Binary Operations Ex 3.5 Q1

$a \times_4 b$ = the remainder when ab is divided by 4.

eg. (i) $2 \times 3 = 6 \Rightarrow 2 \times_4 3 = 2$

[When 6 is divided by 4 we get 2 as remainder]

(ii) $2 \times 3 = 4 \Rightarrow 2 \times_4 2 = 0$

[When 4 is divided by 4 we get 0 as remainder]

The composition table for $\times_4$ on set $S = \{0, 1, 2, 3\}$ *is* :

$\times_4$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	1

Binary Operations Ex 3.5 Q 2

$a +_5 b =$ the remainder when $a + b$ is divided by 5.

eg. $2 + 4 = 6 \Rightarrow 2 +_5 4 = 1$ $\because$ [we get 1 as remainder when 6 is divided by 5]

$2 + 4 = 7 \Rightarrow 3 +_5 4 = 2$ $\because$ [we get 2 as remainder when 7 is divided by 5]

The composition table for $+_5$ on set $S = \{0, 1, 2, 3, 4\}$.

$+_5$	0	1	2	3	4
0	0	1	2	3	4
1	1	2	3	4	0
2	2	3	4	0	1
3	3	4	0	1	2
4	4	0	1	2	3

Binary Operations Ex 3.5 Q3

$a \times_6 b$ = the remainder when the product of ab is divided by 6.

The composition table for $\times_6$ on set $S = \{0, 1, 2, 3, 4, 5\}$.

$\times_6$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Binary Operations Ex 3.5 Q4

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$.

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q5

$a \times_{10} b$ = the remainder when the product of ab is divided by 10.

The composition table for $\times_{10}$ on set $S = \{1, 3, 7, 9\}$

$\times_{10}$	1	3	7	9
1	1	3	7	9
3	3	9	1	7
7	7	1	9	3
9	9	7	3	1

We know that an element $b \in S$ will be the inverse of $a \in S$

if $a \times_{10} b = 1$ $\left[\because 1 \text{ is the identity element with respect to multiplication}\right]$

$\Rightarrow$ $3 \times_{10} b = 1$

From the above table $b = 7$

$\therefore$ Inverse of 3 is 7.

Binary Operations Ex 3.5 Q6

$a \times_7 b$ = the remainder when the product of ab is divided by 7.

The composition table for $\times_7$ on $S = \{1, 2, 3, 4, 5, 6\}$

$\times_7$	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	1	3	5
3	3	6	2	5	1	4
4	4	1	5	2	6	3
5	5	3	1	6	4	2
6	6	5	4	3	2	1

We know that 1 is the identity element with respect to multiplication

Also, b will be the inverse of a
if, $a \times_7 b = e = 1$

$\Rightarrow$ $3 \times_7 b = 1$

From the above table $3 \times_7 5 = 1$

$\therefore$ $b = 3^{-1} = 5$

Now, $3^{-1} \times_7 4 = 5 \times_7 4 = 6$

Binary Operations Ex 3.5 Q7

$a \times_{11} b$ = the remainder when the product of ab is divided by 11.

The composition table for $\times_{11}$ on Z_{11}

$\times_{11}$	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	1	3	5	7	9
3	3	6	9	1	4	7	10	3	5	8
4	4	8	1	5	9	2	6	10	3	7
5	5	10	4	9	3	8	2	7	1	6
6	6	1	7	2	8	3	9	4	10	5
7	7	3	10	6	2	9	5	1	8	4
8	8	5	3	10	7	4	1	9	6	3
9	9	7	5	3	1	10	8	6	4	2
10	10	9	8	7	6	5	4	3	2	1

From the above table

$5 \times_{11} 9 = 1$ $\quad$ [$\because$ 1 is the identity element]

$\therefore$ Inverse of 5 is 9.

Binary Operations Ex 3.5 Q8

$Z_5 = \{0, 1, 2, 3, 4\}$

$a \times_5 b$ = the remainder when the product of ab is divided by 5.

The composition table for $\times_5$ on $Z_5 = \{0, 1, 2, 3, 4\}$

$\times_5$	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

Binary Operations Ex 3.5 Q9

(i)

From the above table we can say that

$a*b=b*a=b$

$a*c=c*a=c$

$a*d=d*a=d$

$b*c=c*b=d$

$b*d=d*b=c$

$c*d=d*c=b$

$\therefore$ '*' is commutative

Again, $a,b,c\in S$

$\Rightarrow$ $(a*b)*c=b*c=d$ and

$a*(b*c)=a*d=d$

$\therefore$ $(a*b)*c=a*(b*c)$

$\therefore$ * is associative

We know that e will be identity element with respect to * if

$a*e=e*a=a$ for all $a\in S$

$\Rightarrow$ $a*a=a,\ a*b=b,\ a*c=c,\ a*d=d$

$\therefore$ a will be the identity element

Again,

b will be the inverse of a if

$b*a=a*b=e$

From the above table

$a*a=a,\quad b*b=b, c*c=c$ and $d*d=d$

$\therefore$ Inverse of $a=a$

$b=b$

$c=c$

$d=d$

(ii)

From the above table, we can observe

$aob = boa, \quad boc = cob$

$aoc = coa, \quad bod = dob$

$aod = doa, \quad cod = doc$

$\therefore$ 'o' is commutative on S

Again, for $a, b, c \in S$

$(aob)oc = aoc = a$ ---(i)

$ao(boc) = aoc = a$ ---(ii)

From (i) & (ii)

$(aob)oc = ao(boc)$

So, 'o' is associative on S

Now, we have,

$aob = a$

$bob = b$

$cob = c$

$dob = d$

$\Rightarrow$ b is the identity element with respect to 'o'

We know that x will be inverse of y

If $xoy = yox = e$

$\Rightarrow$ $xoy = yox = b$ $[\because e = b]$

Now, from the above table we find that

$bob = b$

$cod = b$

$doc = b$

$\therefore$ $b^{-1} = b$, $c^{-1} = d$, and $d^{-1} = c$

Not: a^{-1} does ont exist.

Binary Operations Ex 3.5 Q10

Let $X = \{0, 1, 2, 3, 4, 5\}$.

The operation * on X is defined as:

$$a * b = \begin{cases} a+b & \text{if } a+b<6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

An element $e \in X$ is the identity element for the operation *, if

$a * e = a = e * a \ \forall a \in X$.

For $a \in X$, we observed that:

$$a * 0 = a + 0 = a \qquad [a \in X \Rightarrow a + 0 < 6]$$

$$0 * a = 0 + a = a \qquad [a \in X \Rightarrow 0 + a < 6]$$

$$\therefore a * 0 = a = 0 * a \ \forall a \in X$$

Thus, 0 is the identity element for the given operation *.

An element $a \in X$ is invertible if there exists $b \in X$ such that $a * b = 0 = b * a$.

$$\text{i.e.,} \begin{cases} a+b=0=b+a, & \text{if } a+b<6 \\ a+b-6=0=b+a-6, & \text{if } a+b \geq 6 \end{cases}$$

i.e.,

$a = -b$ or $b = 6 - a$

But, $X = \{0, 1, 2, 3, 4, 5\}$ and $a, b \in X$. Then, $a \neq -b$.

Therefore, $b = 6 - a$ is the inverse of a $a \in X$.

Hence, the inverse of an element $a \in X$, $a \neq 0$ is $6 - a$ i.e., $a^{-1} = 6 - a$.