Chapter Determinants Ex 6.3 Q1(i)

If the vertices of a triangle are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) then the area of the triangle is given by :

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Substituting the values

$$\Delta = \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

expanding the determinant along R_1

$$= \frac{1}{2}\left[3\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} - 8\begin{vmatrix} -4 & 1 \\ 5 & 1 \end{vmatrix} + 1\begin{vmatrix} -4 & 2 \\ 5 & -1 \end{vmatrix}\right]$$

$$= \frac{1}{2}\left[3(3) - 8(-9) + 1(-6)\right]$$

$$= \frac{1}{2}\left[9 + 72 - 6\right] = \frac{75}{2} \text{ sq. units}$$

The area of the Δ is $\frac{75}{2}$ sq. units

Chapter Determinants Ex 6.3 Q1(ii)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$$

expanding along R_1

$$= \frac{1}{2}\left[2(-7) - 7(-9) + 1(-2)\right]$$

$$= \frac{1}{2}\left[-14 + 63 - 2\right]$$

$$= \frac{47}{2} \text{ sq. units}$$

The area of the Δ is $\frac{47}{2}$ sq. units

Chapter Determinants Ex 6.3 Q1(iii)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} -1 & -8 & 1 \\ -2 & -3 & 1 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-1(-5) + 8(-5) + 1(5)\right]$$

$$= \frac{1}{2}\left[5 - 40 + 5\right] = \frac{-30}{2} = 15 \text{ sq. units}$$

$\because$ Area can not be negative, so answer will be 15 sq. units.

The area of the Δ is 15 sq. units.

Chapter Determinants Ex 6.3 Q1(iv)

The area is given by:

$$\Delta = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[0 - 0 + 1(18)\right] = 9 \text{ sq. units}$$

The area is 9 sq. units

Chapter Determinants Ex 6.3 Q2(i)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 5 & 5 & 1 \\ -5 & 1 & 1 \\ 10 & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$= \frac{1}{2}\left[5(-6) - 5(-15) + 1(-35 - 10)\right]$$

$$= \frac{1}{2}\left[-35 + 75 - 45\right]$$

$$= \frac{1}{2}\left[0\right]$$

$$= 0$$

Since the area of the triangle is zero, hence the points are collinear.

Chapter Determinants Ex 6.3 Q2(ii)

If 3 points are collinear, then the area of the triangle then form will be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 4 & 5 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$= \frac{1}{2}\left[1(-4) + 1(-2) + 1(6)\right]$$

$$= 0$$

Since the area of the triangle is zero, hence the points are collinear.

Chapter Determinants Ex 6.3 Q2(iii)

If the points are collinear, then the area of the triangle will be zero.

So

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ 8 & 8 & 1 \\ 5 & 2 & 1 \end{vmatrix} = 0$$

L.H.S

Expanding along R_1

$$= \frac{1}{2}\left[3(6) + 2(3) + 1(-24)\right]$$

$$= \frac{1}{2}\left[18 + 6 - 24\right]$$

$$= \frac{1}{2}\left[0\right]$$

$$= 0$$

Since the area of the triangle is zero, hence given points are collinear.

Chapter Determinants Ex 6.3 Q2(iv)

If given points are collinear, then the area of the triangle must be zero.

Hence

$$= \frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & -2 & 1 \\ 5 & 8 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[2(-10) - 3(-6) + 1(2)\right]$$

$$= \frac{1}{2}\left[-20 + 18 + 2\right]$$

$$= \frac{1}{2}[0]$$

$= 0$

Hence the given points are collinear.

Chapter Determinants Ex 6.3 Q3

If the given points are collinear, the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & 0 & 1 \\ 0 & b & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

Expanding along R_1

$$= \frac{1}{2}\left[a(b-1) - 0(0-1) + 1(-b)\right] = 0$$

$or\ ab - a - 0 - b = 0$

$or\ ab = a + b$

Hence proved

Chapter Determinants Ex 6.3 Q4

If the given points are collinear, then the area of the triangle must be zero.

Hence

$$\frac{1}{2}\begin{vmatrix} a & b & 1 \\ a' & b' & 1 \\ a-a' & b-b' & 1 \end{vmatrix} = 0$$

or

$$\frac{1}{2}\left[a(b'-b+b') - b(a'-a+a') + 1(a'b - a'b' - ab' + a'b')\right] = 0$$

or $\frac{1}{2}\left[ab' - ab + ab' - a'b + ab - a'b + a'b - ab'\right] = 0$

or $ab' - a'b = 0$

$ab' = a'b$

Hence proved

Chapter Determinants Ex 6.3 Q5

If the points are collinear, then the area of the triangle must be zero.

Hence

$$\begin{vmatrix} 1 & -5 & 1 \\ -4 & 5 & 1 \\ \lambda & 7 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$1(-2) + 5(-4 - \lambda) + 1(-28 - 5\lambda) = 0$$

$$-2 - 20 - 5\lambda - 28 - 5\lambda = 0$$

$$-50 - 10\lambda = 0$$

$$\lambda = -5$$

Hence $\lambda = -5$

Chapter Determinants Ex 6.3 Q6

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix} \right|$$

$$\pm 2 \times 35 = \begin{vmatrix} x & 4 & 1 \\ 2 & -6 & 1 \\ 5 & 4 & 1 \end{vmatrix}$$

$$\pm 70 = x(-10) - 4(-3) + 1(38)$$

$$\pm 70 = -10x + 12 + 38$$

$$\pm 70 = -10x + 50 \qquad ---(1)$$

Taking $(+)$ sign

$$+70 = -10x + 50$$

$$10x = -20 \text{ or } x = -2$$

Again taking $(-)$ sign

$$-70 = -10x + 50$$

$$10x = 120 \text{ or } x = 12$$

Hence $x = -2, 12$

Chapter Determinants Ex 6.3 Q7

$$\text{Area} = \frac{1}{2} \begin{vmatrix} 1 & 4 & 1 \\ 2 & 3 & 1 \\ -5 & -3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[1(6) - 4(7) + 1(-6 + 15)\right]$$

$$= \frac{1}{2}\left[6 - 28 + 9\right]$$

$$= \frac{1}{2}\left[-13\right]$$

$$= \frac{13}{2} \text{ sq. units} \qquad [\because \text{Area can not be negative}]$$

Also, since the area of the triangle is non-zero.

Hence these points are non-collinear.

Chapter Determinants Ex 6.3 Q8

$$\text{Area} = \frac{1}{2}\begin{vmatrix} -3 & 5 & 1 \\ 3 & -6 & 1 \\ 7 & 2 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[-3(-8) - 5(-4) + 1(48)\right]$$

$$= \frac{1}{2}\left[24 + 20 + 48\right]$$

$$= 46 \text{ sq. units}$$

Hence the area is 46 sq. units.

Chapter Determinants Ex 6.3 Q9

If the given points are collinear, then the area of the triangle must be zero.

$$\text{so } \frac{1}{2}\begin{vmatrix} k & 2-2k & 1 \\ -k+1 & 2k & 1 \\ -4-k & 6-2k & 1 \end{vmatrix} = 0$$

expanding along R_1

$$k(2k - 6 + 2k) - (2 - 2k)(-k + 1 + 4 + k) + 1(1 - k)\times(6 - 2k) - 2k(-4 - k) = 0$$

$$k(4k - 6) - (2 - 2k)(5) + 1\left[6 - 2k - 6k + 2k^2 + 8k + 2k^2\right] = 0$$

$$4k^2 - 6k - 10 + 10k + 6 + 4k^2 = 0$$

$$8k^2 + 4k - 4 = 0$$

$$8k^2 + 8k - 4k - 4 = 0 \qquad \text{(Middle term splitting)}$$

$$8k(k + 1) - 4(k + 1) = 0$$

$$(8k - 4)(k + 1) = 0$$

If $8k - 4 = 0$ or if $k + 1 = 0$

$k = \frac{1}{2}$ $\qquad$ $k = -1$

Hence $k = -1, \frac{1}{2}$

Chapter Determinants Ex 6.3 Q10

Since the points are collinear, hence the area of the triangle must be zero.

$$\text{so } \frac{1}{2}\begin{vmatrix} x & -2 & 1 \\ 5 & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$\text{or } x(-6) + 2(-3) + 1(24) = 0$$

$$\text{or } -6x - 6 + 24 = 0$$

$$-6x + 18 = 0$$

$$x = 3$$

Hence $x = 3$

Chapter Determinants Ex 6.3 Q11

Since the points are collinear, hence the area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$$

$$3(-6) + 2(x - 8) + 1(8x - 16) = 0$$
$$-18 + 2x - 16 + 8x - 16 = 0$$
$$10x = 50$$
$$x = 5$$

Hence $x = 5$

Chapter Determinants Ex 6.3 Q12(i)

Let $A(x, y)$, $B(1, 2)$ and $C(3, 6)$ are 3 points in a line.

Since these points are collinear, hence area of the triangle must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$x(-4) - y(-2) + 1(0) = 0$$
$$-4x + 2y = 0$$
$$\text{or } 2x - y = 0$$
$$\text{or } y = 2x$$

Hence the equation is $y = 2x$

Chapter Determinants Ex 6.3 Q12(ii)

Let $A(x, y)$, $B(3, 1)$ and $C(9, 3)$ are 3 points in a line.

Since these points are collinear, hence the area of the triangle ABC must be zero.

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along R_1

$$x(-2) - y(-6) + 1(0) = 0$$
$$-2x + 6y = 0$$
$$x - 3y = 0$$

Hence the equation of the line is $x - 3y = 0$

Chapter Determinants Ex 6.3 Q13(i)

$$\text{Area} = \frac{1}{2}\left\|\begin{matrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{matrix}\right\|$$

$$\pm 4 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding along R_1

$$\pm 8 = k(-2) - 0(4-0) + 1(8)$$

$$\pm 8 = -2k + 8$$

Taking positive $(+)$ sign

$+8 = -2k + 8$ or $k = 0$

Taking negative $(-)$ sign

$-8 = -2k + 8$ or $k = 8$

Hence $k = 0, 8$