

Adjoint and Inverse of Matrix Ex 7.2 Q1

$$A = \begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix}$$

For row transformations $A = IA$

$$\begin{bmatrix} 7 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{7}R_1$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 4R_1$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 0 & -\frac{25}{7} \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \left(-\frac{7}{25}\right)R_2$

$$\begin{bmatrix} 1 & \frac{1}{7} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{7} & 0 \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{7}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{21}{175} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix} A$$

Hence, $I = B A$

So, $B = \begin{bmatrix} \frac{3}{25} & \frac{1}{25} \\ \frac{4}{25} & -\frac{7}{25} \end{bmatrix}$ is the inverse of A .

Adjoint and Inverse of Matrix Ex 7.2 Q2

$$A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

For row- transformation $A = IA$

$$\begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{5}R_1$

$$\begin{bmatrix} 1 & \frac{2}{5} \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & \frac{2}{5} \\ 0 & \frac{1}{5} \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -\frac{2}{5} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 5R_2$

$$\begin{bmatrix} 1 & \frac{2}{5} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ -2 & 5 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{2}{5}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix} A$$

$$I = B.A$$

Hence, $B = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ is the inverse of A .

Adjoint and Inverse of Matrix Ex 7.2 Q3

$$\text{Let } A = \begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix}$$

For row transformations $A = IA$

$$\begin{bmatrix} 1 & 6 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + 3R_1$

$$\begin{bmatrix} 1 & 6 \\ 0 & 23 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \frac{1}{23} R_2$

$$\begin{bmatrix} 1 & 6 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{23} & \frac{1}{23} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 6R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{23} & \frac{-6}{23} \\ \frac{3}{23} & \frac{1}{23} \end{bmatrix} A$$

$$I = B.A$$

Hence, $B = \frac{1}{23} \begin{bmatrix} 5 & -6 \\ 3 & 1 \end{bmatrix}$ is the inverse of A .



Adjoint and Inverse of Matrix Ex 7.2 Q4

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

Now, $AI = I$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -\frac{1}{2} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 2R_2$

$$\begin{bmatrix} 1 & \frac{5}{2} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 \\ -1 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

or $I = BA$

Hence, B is the inv. of A .



Adjoint and Inverse of Matrix Ex 7.2 Q5

$$A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$$

Now, $IA = A$

$$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ \frac{-2}{3} & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 3R_2$

$$\begin{bmatrix} 1 & \frac{10}{3} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ -2 & 3 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{10}{3}R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

$$I = B.A$$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q6

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow \frac{R_3}{2}$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 + R_3$, $R_2 \rightarrow R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix} \cdot A$$

$$I = B \cdot A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & 1 \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q7

$$A = I A$$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{R_1}{2}$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 5R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ \frac{5}{2} & -1 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{-5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, $R_2 \rightarrow R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$I = B.A$$

Adjoint and Inverse of Matrix Ex 7.2 Q8

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix}$$

Here, $A = I A$

$$\begin{bmatrix} 2 & 3 & 1 \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 2 & 4 & 1 \\ 3 & 7 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & \frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & \frac{5}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & 1 & 0 \\ \frac{-3}{2} & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{3}{2}R_2$, $R_3 \rightarrow R_3 - \frac{5}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & 0 \\ -1 & 1 & 0 \\ 1 & \frac{-5}{2} & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow 2R_3$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & \frac{-3}{2} & 0 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix} A$$

Hence, $A^{-1} = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 2 & -5 & 2 \end{bmatrix}$

Adjoint and Inverse of Matrix Ex 7.2 Q9

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

Now, $IA = I A$

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & -1 & \frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & -1 & \frac{4}{3} \\ 0 & 1 & -\frac{4}{3} \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ \frac{2}{3} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 + R_2, R_3 \rightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{3} \\ 0 & 0 & -\frac{1}{3} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ \frac{2}{3} & -1 & 1 \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow (-3)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{4}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ \frac{2}{3} & -1 & 0 \\ -2 & 3 & -3 \end{bmatrix} \cdot A$$

$R_2 \rightarrow R_2 + \frac{4}{3}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \cdot A$$

$$I = B \cdot A$$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q10

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - 2R_1$, $R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & -1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow (-1)R_2$

$$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & -3 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 2R_2$, $R_3 \rightarrow R_3 + 3R_2$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow \frac{R_3}{6}$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 2 & -1 & 0 \\ \frac{5}{6} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + 2R_3$, $R_2 \rightarrow R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & -\frac{1}{2} & -\frac{1}{6} \\ \frac{5}{6} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix} A$$

$$\text{Hence, } A^{-1} = \begin{bmatrix} -\frac{4}{3} & 1 & \frac{1}{3} \\ \frac{7}{6} & -\frac{1}{2} & -\frac{1}{6} \\ \frac{5}{6} & -\frac{1}{2} & \frac{1}{6} \end{bmatrix}$$

Adjoint and Inverse of Matrix Ex 7.2 Q11

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A = I A$$

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$\text{Applying } R_1 \rightarrow \frac{R_1}{2}$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 1 & 2 & 4 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$\text{Applying } R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & \frac{5}{2} & \frac{5}{2} \\ 0 & \frac{5}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{2} & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} \cdot A$$

$$\text{Applying } R_2 \rightarrow \left(\frac{2}{5}\right) R_2$$

$$\begin{bmatrix} 1 & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & 1 \\ 0 & \frac{5}{2} & -\frac{7}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -\frac{3}{2} & -1 & 1 \end{bmatrix} \cdot A$$

$$\text{Applying } R_1 \rightarrow R_1 + \frac{1}{2} R_2, R_3 \rightarrow R_3 - \frac{5}{2} R_2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ -1 & -1 & 1 \end{bmatrix} \cdot A$$

$$\text{Applying } R_3 \rightarrow \frac{R_3}{-6}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} & 0 \\ -\frac{1}{5} & \frac{2}{5} & 0 \\ \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} \cdot A$$

$$\text{Applying } R_2 \rightarrow R_2 - R_3, R_1 \rightarrow R_1 - 2R_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{15} & -\frac{2}{15} & -\frac{1}{3} \\ -\frac{1}{11} & \frac{2}{7} & \frac{1}{11} \end{bmatrix} \cdot A$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{30}{6} & \frac{30}{6} & \frac{6}{6} \\ \frac{1}{6} & \frac{1}{6} & \frac{-1}{6} \end{bmatrix} \cdot A$$

$$[I \cdot A = A \cdot I]$$

Ans.

Adjoint and Inverse of Matrix Ex 7.2 Q12

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$$A = I \cdot A$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow R_2 - 3R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -2 & -5 \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow \frac{R_2}{-2}$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ -2 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 - R_2$, $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{-11}{2} \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{-7}{2} & \frac{1}{2} & \frac{-2}{11} \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow R_3 \cdot \left(\frac{-2}{11}\right)$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & 0 \\ \frac{3}{2} & \frac{-1}{2} & 0 \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_3$, $R_2 \rightarrow R_2 - \frac{5}{2}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-2}{11} & \frac{5}{11} & \frac{-1}{11} \\ \frac{-1}{11} & \frac{-3}{11} & \frac{5}{11} \\ \frac{7}{11} & \frac{-1}{11} & \frac{-2}{11} \end{bmatrix} \cdot A$$

Adjoint and Inverse of Matrix Ex 7.2 Q13

$$A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix}$$

Now, $IA = I A$

$$\begin{bmatrix} 2 & -1 & 4 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow \frac{1}{2}R_1$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 4 & 0 & 2 \\ 3 & -2 & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow R_2 - 4R_1$, $R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 0 & 2 & -6 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -2 & 1 & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & -\frac{1}{2} & 2 \\ 0 & 1 & -3 \\ 0 & -\frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{1}{2} & 0 \\ -\frac{3}{2} & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{2}R_2$, $R_3 \rightarrow R_3 + \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & \frac{-1}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ -2 & \frac{1}{4} & 1 \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow (-2)R_3$

$$\begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{4} & 0 \\ -1 & \frac{1}{2} & 0 \\ 4 & \frac{-1}{2} & -2 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 - \frac{1}{2}R_3$, $R_2 \rightarrow R_2 + 3R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & \frac{1}{2} & 1 \\ 11 & -1 & -6 \\ 4 & \frac{-1}{2} & -2 \end{bmatrix} \cdot A$$

$$I = B.A$$

Hence, B is the inv. of A .



Adjoint and Inverse of Matrix Ex 7.2 Q14

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

Now, $IA = I A$

$$\begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow \frac{1}{3}R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 3 & \frac{2}{3} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{3} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_2 \rightarrow \frac{1}{3}R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow R_3 - 4R_2$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & \frac{1}{9} \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ \frac{8}{9} & -\frac{4}{3} & 1 \end{bmatrix} \cdot A$$

Applying $R_3 \rightarrow 9R_3$

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{2}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ -\frac{2}{9} & \frac{1}{3} & 0 \\ 8 & -12 & 9 \end{bmatrix} \cdot A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{3}R_3$, $R_2 \rightarrow R_2 - \frac{2}{9}R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & -4 & 3 \\ -2 & 3 & -2 \\ 8 & -12 & 9 \end{bmatrix} \cdot A$$

or $I = B \cdot A$

Hence, B is the inv. of A .

Adjoint and Inverse of Matrix Ex 7.2 Q15

Consider the given matrix:

$$\text{Let } A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix}$$

We know that $A = IA$

Thus, we have,

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow 3R_1 + R_2$ and $R_3 \rightarrow R_3 - 2R_1$, we have,

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -5 \\ 0 & -5 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 - 3R_2$ and $R_3 \rightarrow R_3 + 5R_2$, we have,

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{-5}{9} \\ 0 & 0 & \frac{11}{9} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{-1}{3} & \frac{5}{9} & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow \frac{R_1}{\frac{11}{9}}$ we have,

$$\begin{bmatrix} 1 & 0 & \frac{-1}{3} \\ 0 & 1 & \frac{-5}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 + \frac{5}{9}R_3$ and $R_1 \rightarrow R_1 + \frac{1}{3}R_3$, we have,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-1}{11} & \frac{-2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix} A$$

$$\Rightarrow \text{Inverse of the given matrix is } \begin{bmatrix} \frac{-1}{11} & \frac{-2}{11} & \frac{3}{11} \\ \frac{2}{11} & \frac{4}{11} & \frac{5}{11} \\ \frac{-3}{11} & \frac{5}{11} & \frac{9}{11} \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} 11 & 11 & 11 & & & \\ & & & & & \\ & & & & & \end{array} \right]$$

Adjoint and Inverse of Matrix Ex 7.2 Q16

Consider the given matrix $\begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$$\text{Let } A = \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

We know that $A = IA$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow (-1)R_1$, we have

$$\begin{bmatrix} 1 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 - 3R_1$, we have

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \rightarrow \frac{R_2}{3}$, we have,

$$\begin{bmatrix} 1 & -1 & -2 \\ 0 & 1 & \frac{5}{3} \\ 0 & 4 & 7 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 3 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + R_2$ and $R_3 \rightarrow R_3 - 4R_2$, we have

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ \frac{5}{3} & -\frac{4}{3} & 1 \end{bmatrix} A$$

Applying $R_3 \rightarrow \frac{R_3}{3}$, we have

$$\begin{bmatrix} 1 & 0 & -\frac{1}{3} \\ 0 & 1 & \frac{5}{3} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -4 & 3 \end{bmatrix} A$$

Applying $R_1 \rightarrow R_1 + \frac{1}{3}R_3$, $R_2 \rightarrow R_2 - \frac{5}{3}R_3$, we have,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix} A$$

Thus, the inverse of the given matrix is $\begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ 5 & -4 & 3 \end{bmatrix}$.