x \neq 2$, we have,

$f(x) = \dfrac{x^4 - 16}{x - 2} = \dfrac{(x^2 + 4)(x^2 - 4)}{x - 2} = \dfrac{(x^2 + 4)(x + 2)(x - 2)}{x - 2} = f(x) = (x^2 + 4)(x + 2)$

which is a polynomial, so the function is continuous when $x < 2$ or $x > 2$

Now, consider the point $x = 2$

LHL $= \lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2-h) = \lim\limits_{h \to 0} \dfrac{(2-h)^4 - 16}{(2-h) - 2}$

$= \lim\limits_{h \to 0} \dfrac{2^4 - 4.8h + 6.4h^2 - 4.2h^3 + h^4 - 16}{-h}$

$= \lim\limits_{h \to 0} \dfrac{16 - 32h + 24h^2 - 8h^3 + h^4 - 16}{-h}$

$= \lim\limits_{h \to 0} 32 - 24h + 8h^2 - h^3 = 32$

RHL $= \lim\limits_{x \to 2^+} f(x) = \lim\limits_{h \to 0} f(2+h) = \lim\limits_{h \to 0} \dfrac{(2+h)^4 - 16}{(2+h) - 2} = \lim\limits_{h \to 0} \dfrac{16 + 32h + 24h^2 + 8h^3 + h^4 - 16}{h}$

$= \lim\limits_{h \to 0} 32 + 24h + 8h^2 + h^3$

$= 32$

Also, $f(2) = 16$

Thus, LHL = RHL $\neq f(2)$

Hence, the function is discontinuous at $x = 2$

**Chapter 9 Continuity Ex 9.2 Q3(iii)**

When $x < 0$, we have, $f(x) = \dfrac{\sin x}{x}$

We know that $\sin x$ and the identity function continuous for $x < 0$, so the quotient function $f(x) = \dfrac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$ $f(x) = 2x + 3$, which is a polynomial of degree 1 so $f(x) = 2x + 3$ is continuous for $x >$

Now, consider the point $x = 0$

LHL $= \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0 - h) = \lim\limits_{h \to 0} \dfrac{\sin(-h)}{-h} = \lim\limits_{h \to 0} \dfrac{-\sin h}{-h} = 1$

RHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h) = \lim\limits_{h \to 0} \dfrac{\sin h}{h} = 1$

$f(0) = 2 \times 0 + 3 = 3$

Thus, L.H.L = R.H.L $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

**Chapter 9 Continuity Ex 9.2 Q3(iv)**

When $x \neq 0$ $f(x) = \dfrac{\sin 3x}{x}$

We know that $\sin 3x$ and the identity function $x$ are continuous for $x < 0$ and $x > 0$.

So, the quotient function $f(x) = \dfrac{\sin 3x}{x}$ is continuous for $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

LHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 - h) = \lim\limits_{h \to 0} \dfrac{\sin 3(-h)}{-h} = \lim\limits_{h \to 0} \dfrac{-\sin 3h}{-h} = 3$

RHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0 + h) = \lim\limits_{h \to 0} \dfrac{\sin 3h}{h} = 3$

$f(0) = 4$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

**Chapter 9 Continuity Ex 9.2 Q3(v)**

When $x \neq 0$, we have, $f(x) = \dfrac{\sin x}{x} + \cos x$

We know that

$\sin x$ and $\cos x$ is continuous for $x < 0$ and $x > 0$.

The identity function $x$ is also continuous for $x < 0$ and $x > 0$.

$\therefore$ The quotient function $f(x) = \dfrac{\sin x}{x}$ is continuous for $x < 0$ and $x > 0$.

And, the sum $\dfrac{\sin x}{x} + \cos x$ is also continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

LHL $= \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\sin(-h)}{-h} + \cos(-h) = \lim\limits_{h \to 0} \dfrac{-\sin h}{-h} + \cos h = 1 + 1 = 2$

RHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} \dfrac{\sin h}{h} + \cos h = 1 + 1 = 2$

$f(0) = 5$

Thus, LHL $=$ RHL $\neq f(0)$

Hence, $f(x)$ is discontinuous at $x = 0$

**Chapter 9 Continuity Ex 9.2 Q3(vi)**

When $x \neq 0$, we have, $f(x) = \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$

We know that a polynomial is continuous for $x < 0$ and $x > 0$, Also the inverse trignometric functio
is continuous in its domain.

Here, $x^4 + x^3 + 2x^2$ is polynomial, so is continuous for $x < 0$ and $x > 0$ and $\tan^{-1} x$ is also continuo
for $x < 0$ and $x > 0$

So, the quotient function $f(x) = \dfrac{x^4 + x^3 + 2x^2}{\tan^{-1} x}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

LHL $= \lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{(-h)^4 + (-h)^3 + 2(-h)^2}{\tan^{-1}(-h)} = \lim\limits_{h \to 0} \dfrac{h^4 - h^3 + 2h^2}{\tan^{-1} h} = 0$

RHL $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{h \to 0} f(0+h) = \lim\limits_{h \to 0} \dfrac{h^4 + h^3 + 2h^2}{\tan^{-1} h} = 0$

$f(0) = 10$

Thus, LHL $=$ RHL $\neq f(0)$

Hence, the function is not continuous at $x = 0$

When $x \neq 0$, we have,

$$f(x) = \frac{e^x - 1}{\log_e (1 + 2x)}$$

We know that $e^x$ and the constant function is continuous for $x < 0$ and $x > 0$

$\Rightarrow \ e^x - 1$ is continuous for $x < 0$ and $x > 0$

Again, logarithmic function is continuous for $x < 0$ and $x > 0$

$\Rightarrow \ \log_e (1 + 2x)$ is continuous for $x > 0$ and $x < 0$

So, the quotient function $f(x) = \dfrac{e^x - 1}{\log_e (1 + 2x)}$ is continuous for each $x < 0$ and $x > 0$.

Now, consider the point $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0 - h) = \lim_{h \to 0} \frac{e^{-h} - 1}{\log_e (1 - 2h)} = \lim_{h \to 0} \frac{\dfrac{e^{-h} - 1}{-h}}{\dfrac{\log_e (1 - 2h)}{-2h} \times -2} = \frac{1}{2}$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0 + h) = \lim_{h \to 0} \frac{e^{h} - 1}{\log_e (1 + 2h)} = \lim_{h \to 0} \frac{\dfrac{e^{h} - 1}{h}}{\dfrac{\log_e (1 + 2h)}{2h} \times 2} = \frac{1}{2}$$

$f(0) = 7$

Thus, LHL = RHL $\neq f(0)$

Hence, $f(x)$ is not continuous at $x = 0$

We know that

(i) The absolute value function $g(x) = |x|$ is continuous on IR

(ii) Polynomial function are every where continuous.

So, the only possible point of discontinuity of $f(x)$ can be $x = 1$

Now

$f(1) = |1 - 3| = |-2| = 2$

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} |x - 3| = 2$

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \left( \dfrac{x^2}{4} - \dfrac{3x}{2} + \dfrac{13}{4} \right)$

$\qquad = \dfrac{1}{4} - \dfrac{3}{2} + \dfrac{13}{4} = \dfrac{8}{4} = 2$

Since

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^+} f(x) = f(1) = 2$

$\therefore \qquad f(x)$ is continuous at $x$

Hence $f(x)$ has no point of discontinuity.

**Chapter 9 Continuity Ex 9.2 Q3(ix)**

When $x < -3$,

$f(x) = |x| + 3$

We know that $|x|$ is continuous for $x < -3$

$\therefore |x| + 3$ is continuous for $x < -3$

When $x > 3$

$f(x) = 6x + 2$ which is a polynomial of degree 1, so $f(x) = 6x + 2$ is continuous for $x > 3$

When $-3 < x < 3$

$f(x) = -2x$ which is again a polynomial so, it is continuous for $-3 < x < 3$

Now, consider the point $x = -3$

LHL $= \lim\limits_{x \to -3^-} f(x) = \lim\limits_{h \to 0} f(-3-h) = \lim\limits_{h \to 0} |-3-h| + 3 = \lim\limits_{h \to 0} |3+h| + 3 = 6$

RHL $= \lim\limits_{x \to -3^+} f(x) = \lim\limits_{h \to 0} f(-3+h) = \lim\limits_{h \to 0} -2(-3+h) = 6$

$f(-3) = |-3| + 3 = 6$

Thus, LHL $=$ RHL $= f(-3) = 6$

So, the function is continuous at $x = -3$

Now, consider the point $x = 3$

LHL $= \lim\limits_{x \to 3^-} f(x) = \lim\limits_{h \to 0} f(3-h) = \lim\limits_{h \to 0} -2(3-h) = -6$

**Chapter 9 Continuity Ex 9.2 Q3(xi)**

The given function is $f(x) = \begin{cases} 2x, & \text{if } x < 0 \\ 0, & \text{if } 0 \le x \le 1 \\ 4x, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let $c$ be a point on the real line.

Case I:

If $c < 0$, then $f(c) = 2c$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (2x) = 2c$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, $f$ is continuous at all points $x$, such that $x < 0$

The given function $f$ is $f(x) = \begin{cases} \sin x - \cos x, & \text{if } x \neq 0 \\ -1 & \text{if } x = 0 \end{cases}$

It is evident that $f$ is defined at all points of the real line.

Let $c$ be a real number.

Case I:

If $c \neq 0$, then $f(c) = \sin c - \cos c$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (\sin x - \cos x) = \sin c - \cos c$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, $f$ is continuous at all points $x$, such that $x \neq 0$

The given function $f$ is $f(x) = \begin{cases} -2, & \text{if } x \leq -1 \\ 2x, & \text{if } -1 < x \leq 1 \\ 2, & \text{if } x > 1 \end{cases}$

The given function is defined at all points of the real line.

Let $c$ be a point on the real line.

Case I:

If $c < -1$, then $f(c) = -2$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (-2) = -2$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, $f$ is continuous at all points $x$, such that $x < -1$

Case II:

If $c = -1$, then $f(c) = f(-1) = -2$

The left hand limit of $f$ at $x = -1$ is,

$\lim\limits_{x \to -1^-} f(x) = \lim\limits_{x \to -1^-} (-2) = -2$

The right hand limit of $f$ at $x = -1$ is,

$\lim\limits_{x \to -1^+} f(x) = \lim\limits_{x \to -1^+} (2x) = 2 \times (-1) = -2$

$\therefore \lim\limits_{x \to -1} f(x) = f(-1)$

Therefore, $f$ is continuous at $x = -1$

Case III:

If $-1 < c < 1$, then $f(c) = 2c$

$\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} (2x) = 2c$

$\therefore \lim\limits_{x \to c} f(x) = f(c)$

Therefore, $f$ is continuous at all points of the interval $(-1, 1)$.

Case IV:

If $c = 1$, then $f(c) = f(1) = 2 \times 1 = 2$

The left hand limit of $f$ at $x = 1$ is,

$\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-}(2x) = 2 \times 1 = 2$

The right hand limit of $f$ at $x = 1$ is,

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} 2 = 2$

$\therefore \lim\limits_{x \to 1} f(x) = f(c)$

Therefore, $f$ is continuous at $x = 2$

Case V:

If $c > 1$, then $f(c) = 2$ and $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c}(2) = 2$

$\lim\limits_{x \to c} f(x) = f(c)$

Therefore, $f$ is continuous at all points $x$, such that $x > 1$

Thus, from the above observations, it can be concluded that $f$ is continuous at all points of the real line

### Chapter 9 Continuity Ex 9.2 Q4(i)

We have given that the function is continuous at $x = 0$

$\therefore$ LHL = RHL = $f(0)$ .... (1)

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} \dfrac{\sin(-2h)}{5(-h)} = \lim\limits_{h \to 0} \dfrac{-\sin 2h}{-5h} = \lim\limits_{h \to 0} \dfrac{\sin 2h}{2h} \times \dfrac{2h}{5h} = \dfrac{2}{5}$

$f(0) = 3k$

So, using (1) we get,

$\dfrac{2}{5} = 3k$

$k = \dfrac{2}{15}$

### Chapter 9 Continuity Ex 9.2 Q4(ii)

It is given that the function is continuous

$\therefore$ LHL = RHL = $f(2)$ ....(1)

LHL = $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2-h) = \lim\limits_{h \to 0} k(2-h) + 5 \quad = 2k + 5$

RHL = $\lim\limits_{x \to 2^+} f(x) = \lim\limits_{h \to 0} f(2+h) = \lim\limits_{h \to 0}(2+h) - 1 = 1$

Thus, using (1), we get,

$2k + 5 = 1$

$k = -2$

**Chapter 9 Continuity Ex 9.2 Q4(iii)**

It is given that the function is continuous

LHL = RHL = $f(0)$ .... (1)

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} k\left((-h)^2 + 3(-h)\right) = \lim\limits_{h \to 0} k\left(h^2 - 3h\right) = 0$

$f(0) = \cos 2 \times 0 = \cos 0° = 1$

LHL $\neq f(0)$

Hence, no value of $k$ can make $f$ continuous

**Chapter 9 Continuity Ex 9.2 Q4(iv)**

First check the continuity of the function at $x = 3$

$f(3) = 2$ .....(A)

RHL = $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{h \to 0} f(3+h) = \lim\limits_{h \to 0} a(3+h) + b = 3a + b$ .....(B)

$\therefore f(x)$ will be continuous at $x = 3$ if $3a + b = 2$ .....(1)

Now, check the continuity at $x = 5$

$f(5) = 9$ .... (C)

LHL = $\lim\limits_{x \to 5^-} f(x) = \lim\limits_{h \to 0} f(5-h) = \lim\limits_{h \to 0} a(5-h) + b = 5a + b$

$f(x)$ will be continuous at $x = 5$ if $5a + b = 9$ .... (2)

Solving (1) & (2), we get

$a = \dfrac{7}{2}$ and $b = \dfrac{-17}{2}$

**Chapter 9 Continuity Ex 9.2 Q4(v)**

It is given that the function is continuous

At $x = -1$

$f(-1) = 4$

RHL = $\lim\limits_{x \to -1^+} f(x) = \lim\limits_{h \to 0} f(-1+h) = \lim\limits_{h \to 0} a(-1+h)^2 + b = a + b$

Since, $f(x)$ is continuous at $x = -1$

$\therefore a + b = 4$ .... (A)

Now, at $x = 0$,

$f(0) = \cos 0° = 1$

LHL = $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{h \to 0} f(0-h) = \lim\limits_{h \to 0} a(-h)^2 + b = b$

Since, $f(x)$ is continuous at $x = 0$

$\therefore f(0) = $ LHL

$\Rightarrow b = 1$

$\therefore$ from (A)

$a = 3$

Thus, $a = 3$, $b = 1$

### Chapter 9 Continuity Ex 9.2 Q4(vi)

It is given that the function is continuous.

At $x = 0$

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sqrt{1-Ph} - \sqrt{1+Ph}}{-h} = \lim_{h \to 0} \frac{\left(\sqrt{1-Ph} - \sqrt{1+Ph}\right)}{-h} \times \frac{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}{\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)}$$

$$= \lim_{h \to 0} \frac{(1-Ph) - (1+Ph)}{-h\left(\sqrt{1-Ph} + \sqrt{1+Ph}\right)} = \frac{2P}{2} = P$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} \frac{2h+1}{h-2} = \frac{-1}{2}$$

Since, $f(x)$ is continuous so,

$$P = \frac{-1}{2}$$

The given function $f$ is $f(x) = \begin{cases} 5, & \text{if } x \le 2 \\ ax+b, & \text{if } 2 < x < 10 \\ 21, & \text{if } x \ge 10 \end{cases}$

It is evident that the given function $f$ is defined at all points of the real line.

If $f$ is a continuous function, then $f$ is continuous at all real numbers.

In particular, $f$ is continuous at $x = 2$ and $x = 10$

Since $f$ is continuous at $x = 2$, we obtain

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) = f(2)$

$\Rightarrow \lim\limits_{x \to 2^-} (5) = \lim\limits_{x \to 2^+} (ax+b) = 5$

$\Rightarrow 5 = 2a+b = 5$

$\Rightarrow 2a+b = 5 \qquad \ldots(1)$

Since $f$ is continuous at $x = 10$, we obtain

$\lim\limits_{x \to 10^-} f(x) = \lim\limits_{x \to 10^+} f(x) = f(10)$

$\Rightarrow \lim\limits_{x \to 10^-} (ax+b) = \lim\limits_{x \to 10^+} (21) = 21$

$\Rightarrow 10a+b = 21 = 21$

$\Rightarrow 10a+b = 21 \qquad \ldots(2)$

On subtracting equation (1) from equation (2), we obtain

$8a = 16$

$a = 2$

By putting $a = 2$ in equation (1), we obtain

$2 \times 2 + b = 5$

$4 + b = 5$

$b = 1$

Therefore, the values of $a$ and $b$ for which $f$ is a continuous function are 2 and 1 respectively.

**Chapter 9 Continuity Ex 9.2 Q4(viii)**

Since the function is continuous at $x = \dfrac{\pi}{2}$ therefore

LHL of $f(x)$ at $x = \dfrac{\pi}{2}$ is

$$= \lim_{x \to \frac{\pi}{2}^-} f(x)$$

$$= \lim_{h \to 0} f\left(h - \dfrac{\pi}{2}\right)$$

$$= \lim_{h \to 0} \dfrac{k\cos\left(h - \dfrac{\pi}{2}\right)}{\pi - 2\left(h - \dfrac{\pi}{2}\right)}$$

$$= \lim_{h \to 0} \dfrac{k\sinh}{2\pi - 2h}$$

$$= \dfrac{k}{2} \lim_{h \to 0} \dfrac{\sin(\pi - h)}{(\pi - h)}$$

$$= \dfrac{k}{2}$$

Again

$$f\left(\dfrac{\pi}{2}\right) = 3$$

Hence

$$LHL = f\left(\dfrac{\pi}{3}\right)$$

$$\dfrac{k}{2} = 3$$

$$k = 6$$

**Chapter 9 Continuity Ex 9.2 Q5**

We have given that $f(x)$ is continuous on $[0, \infty]$

$\therefore f(x)$ is continuous at $x = 1$ and $x = \sqrt{2}$

$\therefore$ At $x = 1$, LHL = RHL = $f(1)$         ..... (A)

$f(1) = a$           ..... (1)

LHL = $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{h \to 0} f(1 - h) = \lim\limits_{h \to 0} \dfrac{(1-h)^2}{a} = \dfrac{1}{a}$

Using (A) we get,

$a = \dfrac{1}{a} \Rightarrow a^2 = 1 \Rightarrow a = \pm 1$

At $x = \sqrt{2}$ LHL = RHL = $f(\sqrt{2})$       ..... (B)

$f(\sqrt{2}) = \dfrac{2b^2 - 4b}{(\sqrt{2})^2} = \dfrac{2b^2 - 4b}{2} = b^2 - 2b$       ......(2)

LHL = $\lim\limits_{x \to \sqrt{2}^-} f(x) = \lim\limits_{h \to 0} f(\sqrt{2} - h) = \lim\limits_{h \to 0} a = a.$

So, using (B), we get,

$b^2 - 2b = a$

For $a = 1$,       $b^2 - 2b - 1 = 0$

$\Rightarrow b = \dfrac{2 \pm \sqrt{4 + 4}}{2} = 1 \pm \sqrt{2}$

For $a = -1$       $b^2 - 2b + 1 = 0$

$\Rightarrow (b - 1)^2 = 0 \Rightarrow b = 1$

Thus, $a = -1$, $b = 1$ or $a = 1$, $b = 1 \pm \sqrt{2}$

**Chapter 9 Continuity Ex 9.2 Q6**

Since, $f(x)$ is continuous on $[0, \pi]$

$f(x)$ is continuous at $x = \frac{\pi}{4}$ and $x = \frac{\pi}{2}$

At $x = \frac{\pi}{4}$,

LHL = RHL = $f\left(\frac{\pi}{4}\right)$ .....(A)

Now, $f\left(\frac{\pi}{4}\right) = 2\frac{\pi}{4} \cdot \cot\left(\frac{\pi}{4}\right) + b = \frac{\pi}{2} \cdot 1 + b = \frac{\pi}{2} + b$ ....(1)

LHL = $\lim_{x \to \frac{\pi}{4}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{4} - h\right) = \lim_{h \to 0}\left(\frac{\pi}{4} - h\right) + a\sqrt{2}\sin\left(\frac{\pi}{4} - h\right) = \frac{\pi}{4} + a\sqrt{2}\cdot\frac{1}{\sqrt{2}} = \frac{\pi}{4} + a$

Thus, using (A)

$\frac{1}{2} + b = \frac{\pi}{4} + a$

$a - b = \frac{\pi}{4}$ .....(B)

At $x = \frac{\pi}{2}$

LHL = RHL = $f\left(\frac{\pi}{2}\right)$ .....(C)

Now, $f\left(\frac{\pi}{2}\right) = a\cos 2 \cdot \frac{\pi}{2} - b\sin\frac{\pi}{2} = -a - b$ ....(2)

LHL = $\lim_{x \to \frac{\pi}{2}^-} f(x) = \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right)$

$= \lim_{h \to 0} 2\left(\frac{\pi}{2} - h\right)\cot\left(\frac{\pi}{2} - h\right) + b = \pi \times 0 + b = b$

using (C), we get,

$-a - b = b \quad \Rightarrow 2b = -a \quad \Rightarrow b = \frac{-a}{2}$

from (B), $a + \frac{a}{2} = \frac{\pi}{4}$

$\Rightarrow \quad \frac{3}{2}a = \frac{\pi}{4}$

$\Rightarrow \quad a = \frac{\pi}{6}$

and $b = \frac{-a}{2} = \frac{-\pi}{12}$

Thus, $a = \frac{\pi}{6}, \quad b = \frac{-\pi}{12}$

**Chapter 9 Continuity Ex 9.2 Q7**

It is given that the $f(x)$ is continuous on $[0,8]$

$f(x)$ is continuous at $x = 2$ and $x = 4$.

Now, At $x = 2$

LHL = RHL = $f(2)$....(A)

$f(2) = 3 \times 2 + 2 = 8$....(1)

LHL = $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2-h) = \lim\limits_{h \to 0} (2-h)^2 + a(2-h) + b = 4 + 2a + b$

from $(A)$

$4 + 2a + b = 8$

$2a + b = 4$.....(B)

Now, At $x = 4$

LHL = RHL = $f(4)$....(C)

$f(4) = 3 \times 4 + 2 = 14$.....(2)

RHL = $\lim\limits_{x \to 4^+} f(x) = \lim\limits_{h \to 0} f(4+h) = \lim\limits_{h \to 0} 2a(4+h) + 5b = 8a + 5b$

From $(C)$, we get,

$8a + 5b = 14$....(D)

Solving $(B)$ and $(D)$, we get,

$a = 3$ and $b = -2$

**Chapter 9 Continuity Ex 9.2 Q8**

The function will be continuous on $\left[0, \dfrac{\pi}{2}\right]$ if it is continuous at every point in $\left[0, \dfrac{\pi}{2}\right]$

Let us consider the point $x = \dfrac{\pi}{4}$,

We must have,

LHL = RHL = $f\left(\dfrac{\pi}{4}\right)$....(A)

LHL = $\lim\limits_{x \to \frac{\pi}{4}^-} f(x) = \lim\limits_{h \to 0} f\left(\dfrac{\pi}{4} - h\right) = \lim\limits_{h \to 0} \dfrac{\tan\left(\dfrac{\pi}{4} - \dfrac{\pi}{4} + h\right)}{\cot + 2\left(\dfrac{\pi}{4} - h\right)} = \lim\limits_{h \to 0} \dfrac{\tan h}{\tan 2h}$  $\left[\because \cot\left(\dfrac{\pi}{2} - \theta\right) = \tan\theta\right]$

$= \lim\limits_{h \to 0} \dfrac{\dfrac{\tan h}{h}}{\dfrac{\tan 2h}{h}} = \dfrac{1}{2}$

Thus, using $(A)$ we get,

$f\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2}$

Hence, $f(x)$ will be continuous on $\left[0, \dfrac{\pi}{2}\right]$ if $f\left(\dfrac{\pi}{4}\right) = \dfrac{1}{2}$.

**Chapter 9 Continuity Ex 9.2 Q9**

When $x < 2$, we have

$f(x) = 2x - 1$, which is a polynomial of degree 1.

So, $f(x)$ is continuous for $x < 2$.

When $x > 2$, we have

$f(x) = \dfrac{3x}{2}$, which is again a polynomial of degree 1.

So, $f(x)$ is continuous for $x > 2$.

Now, consider the point $x = 2$

$f(2) = \dfrac{3 \times 2}{2} = 3$

$\text{LHL} = \lim\limits_{x \to 2^-} f(x) = \lim\limits_{h \to 0} f(2 - h) = \lim\limits_{h \to 0} 2(2 - h) - 1 = 3$

$\text{RHL} = \lim\limits_{x \to 2^+} f(x) = \lim\limits_{h \to 0} f(2 + h) = \lim\limits_{h \to 0} \dfrac{3(2 + h)}{2} = 3$

$\text{LHL} = \text{RHL} = f(2) = 3$

Thus, $f(x)$ is continuous at $x = 2$

Hence, $f(x)$ is continuous every where.

**Chapter 9 Continuity Ex 9.2 Q10**

Let $f(x) = \sin|x|$

This function $f$ is defined for every real number and $f$ can be written as the composition of two functions as,

$f = g \circ h$, where $g(x) = |x|$ and $h(x) = \sin x$

$\left[ \because (g \circ h)(x) = g(h(x)) = g(\sin x) = |\sin x| = f(x) \right]$

It has to be proved first that $g(x) = |x|$ and $h(x) = \sin x$ are continuous functions.

$g(x) = |x|$ can be written as

$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

Clearly, $g$ is defined for all real numbers.

Let $c$ be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} (-x) = -c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim_{x \to c} g(x) = \lim_{x \to c} x = c$

$\therefore \lim_{x \to c} g(x) = g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$\lim_{x \to 0^-} g(x) = \lim_{x \to 0^-} (-x) = 0$

$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} (x) = 0$

$\therefore \lim_{x \to 0^-} g(x) = \lim_{x \to 0^+} (x) = g(0)$

Therefore, $g$ is continuous at $x = 0$

From the above three observations, it can be concluded that $g$ is continuous at all points.

$h(x) = \sin x$

It is evident that $h(x) = \sin x$ is defined for every real number.

Let $c$ be a real number. Put $x = c + k$

If $x \to c$, then $k \to 0$

$h(c) = \sin c$

$$h(c) = \sin c$$

$$\lim_{x \to c} h(x) = \lim_{x \to c} \sin x$$

$$= \lim_{k \to 0} \sin(c+k)$$

$$= \lim_{k \to 0} \left[ \sin c \cos k + \cos c \sin k \right]$$

$$= \lim_{k \to 0} (\sin c \cos k) + \lim_{h \to 0} (\cos c \sin k)$$

$$= \sin c \cos 0 + \cos c \sin 0$$

$$= \sin c + 0$$

$$= \sin c$$

$$\therefore \quad \lim_{x \to c} h(x) = g(c)$$

Therefore, $h$ is a continuous function.

It is known that for real valued functions $g$ and $h$, such that $(g \circ h)$ is defined at $c$, if $g$ is continuous at $c$ and if $f$ is continuous at $g(c)$, then $(f \circ g)$ is continuous at $c$.

Therefore, $f(x) = (goh)(x) = g(h(x)) = g(\sin x) = |\sin x|$ is a continuous function.

**Chapter 9 Continuity Ex 9.2 Q11**

When $x < 0$, we have,

$$f(x) = \frac{\sin x}{x}$$

We know that the $\sin x$ and the identity function $x$ are continuous for $x < 0$.

So, the quotient function $f(x) = \frac{\sin x}{x}$ is continuous for $x < 0$.

When $x > 0$, we have,

$f(x) = x + 1$, which is a polynomial of degree 1. So, $f(x)$ is continuous for $x > 0$

Now, consider the point $x = 0$.

$f(0) = 0 + 1 = 1$.

$$\text{LHL} = \lim_{x \to 0^-} f(x) = \lim_{h \to 0} f(0-h) = \lim_{h \to 0} \frac{\sin(-h)}{-h} = \lim_{h \to 0} \frac{-\sin h}{-h} = 1$$

$$\text{RHL} = \lim_{x \to 0^+} f(x) = \lim_{h \to 0} f(0+h) = \lim_{h \to 0} h + 1 = 1$$

Thus, LHL = RHL = $f(0) = 1$

So, $f(x)$ is continuous at $x = 0$.

Hence, $f(x)$ is continuous everywhere

**Chapter 9 Continuity Ex 9.2 Q12**

The given function is $g(x) = x - [x]$

It is evident that $g$ is defined at all integral points.

Let $n$ be an integer.

Then,

$g(n) = n - [n] = n - n = 0$

The left hand limit of $f$ at $x = n$ is,

$\lim\limits_{x \to n^-} g(x) = \lim\limits_{x \to n^-} (x - [x]) = \lim\limits_{x \to n^-} (x) - \lim\limits_{x \to n^-} [x] = n - (n-1) = 1$

The right hand limit of $f$ at $x = n$ is,

$\lim\limits_{x \to n^+} g(x) = \lim\limits_{x \to n^+} (x - [x]) = \lim\limits_{x \to n^+} (x) - \lim\limits_{x \to n^+} [x] = n - n = 0$

It is observed that the left and right hand limits of $f$ at $x = n$ do not coincide.

Therefore, $f$ is not continuous at $x = n$

Hence, $g$ is discontinuous at all integral points

**Chapter 9 Continuity Ex 9.2 Q13**

It is known that if $g$ and $h$ are two continuous functions, then

$g + h,\ g - h,$ and $g.h$ are also continuous.

It has to proved first that $g(x) = \sin x$ and $h(x) = \cos x$ are continuous functions.

Let $g(x) = \sin x$

It is evident that $g(x) = \sin x$ is defined for every real number.

Let $c$ be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$g(c) = \sin c$

$$\lim_{x \to c} g(x) = \lim_{x \to c} \sin x$$

$$= \lim_{h \to 0} \sin(c + h)$$

$$= \lim_{h \to 0} \left[ \sin c \cos h + \cos c \sin h \right]$$

$$= \lim_{h \to 0} (\sin c \cos h) + \lim_{h \to 0} (\cos c \sin h)$$

$$= \sin c \cos 0 + \cos c \sin 0$$

$$= \sin c + 0$$

$$= \sin c$$

$\therefore\ \lim_{x \to c} g(x) = g(c)$

Therefore, $g$ is a continuous function.

Let $h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let $c$ be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\lim_{x \to c} h(x) = \lim_{x \to c} \cos x$$

$$= \lim_{h \to 0} \cos(c + h)$$

$$= \lim_{h \to 0} \left[ \cos c \cos h - \sin c \sin h \right]$$

$$= \lim_{h \to 0} \cos c \cos h - \lim_{h \to 0} \sin c \sin h$$

$$= \cos c \cos 0 - \sin c \sin 0$$

$$= \cos c \times 1 - \sin c \times 0$$

$$= \cos c$$

$\therefore \lim_{x \to c} h(x) = h(c)$

Therefore, $h$ is a continuous function.

Therefore, it can be concluded that

(a) $f(x) = g(x) + h(x) = \sin x + \cos x$ is a continuous function

(b) $f(x) = g(x) - h(x) = \sin x - \cos x$ is a continuous function

(c) $f(x) = g(x) \times h(x) = \sin x \times \cos x$ is a continuous function

The given function is $f(x) = \cos(x^2)$

This function $f$ is defined for every real number and $f$ can be written as the composition of two functions as,

$f = g \ o \ h$, where $g(x) = \cos x$ and $h(x) = x^2$

$\left[ \because (goh)(x) = g(h(x)) = g(x^2) = \cos(x^2) = f(x) \right]$

It has to be first proved that $g(x) = \cos x$ and $h(x) = x^2$ are continuous functions.

It is evident that $g$ is defined for every real number.

Let $c$ be a real number.

Then, $g(c) = \cos c$

Put $x = c + h$

If $x \to c$, then $h \to 0$

$\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} \cos x$

$\qquad = \lim\limits_{h \to 0} \cos(c + h)$

$\qquad = \lim\limits_{h \to 0} \left[ \cos c \cos h - \sin c \sin h \right]$

$\qquad = \lim\limits_{h \to 0} \cos c \cos h - \lim\limits_{h \to 0} \sin c \sin h$

$\qquad = \cos c \cos 0 - \sin c \sin 0$

$\qquad = \cos c \times 1 - \sin c \times 0$

$\qquad = \cos c$

$\therefore \lim\limits_{x \to c} g(x) = g(c)$

Therefore, $g(x) = \cos x$ is continuous function.

$h(x) = x^2$

Clearly, $h$ is defined for every real number.

Let $k$ be a real number, then $h(k) = k^2$

$\lim\limits_{x \to k} h(x) = \lim\limits_{x \to k} x^2 = k^2$

$\therefore \lim\limits_{x \to k} h(x) = h(k)$

Therefore, $h$ is a continuous function.

It is known that for real valued functions $g$ and $h$, such that $(g \ o \ h)$ is defined at $c$, if $g$ is continuous at $c$ and if $f$ is continuous at $g(c)$, then $(f \ o \ g)$ is continuous at $c$.

Therefore, $f(x) = (goh)(x) = \cos(x^2)$ is a continuous function.

The given function is $f(x) = |\cos x|$

This function $f$ is defined for every real number and $f$ can be written as the composition of two functions as,

$f = g \; o \; h$, where $g(x) = |x|$ and $h(x) = \cos x$

$\left[ \because (goh)(x) = g(h(x)) = g(\cos x) = |\cos x| = f(x) \right]$

It has to be first proved that $g(x) = |x|$ and $h(x) = \cos x$ are continuous functions.

$g(x) = |x|$ can be written as

$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$

Clearly, $g$ is defined for all real numbers.

Let $c$ be a real number.

Case I:

If $c < 0$, then $g(c) = -c$ and $\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c}(-x) = -c$

$\therefore \lim\limits_{x \to c} g(x) = g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x < 0$

Case II:

If $c > 0$, then $g(c) = c$ and $\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} x = c$

$\therefore \lim\limits_{x \to c} g(x) = g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} (-x) = 0$

$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} (x) = 0$

$\therefore \lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^+} (x) = g(0)$

Therefore, $g$ is continuous at $x = 0$

From the above three observations, it can be concluded that $g$ is continuous at all points.

$h(x) = \cos x$

It is evident that $h(x) = \cos x$ is defined for every real number.

Let $c$ be a real number. Put $x = c + h$

If $x \to c$, then $h \to 0$

$h(c) = \cos c$

$$\lim\limits_{x \to c} h(x) = \lim\limits_{x \to c} \cos x$$
$$= \lim\limits_{h \to 0} \cos(c + h)$$
$$= \lim\limits_{h \to 0} \left[ \cos c \cos h - \sin c \sin h \right]$$
$$= \lim\limits_{h \to 0} \cos c \cos h - \lim\limits_{h \to 0} \sin c \sin h$$
$$= \cos c \cos 0 - \sin c \sin 0$$
$$= \cos c \times 1 - \sin c \times 0$$
$$= \cos c$$

$\therefore \lim\limits_{x \to c} h(x) = h(c)$

Therefore, $h(x) = \cos x$ is a continuous function.

It is known that for real valued functions $g$ and $h$, such that $(g \circ h)$ is defined at $c$, if $g$ is continuous at $c$ and if $f$ is continuous at $g(c)$, then $(f \circ g)$ is continuous at $c$.

Therefore, $f(x) = (g o h)(x) = g(h(x)) = g(\cos x) = |\cos x|$ is a continuous function

**Chapter 9 Continuity Ex 9.2 Q16**

The given function is $f(x) = |x| - |x+1|$

The two functions, $g$ and $h$, are defined as

$g(x) = |x|$ and $h(x) = |x+1|$

Then, $f = g - h$

The continuity of $g$ and $h$ is examined first.

$g(x) = |x|$ can be written as

$$g(x) = \begin{cases} -x, & \text{if } x < 0 \\ x, & \text{if } x \geq 0 \end{cases}$$

Clearly, $g$ is defined for all real numbers.

Let $c$ be a real number.

  Case I:

  If $c < 0$, then $g(c) = -c$ and $\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} (-x) = -c$

  $\therefore \lim\limits_{x \to c} g(x) = g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x < 0$

  Case II:

  If $c > 0$, then $g(c) = c$ and $\lim\limits_{x \to c} g(x) = \lim\limits_{x \to c} x = c$

  $\therefore \lim\limits_{x \to c} g(x) = g(c)$

Therefore, $g$ is continuous at all points $x$, such that $x > 0$

Case III:

If $c = 0$, then $g(c) = g(0) = 0$

$\lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^-} (-x) = 0$

$\lim\limits_{x \to 0^+} g(x) = \lim\limits_{x \to 0^+} (x) = 0$

$\therefore \lim\limits_{x \to 0^-} g(x) = \lim\limits_{x \to 0^+} (x) = g(0)$

Therefore, $g$ is continuous at $x = 0$

From the above three observations, it can be concluded that $g$ is continuous at all points.

$h(x) = |x + 1|$ can be written as

$h(x) = \begin{cases} -(x+1), & \text{if, } x < -1 \\ x+1, & \text{if } x \geq -1 \end{cases}$

Clearly, $h$ is defined for every real number

Let $c$ be a real number.

Case I:

If $c < -1$, then $h(c) = -(c+1)$ and $\lim\limits_{x \to c} h(x) = \lim\limits_{x \to c}\left[-(x+1)\right] = -(c+1)$

$\therefore \lim\limits_{x \to c} h(x) = h(c)$

Therefore, $h$ is continuous at all points $x$, such that $x < -1$

Case II:

If $c > -1$, then $h(c) = c+1$ and $\lim\limits_{x \to c} h(x) = \lim\limits_{x \to c}(x+1) = c+1$

$\therefore \lim\limits_{x \to c} h(x) = h(c)$

Therefore, $h$ is continuous at all points $x$, such that $x > -1$

Case III:

If $c = -1$, then $h(c) = h(-1) = -1 + 1 = 0$

$$\lim_{x \to -1^-} h(x) = \lim_{x \to -1^-} \left[ -(x+1) \right] = -(-1+1) = 0$$

$$\lim_{x \to -1^+} h(x) = \lim_{x \to -1^+} (x+1) = (-1+1) = 0$$

$$\therefore \lim_{x \to -1^-} h(x) = \lim_{h \to -1^+} h(x) = h(-1)$$

Therefore, $h$ is continuous at $x = -1$

From the above three observations, it can be concluded that $h$ is continuous at all points of the real line.

$g$ and $h$ are continuous functions. Therefore, $f = g - h$ is also a continuous function.

Therefore, $f$ has no point of discontinuity.

**Chapter 9 Continuity Ex 9.2 Q17**

He given function $f$ is $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

It is evident that $f$ is defined at all points of the real line.

Let $c$ be a real number.

Case I:

If $c \neq 0$, then $f(c) = c^2 \sin \dfrac{1}{c}$

$$\lim_{x \to c} f(x) = \lim_{x \to c} \left( x^2 \sin \frac{1}{x} \right) = \left( \lim_{x \to c} x^2 \right) \left( \lim_{x \to c} \sin \frac{1}{x} \right) = c^2 \sin \frac{1}{c}$$

$$\therefore \lim_{x \to c} f(x) = f(c)$$

Therefore, $f$ is continuous at all points $x \neq 0$

Case II:

If $c = 0$, then $f(0) = 0$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left( x^2 \sin \frac{1}{x} \right) = \lim_{x \to 0} \left( x^2 \sin \frac{1}{x} \right)$$

It is known that, $-1 \leq \sin \frac{1}{x} \leq 1, \ x \neq 0$

$$\Rightarrow -x^2 \leq \sin \frac{1}{x} \leq x^2$$

$$\Rightarrow \lim_{x \to 0} (-x^2) \leq \lim_{x \to 0} \left( x^2 \sin \frac{1}{x} \right) \leq \lim_{x \to 0} x^2$$

$$\Rightarrow 0 \leq \lim_{x \to 0} \left( x^2 \sin \frac{1}{x} \right) \leq 0$$

$$\Rightarrow \lim_{x \to 0} \left( x^2 \sin \frac{1}{x} \right) = 0$$

$$\therefore \lim_{x \to 0^-} f(x) = 0$$

Similarly, $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left( x^2 \sin \frac{1}{x} \right) = \lim_{x \to 0} \left( x^2 \sin \frac{1}{x} \right) = 0$

$$\therefore \lim_{x \to 0^-} f(x) = f(0) = \lim_{x \to 0^+} f(x)$$

Therefore, $f$ is continuous at $x = 0$

From the above observations, it can be concluded that $f$ is continuous at every point of the real line.

Thus, $f$ is a continuous function.

**Chapter 9 Continuity Ex 9.2 Q18**

$$f(x) = \frac{1}{x+2}$$

$$\lim_{x \to -2^-} f(x) = \lim_{h \to 0} \frac{1}{-2-h+2} = \lim_{h \to 0} -\frac{1}{h} \to -\infty$$

$$\lim_{x \to -2^+} f(x) = \lim_{h \to 0} \frac{1}{-2+h+2} = \lim_{h \to 0} \frac{1}{h} \to \infty$$

$\therefore$ $f(x)$ is discontinuous at $x = -2$

Let $g(x) = f(f(x)) = \frac{x+2}{2x+5}$

$$\lim_{x \to -\frac{5}{2}^-} g(x) = \lim_{h \to 0} \frac{-\frac{5}{2}-h+2}{-5-h+5} = \lim_{h \to 0} -\frac{-\frac{5}{2}-h+2}{h} \to -\infty$$

$$\lim_{x \to -\frac{5}{2}^+} f(x) = \lim_{h \to 0} \frac{-\frac{5}{2}+h+2}{-5+h+5} = \lim_{h \to 0} \frac{-\frac{5}{2}-h+2}{h} \to \infty$$

$\therefore$ $g(x)$ is discontinuous at $x = -\dfrac{5}{2}$

$\therefore$ $f(f(x))$ is discontinuous at $x = -\dfrac{5}{2}$

$\therefore$ $f(x)$ is discontinuous at $x = -2$ and $-\dfrac{5}{2}$

**Chapter 9 Continuity Ex 9.2 Q19**

$$f(t) = \frac{1}{t^2+t-2}, \text{ where } t = \frac{1}{x-1}$$

Clearly $t = \dfrac{1}{x-1}$ is discontinuous at $x = 1$.

For $x \neq 1$, we have

$$f(t) = \frac{1}{t^2+t-2} = \frac{1}{(t+2)(t-1)}$$

This is discontinuous at $t = -2$ and $t = 1$

For $t = -2$, $t = \dfrac{1}{x-1} \Rightarrow x = \dfrac{1}{2}$

For $t = 1$, $t = \dfrac{1}{x-1} \Rightarrow x = 2$

Hence $f$ is discontinuous at $x = \dfrac{1}{2}$, $x = 1$ and $x = 2$.