EXERCISE NO: 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \frac{1}{2}, 1, 2$ $2x^3 + x^2 - 5x + 2$; ½, 1, -2

(ii) $x^3 - 4x^2 + 5x - 2; 2, 1, 1$

Solution 1:

(i)p(x) = $2x^3 + x^2 - 5x + 2$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$

$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$

$$= 0$$

$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$

$$= 0$$

$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$

$$= -16 + 4 + 10 + 2 = 0$$

Therefore, $\frac{1}{2}$, 1, and −2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain *a* = 2, *b* = 1, *c* = −5, *d* = 2

We can take $\alpha = \frac{1}{2}, \beta = 1, y = -2$

$$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1

$p(2) = 2^3 - 4(2^2) + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 0$

$p(1) = 1^3 - 4(1^2) + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 0$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we obtain $a = 1$, $b = -4$, $c = 5$, $d = -2$.

Verification of the relationship between zeroes and coefficient of the given polynomial

Sum of zeroes $= 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

Multiplication of zeroes taking two at a time = (2)(1) + (1)(1) + (2)(1)

$= 2 + 1 + 2 = 5 = \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes $= 2 \times 1 \times 1 = 2 = \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Solution 2:

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be α, β, and γ.

It is given that

$\alpha + \beta + \gamma = \frac{2}{1} = \frac{-b}{a}$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{-7}{1} = \frac{c}{a}$$

$$\alpha\beta\gamma = \frac{-14}{1} = \frac{-d}{a}$$

If *a* = 1, then *b* = −2, *c* = −7, *d* = 14

Hence, the polynomial is $x^3 - 2x^2 - 7x + 14$.

Question 3:

If the zeroes of polynomial $x^3 - 3x^2 + x + 1$ are $a - b, a, a + b$, find *a* and *b*.

Solution 3:

$p(x) = x^3 - 3x^2 + x + 1$

Zeroes are *a* − *b*, *a* + *a* + *b*

Comparing the given polynomial with $px^3 + qx^2 + rx + t$, we obtain

p = 1, *q* = −3, *r* = 1, *t* = 1

Sum of zeroes = a – b + a + a + b

$$\frac{-q}{p} = 3a$$

$$\frac{-(-3)}{1} = 3a$$

$$3 = 3a$$

$$a = 1$$

The zeroes are 1 – b, 1 + b.

Multiplication of zeroes = 1(1 – b)(1 + b)

$$\frac{-t}{p} = 1 - b^2$$

$$\frac{-1}{1} = 1 - b^2$$

$$1 - b^2 = -1$$

$$1 + 1 = b^2$$

$$b = \pm\sqrt{2}$$

Hence, *a* = 1 and *b* = $\sqrt{2}$ or $-\sqrt{2}$.

Question 4:

It two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find other zeroes.

Solution 4:

Given that $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

Therefore, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find the quotient by dividing $x^4 - 6x^3 - 26x^2 + 138x - 35$ by $x^2 - 4x + 1$.

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

Clearly, $= x^4 - 6x^3 - 26x^2 + 138x - 35 = \left(x^2 - 4x + 1\right)\left(x^2 - 2x - 35\right)$

It can be observed that $\left(x^2 - 2x - 35\right)$ is also a factor of the given polynomial.

And $= \left(x^2 - 2x - 35\right) = (x - 7)(x + 5)$

Therefore, the value of the polynomial is also zero when or $x - 7 = 0$

Or $x + 5 = 0$

Or $x = 7$ or -5

Hence, 7 and -5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4 - 6x^3 + 16x^2 - 25x - 10$ is divided by another Polynomial $x^2 - 2x + k$, the remainder comes out to be *x* + *a*, find *k* and *a*.

Solution 5:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x - 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be perfectly divisible by $x^2 - 2x + k$.

Let us divide by $x^4 - 6x^3 + 16x^2 - 26x - 10 - a$ by $x^2 - 2x + k$

$$\text{Quotient: } x^2 - 4x + (8 - k)$$

$$x^2 - 2x + k \,\big)\, x^4 - 6x^3 + 16x^2 - 26x + 10 - a$$

$$x^4 - 2x^3 + kx^2$$

$$- \quad + \quad -$$

$$-4x^3 + (16 - k)x^2 - 26x$$

$$-4x^3 + 8x^2 - 4kx$$

$$+ \quad - \quad +$$

$$(8 - k)x^2 - (26 - 4k)x + 10 - a$$

$$(8 - k)x^2 - (16 - 2k)x + (8k - k^2)$$

$$- \quad + \quad -$$

$$(-10 + 2k)x + (10 - a - 8k + k^2)$$

$$(x^2 - 4x + 1)(x^2 - 2x - 35) = (x - 7)(x + 5)$$

It can be observed that $(-10 + 2k)x + (10 - a - 8k + k^2)$ will be 0.

Therefore, $(-10 + 2k) = 0$ and $(10 - a - 8k + k^2) = 0$

For $(-10 + 2k) = 0$,

$2k = 10$

And thus, $k = 5$

For $\left(10 - a - 8k + k^2\right) = 0$

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$
