Determinants Ex 6.1 Q1(i)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = -1$ $\left[\begin{array}{l}\text{In a } 2\times 2 \text{ matrix, the minor is obtained for a particular element, by} \\ \text{deleting that row and column where the element is present.}\end{array}\right]$

$M_{21} = 20$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\left[\because C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$= (+1)(-1)$

$= -1$

$C_{21} = (-1)^{2+1} M_{21}$

$= (-1)^3 \times 20$

$= -20$

Also,

$|A| = 5(-1) - (0) \times (20)$ $\left[\text{If } A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ then } |A| = a_{11}a_{22} - a_{21}a_{12}\right]$

$= -5$

Determinants Ex 6.1 Q1(ii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is present at the i^{th} row and j^{th} column.

Now,

$M_{11} = 3$ $\left[\begin{array}{l}\text{In a } 2\times 2 \text{ matrix, the minor of an element is obtained by} \\ \text{deleting that row and that column in which it is present.}\end{array}\right]$

$M_{21} = 4$

$C_{11} = (-1)^{1+1} \times M_{11}$ $\left[C_{ij} = (-1)^{i+j} \times M_{ij}\right]$

$C_{21} = (-1)^{2+1} \times M_{21}$

$= (-1)^3 \times 4$

$= -4$

Also,

$|A| = (-1) \times (3) - (2) \times (4)$

$= -3 - 8$

$= -11$

Determinants Ex 6.1 Q1(iii)

Let M_{ij} and C_{ij} represents the minor and co-factor respectively of an element which is placed at the i^{th} row and j^{th} column.

Now,

$M_{11} = \begin{bmatrix} -1 & 2 \\ 5 & 2 \end{bmatrix}$ $\left[\text{In a } 3\times 3 \text{ matrix, } M_{ij} \text{ equals to the determinant of the } 2\times 2 \text{ sub-matrix obtained by leaving the } i^{th} \text{ row and } j^{th} \text{ column of } A.\right]$

$= (-1)\times(2)-(5)\times(2)$

$= -2-10$

$= -12$

$M_{21} = \begin{bmatrix} -3 & 2 \\ 5 & 2 \end{bmatrix} = (-3)\times(2)-(5)\times(2) = -6-10 = -16$

$M_{31} = \begin{bmatrix} -3 & 2 \\ -1 & 2 \end{bmatrix} = (-3)(2)-(-1)(2) = -6+2 = -4$

$C_{11} = (-1)^{1+1} M_{11}$ $\left(C_{ij} = (-1)^{i+j} \times M_{ij}\right)$

$= (+)(-12) = -12$

$C_{21} = (-1)^{2+1} M_{21} = (-1)^3(-16) = 16$

$C_{31} = (-1)^{3+1} M_{31} = (-1)^4(-4) = -4$

Also, expanding the determinant along the first column.

$|A| = a_{11}\times\left((-1)^{1+1}\times M_{11}\right) + a_{21}\times\left((-1)^{2+1}\times M_{21}\right) + a_{31}\times\left((-1)^{3+1}\times M_{31}\right)$

$= a_{11}\times C_{11} + a_{21}\times C_{21} + a_{31}\times C_{31}$

$= 1\times(-12) + 4\times 16 + 3\times(-4)$

$= -12+48-12 = 24$

Determinants Ex 6.1 Q1(iv)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & ca \\ c & ab \end{bmatrix}$$
$$= ab^2 - ac^2$$

$$M_{21} = \begin{bmatrix} a & bc \\ c & ab \end{bmatrix}$$
$$= a^2b - c^2b$$

$$M_{31} = \begin{bmatrix} a & bc \\ b & ca \end{bmatrix}$$
$$= a^2c - b^2c$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +\left(ab^2 - ac^2\right)$$
$$C_{21} = (-1)^{2+1} \times M_{21} = -\left(a^2b - c^2b\right)$$
$$C_{31} = (-1)^{3+1} \times M_{31} = +\left(a^2c - b^2c\right)$$

Also, expanding the determinant, along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= 1\left(ab^2 - ac^2\right) + 1\left(c^2b - a^2b\right) + 1 \times \left(a^2c - b^2c\right)$$
$$= ab^2 - ac^2 + c^2b - a^2b + a^2c - b^2c$$

Determinants Ex 6.1 Q1(v)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} 5 & 0 \\ 7 & 1 \end{bmatrix} = 5 - 0 = 5$$

$$M_{21} = \begin{bmatrix} 2 & 6 \\ 7 & 1 \end{bmatrix} = 2 - 42 = -40$$

$$M_{31} = \begin{bmatrix} 2 & 6 \\ 5 & 0 \end{bmatrix} = 0 - 30 = -30$$

$$C_{11} = (-1)^{1+1} \times M_{11} = +5$$

$$C_{21} = (-1)^{2+1} \times M_{21} = (-)(-40) = 40$$

$$C_{31} = (-1)^{3+1} \times M_{31} = +(-30) = -30$$

Now, expanding the determinant along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= 0 \times 5 + 1 \times (40) + 3 \times (-30)$$
$$= 40 - 90$$
$$= -50$$

Determinants Ex 6.1 Q1(vi)

Let M_{ij} and C_{ij} are respectively the minor and co-factor of the element a_{ij}.

Now,

$$M_{11} = \begin{bmatrix} b & f \\ f & c \end{bmatrix} = bc - f^2$$

$$M_{21} = \begin{bmatrix} h & g \\ f & c \end{bmatrix} = hc - gf$$

$$M_{31} = \begin{bmatrix} h & g \\ b & f \end{bmatrix} = hf - bg$$

$$Also\ C_{11} = (-1)^{1+1} M_{11} = bc - f^2$$

$$C_{21} = (-1)^{2+1} M_{21} = -(hc - gf)$$

$$C_{31} = (-1)^{3+1} M_{31} = hf - bg$$

Also, expanding along the first column.

$$|A| = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$$
$$= a(bc - f^2) + h(-)(hc - gf) + g(hf - bg)$$
$$= abc - af^2 + hgf - h^2c + ghf - bg^2$$

Determinants Ex 6.1 Q1(vii)

We have,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix}$$

Here, $M_{11} = \begin{bmatrix} 0 & 1 & -2 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = -1(0+10) - 1(1-2) = -9$

$$M_{21} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 1 \\ -1 & 5 & 0 \end{bmatrix} = 9$$

$$M_{31} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ -1 & 5 & 0 \end{bmatrix} = -9$$

$$M_{41} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & -2 \\ 1 & - & 1 \end{bmatrix} = 0$$

$\therefore$ $C_{11} = (-1)^{1+1} M_{11} = -9$

$C_{21} = (-1)^{3} M_{21} = -9$

$C_{31} = (-4)^{4} M_{31} = -9$

$C_{41} = (-1)^{5} M_{41} = 0$

Hence,

$$\begin{bmatrix} 2 & -1 & 0 & 1 \\ -3 & 0 & 1 & -2 \\ 1 & 1 & -1 & 1 \\ 2 & -1 & 5 & 0 \end{bmatrix} = 2 \times C_{11} + (-3) C_{21} + 1 \times C_{31} + 2 \times C_{41} = -9[2-3+1] = 0$$

Determinants Ex 6.1 Q2(i)

Let $A = \begin{vmatrix} x & -7 \\ x & 5x+1 \end{vmatrix}$

$|A| = x(5x+1) + 7 \times x$

$= 5x^2 + x + 7x$

$= 5x^2 + 8x$

Hence $|A| = 5x^2 + 8x$

Determinants Ex 6.1 Q2(ii)

Let $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$|A| = \cos\theta \times \cos\theta + \sin\theta \times \sin\theta$

$= \cos^2\theta + \sin^2\theta$

$= 1$

Hence $|A| = 1$

Determinants Ex 6.1 Q2(iii)

Let $A = \begin{vmatrix} \cos 15^\circ & \sin 15^\circ \\ \sin 75^\circ & \cos 75^\circ \end{vmatrix}$

$$|A| = \cos 15^\circ \cos 75^\circ - \sin 15^\circ \sin 75^\circ$$

$= \cos(75 + 15)$ $\quad (\because \cos A \cos B - \sin A \sin B = \cos(A+B))$

$= \cos 90^\circ$

$= 0$

Hence $|A| = 0$

Determinants Ex 6.1 Q2(iv)

Let $A = \begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix}$

$$|A| = (a+ib)(a-ib) - (c+id)(-c+id)$$

$= (a^2+b^2) + (c+id)(c-id)$ $\quad$ (Taking $(-)$ sign common from $-c+id$)

$\quad$ $(Also\ (a+ib)(a-ib) = a^2+b^2)$

$= a^2+b^2+c^2+d^2$

Hence $|A| = a^2+b^2+c^2+d^2$

Determinants Ex 6.1 Q3

Since $|AB| = |A| \times |B|$

Hence $|A|^2 = |A| \times |A|$ $\quad$ ---(1)

Now let $A = \begin{vmatrix} 2 & 3 & 7 \\ 13 & 17 & 5 \\ 15 & 20 & 12 \end{vmatrix}$

Expanding along the first column, we get

$$|A| = 2\begin{vmatrix} 17 & 5 \\ 20 & 12 \end{vmatrix} - 3\begin{vmatrix} 13 & 5 \\ 15 & 12 \end{vmatrix} + 7\begin{vmatrix} 13 & 17 \\ 15 & 20 \end{vmatrix}$$

$= 2(204-100) - 3(156-75) + 7(260-255)$

$= 2(104) - 3(81) + 7(5)$

$= 208 - 243 + 35$

$= 243 - 243$

$= 0$

Hence from eq.(1)

$|A|^2 = |A| \times |A| = 0 \times 0 = 0$

Determinants Ex 6.1 Q4

Evaluating the given determinant

$\sin 10^\circ \times \cos 80^\circ + \cos 10^\circ \sin 80^\circ$

$= \sin(10^\circ + 80^\circ)$ $\quad [\because \sin A \cos B + \cos A \sin B = \sin(A+B)]$

$= \sin 90^\circ$

$= 1$

Hence proved

Determinants Ex 6.1 Q5

We will evaluate the given determinant

(i) Along the first row

(ii) Along the first column

(i) Along the first row

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 7 & -2 \\ -3 & 1 \end{vmatrix} - 5\begin{vmatrix} 7 & 1 \\ -3 & 4 \end{vmatrix}$$

$= 2(1+8) - 3(7-6) - 5(28+3)$

$= 2(9) - 3(1) - 5(31)$

$= 18 - 3 - 155 = -140$

(ii) Along the first column

$$|A| = 2\begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 7\begin{vmatrix} 3 & -5 \\ 4 & 1 \end{vmatrix} - 3\begin{vmatrix} 3 & -5 \\ 1 & -2 \end{vmatrix}$$

$= 2(1+8) - 7(3+20) - 3(-6+5)$

$= 18 - 7(23) - 3(-1)$

$= 18 - 161 + 3$

$= 21 - 161$

$= -140$

We can see, the answer is same with both the methods.

Determinants Ex 6.1 Q6

$$\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$$

$= -\sin\alpha(-\sin\beta\cos\alpha) - \cos\alpha(\sin\alpha\sin\beta)$

$= 0$

Determinants Ex 6.1 Q7

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding along C_3, we have:

$$\begin{aligned}\Delta &= -\sin\alpha\left(-\sin\alpha\sin^2\beta - \cos^2\beta\sin\alpha\right) + \cos\alpha\left(\cos\alpha\cos^2\beta + \cos\alpha\sin^2\beta\right) \\ &= \sin^2\alpha\left(\sin^2\beta + \cos^2\beta\right) + \cos^2\alpha\left(\cos^2\beta + \sin^2\beta\right) \\ &= \sin^2\alpha(1) + \cos^2\alpha(1) \\ &= 1\end{aligned}$$

Determinants Ex 6.1 Q8

Let $A = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}$

$\Rightarrow |A| = 2 - 10 = -8$

$B = \begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$

$\Rightarrow |B| = 20 + 6 = 26$

$$\text{Now } AB = \begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times4+5\times2 & 2\times(-3)+5\times5 \\ 2\times4+1\times2 & 2\times(-3)+1\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 8+10 & -6+25 \\ 8+2 & -6+5 \end{bmatrix}$$

$$= \begin{bmatrix} 18 & 19 \\ 10 & -1 \end{bmatrix}$$

$$\Rightarrow |AB| = 18\times(-1) - (10)(19)$$

$$= -18 - 190 = -208$$

$$\text{Now } |AB| = |A|\times|B|$$

$$-208 = (-8)\times(26)$$

$$-208 = -208$$

Hence verified.

Determinants Ex 6.1 Q9

Let $A = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$

Evaluating the determinant along the first column

$$|A| = 1\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} - 0\begin{vmatrix} 0 & 1 \\ 0 & 4 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 1 \times (4 - 0) - 0 + 0$$

$$= 4$$

Again $3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$ (every element of A will be multiplied by 3)

Now, evaluating this determinant

$$|3A| = 3\begin{vmatrix} 3 & 6 \\ 0 & 12 \end{vmatrix} - 0\begin{vmatrix} 0 & 3 \\ 0 & 12 \end{vmatrix} + 0\begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}$$

$$= 3(36 - 0) - 0 + 0$$

$$= 108$$

Now, according to the question

$|3A| = 27|A|$

$108 = 27(4)$ (Substituting values)

$108 = 108$

Hence proved

Determinants Ex 6.1 Q10

(i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2\times1-5\times4 = 2x\times x-6\times4$

$\Rightarrow 2-20 = 2x^2-24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

(ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2\times5-3\times4 = x\times5-3\times2x$

$\Rightarrow 10-12 = 5x-6x$

$\Rightarrow -2 = -x$

$\Rightarrow x = 2$

(iii)

$\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

$3-x^2 = 3-8$

$x^2 = 8$

$x = \pm2\sqrt{2}$

(iv)

$\begin{vmatrix} 3x & 7 \\ 2 & 4 \end{vmatrix} = 10$

$12x-14 = 10$

$12x = 24$

$x = 2$

Determinants Ex 6.1 Q11

Let $A = \begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix}$

Expanding the given determinant along the first column

$|A| = x^2\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} - 0\begin{vmatrix} x & 1 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} x & 1 \\ 2 & 1 \end{vmatrix}$

$28 = x^2(8-1)-0(4x-1)+3(x-2)$

$28 = 7x^2+3x-6$

or

$7x^2+3x-6 = 28$

$7x^2+3x-34 = 0$

Solving using quadratic formula, we get x = 2.

Determinants Ex 6.1 Q12(i)

A matrix A is called singular if $|A| = 0$

Now expanding along the first row $|A|$

$$= (x-1)\begin{vmatrix} x-1 & 1 \\ 1 & x-1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & x-1 \end{vmatrix} + 1\begin{vmatrix} 1 & 1 \\ x-1 & 1 \end{vmatrix}$$

$$= (x-1)\left[(x-1)^2 - 1\right] - 1\left[x-1-1\right] + 1\left[1-x+1\right]$$

$$= (x-1)\left(x^2+1-2x-1\right) - 1(x-2) + 1(2-x)$$

$$= (x-1)\left(x^2-2x\right) - x + 2 + 2 - x$$

$$= (x-1) \times x \times (x-2) + (4-2x)$$

$$= (x-1) \times x \times (x-2) + 2(2-x)$$

$$= (x-1) \times x \times (x-2) - 2(x-2)$$

$$= (x-2)\left[x(x-1) - 2\right] \quad \text{(Taking } (x-2) \text{ common)}$$

Since A is a singular matrix, so $|A| = 0$

$i.e\ (x-2)\left(x^2 - x - 2\right) = 0$

either $(x-2) = 0$ or $x^2 - x - 2 = 0$

$x = 2$ or $x^2 - 2x + x - 2 = 0$

$x(x-2) + 1(x-2) = 0$

$(x-2)(x+1) = 0$

$x = 2, -1$

$x = 2 \text{ or } -1$

Determinants Ex 6.1 Q12(ii)

A matrix A is said to be singular if $|A| = 0$

Now

$$\begin{vmatrix} 1+x & 7 \\ 3-x & 8 \end{vmatrix} = 0$$

$$8 + 8x - 21 + 7x = 0$$

$$15x = 13$$

$$x = \frac{13}{15}$$

Chapter 6 Determinants Ex 6.2 Q1-i

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} = 2\begin{vmatrix} 1 & 3 & 5 \\ 1 & 3 & 5 \\ 31 & 11 & 38 \end{vmatrix} = 0$$