Solution of Simultaneous Linear Equations Ex 8.2 Q1

$2x - y + z = 0$
$3x + 2y - z = 0$
$x + 4y + 3z = 0$

The systm can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$A \quad x = 0$$

Now $|A| = 2(10) + 1(10) + 1(10)$
$= 40$
$\neq 0$

Since $|A| \neq 0$, hence $x = y = z = 0$ is the only solution of this homogeneous system.

Solution of Simultaneous Linear Equations Ex 8.2 Q2

$2x - y + 2z = 0$
$5x + 3y - z = 0$
$x + 5y - 5z = 0$

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$\text{or } A \quad x = 0$$

$|A| = 2(-10) + 1(-24) + 2(22)$
$= -20 - 24 + 44$
$= 0$

Hence, the system has infinite solutions.

Let $z = k$
$2x - y = -2k$
$5x + 3y = k$

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$
$$A \quad x = B$$

$|A| = 6 + 5 = 11 \neq 0$ so A^{-1} exist

Now $adj\ A = \begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$

$$x = A^{-1}.B = \frac{1}{|A|}(adj\ A)B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -2k \\ k \end{bmatrix} = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$$

Hence, $x = \frac{-5k}{11}$, $y = \frac{12k}{11}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q3

$$3x - y + 2z = 0$$
$$4x + 3y + 3z = 0$$
$$5x + 7y + 4z = 0$$

$$|A| = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}$$

$$= B(-9) + 1(1) + 2(13) = -27 + 1 + 26 = -27 + 27$$
$$= 0$$

Hence, it has infinite solutions.

Let $z = k$

$$3x - y = -2k$$
$$4x + 3y = -3k$$

or $\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$

or $A \quad x = B$

$|A| = 9 + 4 = 13 \neq 0$ hence A^{-1} exists

$$adj\ A = \begin{bmatrix} 3 & -4 \\ 1 & 3 \end{bmatrix}' = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$$

Now $x = A^{-1}B = \frac{1}{|A|}(adj\ A)B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13} \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} -2k \\ -3k \end{bmatrix} = \frac{1}{13} \begin{bmatrix} -9k \\ -k \end{bmatrix}$$

Hence, $x = \frac{-9k}{13}$, $y = \frac{-k}{13}$, $z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q4

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$x + y - 6z = 0$
$x - y + 2z = 0$
$-3x + y + 2z = 0$

$$\text{Hence, } |A| = \begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\begin{aligned} &= 1(-4) - 1(8) - 6(-2) \\ &= -4 - 8 + 12 \\ &= 0 \end{aligned}$$

Hence, the system has infinite solutions.

Let $z = k$
$x + y = 6k$
$x - y = -2k$

$$\text{or} \quad \begin{bmatrix} 1 & +1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$\text{or} \quad A \quad x = B$$

$|A| = -1 - 1 = -2 \neq 0$ hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$x = A^{-1}B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & +1 \end{bmatrix}\begin{bmatrix} 6k \\ -2k \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-2}\begin{bmatrix} -6k + 2k \\ -6k - 2k \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -4k \\ -8k \end{bmatrix} = \begin{bmatrix} 2k \\ 4k \end{bmatrix}$$

Hence, $x = 2k$, $y = 4k$, $z = k$

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Solution of Simultaneous Linear Equations Ex 8.2 Q5

$x + y + z = 0$
$x - y - 5z = 0$
$x + 2y + 4z = 0$

$$|A| = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -5 \\ 1 & 2 & 4 \end{bmatrix}$$
$$= 1(6) - 1(9) + 1(3) = 9 - 9 = 0$$

Hence, the system has infinite solutions.

Let $z = k$

$x + y = -k$

$x - y = 5k$

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -k \\ 5k \end{bmatrix}$$

or $A \quad X = B$

$|A| = -2 \neq 0$, hence A^{-1} exists.

$$adj\ A = \begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}$$

so, $X = A^{-1} B = \frac{1}{|A|}(adj\ A)B = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} -k \\ 5k \end{bmatrix}$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \left(\frac{1}{-2}\right)\begin{bmatrix} k - 5k \\ k + 5k \end{bmatrix} = \begin{bmatrix} 2k \\ -3k \end{bmatrix}$$

$x = 2k,\ y = -3k, z = k$

Solution of Simultaneous Linear Equations Ex 8.2 Q6

$x + y - z = 0$
$x - 2y + z = 0$
$3x + 6y - 5z = 0$

Hence, $|A| = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ 3 & 6 & -5 \end{bmatrix}$

$= 1(4) - 1(-8) - 1(12)$
$= 4 + 8 - 12 = 0$

Hence, the system will have infinite solutions.

Let $z = k$
$x + y = -k$
$x - 2y = -k$

or $\begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} k \\ -k \end{bmatrix}$

or $A \quad X = B$

$|A| = -3 \neq 0$, hence A^{-1} exists.

Now, $adj\ A = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}' = \begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}$

Next $x = A^{-1}B$

$$= \frac{1}{|A|}(adj\ A)(B) = \frac{1}{-3}\begin{bmatrix} -2 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} k \\ -k \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} -2k + k \\ -2k \end{bmatrix}$$

$$= \frac{-1}{3}\begin{bmatrix} -k \\ -2k \end{bmatrix} = \begin{bmatrix} \frac{k}{3} \\ \frac{2k}{3} \end{bmatrix}$$

Hence, $x = \frac{k}{3}$, $y = \frac{2k}{3}$, $z = k$

or $x = k$, $y = 2k$, $z = 3k$

Solution of Simultaneous Linear Equations Ex 8.2 Q7

$3x + y - 2z = 0$
$x + y + z = 0$
$x - 2y + z = 0$

$$\text{Hence, } |A| = \begin{bmatrix} 3 & 1 & -2 \\ 1 & 1 & 1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\begin{aligned} |A| &= B(1+2) - 1(1-1) - 2(-3) \\ &= 9 - 0 + 6 \\ &= 15 \neq 0 \end{aligned}$$

Hence, the given system has only trivial solutions given by $x = y = z = 0$

Solution of Simultaneous Linear Equations Ex 8.2 Q8

$2x + 3y - z = 0$
$x - y - 2z = 0$
$3x + y + 3z = 0$

$$\text{Hence, } A = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$|A| = \begin{bmatrix} 2 & 3 & -1 \\ 1 & -1 & -2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$\begin{aligned} &= 2(-3+2) - 3(3+6) - 1(4) \\ &= -2 - 27 - 4 \\ &\neq 0 \end{aligned}$$

Hence, the system has only trivial solutions given by $x = y = z = 0$