

Chapter 6 Determinants Ex 6.5 Q1

$$\begin{aligned}\text{Here } D &= \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix} \\ &= 1(3) - 1(-3) - 2(3) \\ &= 3 + 3 - 6 \\ &= 0\end{aligned}$$

Since $D = 0$, so the system has infinite solutions:

Now let $z = k$,

$$x + y = 2k$$

$$2x + y = 3k$$

Solving these equations by Cramer's Rule

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}} = \frac{-k}{-1} = k$$

thus, we have $x = k, y = k, z = k$

and these values satisfy eq. (3)

Hence $x = k, y = k, z = k$

Chapter 6 Determinants Ex 6.5 Q2

$$\begin{aligned}\text{Here } D &= \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{vmatrix} \\ &= 2(4) - 3(1) + 4(-3) \\ &= 8 - 3 - 7 \\ &= -2 \\ &\neq 0\end{aligned}$$

So, the given system of equations has only the trivial solutions i.e. $x = 0 = y = z$:

$$\text{Hence } x = 0$$

$$y = 0$$

$$z = 0$$

Chapter 6 Determinants Ex 6.5 Q3

$$\begin{aligned}\text{Here } D &= \begin{vmatrix} 3 & 1 & 1 \\ 1 & -4 & 3 \\ 2 & 5 & -2 \end{vmatrix} \\ &= 3(8 - 15) - 1(-2 - 6) + 1(13) \\ &= -21 + 8 + 13 \\ &= 0\end{aligned}$$

So, the system has infinite solutions:

$$\text{Let } z = k,$$

$$\text{so, } 3x + y = -k$$

$$x - 4y = -3k$$

Now,

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -k & 1 \\ -3k & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{7k}{-13}$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 3 & -k \\ 1 & -3k \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 1 & -4 \end{vmatrix}} = \frac{-8k}{-13}$$

$$x = \frac{-7k}{13}, y = \frac{8k}{13}, z = k$$

and these values satisfy eq. (3)

$$\text{Hence } x = -7k, y = 8k, z = 13k$$

Chapter 6 Determinants Ex 6.5 Q4

$$D = \begin{vmatrix} 2\lambda & -2 & 3 \\ 1 & \lambda & 2 \\ 2 & 0 & \lambda \end{vmatrix}$$

$$= 3\lambda^3 + 2\lambda - 8 - 6\lambda$$

$$= 2\lambda^3 - 4\lambda - 8$$

which is satisfied by $\lambda = 2$ [$\therefore$ for non-trivial solutions $\lambda = 2$]

Now Let $z = k$,

$$4x - 2y = -3k$$

$$x + 2y = -3k$$

$$x = \frac{D_1}{D} = \frac{\begin{vmatrix} -3k & -2 \\ -2k & 2 \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-10k}{10} = -k$$

$$y = \frac{D_2}{D} = \frac{\begin{vmatrix} 4 & -3k \\ 1 & -2k \end{vmatrix}}{\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix}} = \frac{-5k}{10} = \frac{-k}{2}$$

Hence solution is $x = -k, y = \frac{-k}{2}, z = k$

Chapter 6 Determinants Ex 6.5 Q5

$$D = \begin{vmatrix} (a-1) & -1 & -1 \\ -1 & (b-1) & -1 \\ -1 & -1 & (c-1) \end{vmatrix}$$

Now for non-trivial solution, $D = 0$

$$0 = (a-1)[(b-1)(c-1) - 1] + 1[-c + 1 - 1] - [1 + b - 1]$$

$$0 = (a-1)[bc - b - c + 1 - 1] - c - b$$

$$0 = abc - ab - ac + 1 - 1 - c - b$$

$$abc - ab - ac = b + c$$

Hence proved