Functions Ex2.2 Q1(i)

Since, $f : R \to R$ and $g : R \to R$

$\therefore \quad f \circ g : R \to R$ and $gof : R \to R$

Now, $f(x) = 2x + 3$ and $g(x) = x^2 + 5$

$$g \circ f(x) = g(2x+3) = (2x+3)^2 + 5$$

$$\Rightarrow g \circ f(x) = 4x^2 + 12x + 14$$

$$f \circ g(x) = f(g(x)) = f(x^2+5) = 2(x^2+5) + 3$$

$$\Rightarrow f \circ g(x) = 2x^2 + 13$$

Functions Ex2.2 Q1(ii)

$$f(x) = 2x + x^2 \quad \text{and} \quad g(x) = x^3$$

$$g \circ f(x) = g(f(x)) = g(2x + x^2)$$

$$g \circ f(x) = (2x + x^2)^3$$

$$f \circ g(x) = f(g(x)) = f(x^3)$$

$$\therefore \quad f \circ g(x) = 2x^3 + x^6$$

Functions Ex2.2 Q1(iii)

$f(x) = x^2 + 8 \text{ and } g(x) = 3x^3 + 1$

$Thus, \ g \circ f(x) = g[f(x)]$

$\Rightarrow \ g \circ f(x) = g[x^2 + 8]$

$\Rightarrow \ g \circ f(x) = 3[x^2 + 8]^3 + 1$

$Similarly, f \circ g(x) = f[g(x)]$

$\Rightarrow \ f \circ g(x) = f[3x^3 + 1]$

$\Rightarrow \ f \circ g(x) = [3x^3 + 1]^2 + 8$

$\Rightarrow \ f \circ g(x) = [9x^6 + 1 + 6x^3] + 8$

$\Rightarrow \ f \circ g(x) = 9x^6 + 6x^3 + 9$

Functions Ex2.2 Q1(iv)

$f(x) = x$ and $g(x) = |x|$

Now, $g \circ f(x) = g(f(x)) = g(x)$

$\therefore \quad g \circ f(x) = |x|$

and, $f \circ g(x) = f(g(x)) = f(|x|)$

$\therefore \quad f \circ g(x) = |x|$

Functions Ex2.2 Q1(v)

$f(x) = x^2 + 2x - 3$ and $g(x) = 3x - 4$

Now, $g \circ f(x) = g(f(x)) = g(x^2 + 2x - 3)$

$\therefore \quad g \circ f(x) = 3(x^2 + 2x - 3) - 4$

$\Rightarrow \quad g \circ f(x) = 3x^2 + 6x - 13$

and, $f \circ g(x) = f(g(x)) = f(3x - 4)$

$\therefore \quad f \circ g(x) = (3x - 4)^2 + 2(3x - 4) - 3$

$= gx^2 + 16 - 24x + 6x - 8 - 3$

$\therefore \quad f \circ g(x) = 9x^2 - 18x + 5$

Functions Ex2.2 Q1(vi)

$f(x) = 8x^3$ and $g(x) = x^{1/3}$

Now, $g \circ f(x) = g(f(x)) = g(8x^3)$

$= (8x^3)^{1/3}$

$\therefore \quad g \circ f(x) = 2x$

and, $f \circ g(x) = f(g(x)) = f\left(x^{1/3}\right)$

$= 8\left(x^{1/3}\right)^3$

$\therefore \quad f \circ g(x) = 8x$

Functions Ex2.2 Q2

Let $f = \{(3,1),(9,3),(12,4)\}$ and

$g = \{(1,3),(3,3),(4,9),(5,9)\}$

Now,

$$\text{range of } f = \{1,3,4\}$$
$$\text{domain of } f = \{3,9,12\}$$
$$\text{range of } g = \{3,9\}$$
$$\text{domain of } g = \{1,3,4,5\}$$

since, range of $f \subset$ domain of g

$\therefore \quad g \circ f$ in well defined.

Again, range of $g \subseteq$ domain of f

$\therefore \quad f \circ g$ in well defined.

Now $g \circ f = \{(3,3),(9,3),(12,9)\}$

$f \circ g = \{(1,1),(3,1)(4,3),(5,3)\}$

Functions Ex2.2 Q3

We have,

$$f = \{(1,-1),(4,-2),(9,-3),(16,4)\} \text{ and}$$
$$g = \{(-1,-2),(-2,-4),(-3,-6),(4,8)\}$$

Now,

$$\text{Domain of } f = \{1,4,9,16\}$$
$$\text{Range of } f = \{-1,-2,-3,4\}$$
$$\text{Domain of } g = \{-1,-2,-3,4\}$$
$$\text{Range of } g = \{-2,-4,-6,8\}$$

Clearly range of f = domain of g

$\therefore\ g \circ f$ is defined.

but, range of $g \neq$ domain of f

$\therefore \quad f \circ g$ in not defined.

Now,

$$g \circ f(1) = g(-1) = -2$$
$$g \circ f(4) = g(-2) = -4$$
$$g \circ f(9) = g(-3) = -6$$
$$g \circ f(16) = g(4) = 8$$

$\therefore \quad g \circ f = \{(1,-2),(4,-4),(9,-6),(16,8)\}$

Functions Ex2.2 Q4

$A = \{a, b, c\}$, $B = \{u, v, w\}$ and

$f = A \to B$ and $g : B \to A$ defined by

$f = \{(a, v), (b, u), (c, w)\}$ and

$g = \{(u, b), (v, a), (w, c)\}$

For both f and g, different elements of domain have different images

$\therefore$ f and g are one-one

Again for each element in co-domain of f and g, there in a pre image in domain

$\therefore$ f and g are onto

Thus, f and g are bijectives.

Now,

$$g \circ f = \{(a, a), (b, b), (c, c)\} \text{ and}$$

$$f \circ g = \{(u, u), (v, v), (w, w)\}$$

Functions Ex2.2 Q5

We have, $f : R \to R$ given by $f(x) = x^2 + 8$ and

$g : R \to R$ given by $g(x) = 3x^3 + 1$

$$\therefore \ f \circ g(x) = f(g(x)) = f(3x^3 + 1)$$

$$= (3x^3 + 1)^2 + 8$$

$$\therefore \ f \circ g(2) = (3 \times 8 + 1)^2 + 8 = 625 + 8 = 633$$

Again

$$g \circ f(x) = g(f(x)) = g(x^2 + 8)$$

$$= 3(x^2 + 8)^3 + 1$$

$$\therefore \ g \circ f(1) = 3(1 + 8)^3 + 1 = 2188$$

Functions Ex2.2 Q6

We have, $f : R^+ \to R^+$ given by

$f(x) = x^2$

$g : R^+ \to R^+$ given by

$g(x) = \sqrt{x}$

$$\therefore \ f \circ g(x) = f(g(x)) = f(\sqrt{x}) = (\sqrt{x})^2 = x$$

Also,

$$g \circ f(x) = g(f(x)) = g(x^2) = \sqrt{x^2} = x$$

Thus,

$$f \circ g(x) = g \circ f(x)$$

Functions Ex2.2 Q7

We have, $f: R \to R$ and $g: R \to R$ are two functions defined by

$f(x) = x^2$ and $g(x) = x + 1$

Now,

$f \circ g(x) = f(g(x)) = f(x+1) = (x+1)^2$

$\therefore f \circ g(x) = x^2 + 2x + 1$ .... (i)

$g \circ f(x) = g(f(x)) = g(x^2) = x^2 + 1$ ....(ii)

from (i)& (ii)

$f \circ g \neq g \circ f$

Functions Ex2.2 Q8

Let $f: R \to R$ and $g: R \to R$ are defined as

$f(x) = x + 1$ and $g(x) = x - 1$

Now,

$f \circ g(x) = f(g(x)) = f(x-1) = x - 1 + 1$

$= x = I_R$ .... (i)

Again,

$f \circ g(x) = f(g(x)) = g(x+1) = x + 1 - 1$

$= x = I_R$ .... (ii)

from (i)& (ii)

$f \circ g = g \circ f = I_R$

Functions Ex2.2 Q9

We have, $f: N \to Z_0$, $g: Z_0 \to Q$ and

$h: Q \to R$

Also, $f(x) = 2x$, $g(x) = \frac{1}{x}$ and $h(x) = e^x$

Now, $f: N \to Z_0$ and $h \circ g: Z_0 \to R$

$\therefore$ $(h \circ g) \circ f: N \to R$

also, $g \circ f: N \to Q$ and $h: Q \to R$

$\therefore$ $h \circ (g \circ f): N \to R$

Thus, $(h \circ g) \circ f$ and $h \circ (g \circ f)$ exist and are function from N to *set* R.

Finally. $(h \circ g) \circ f\ (x) = (h \circ g)(f(x)) = (h \circ g)(2x)$

$= h\left(\frac{1}{2x}\right)$

$= e^{\frac{1}{2x}}$

now, $h \circ (g \circ f)(x) = h \circ (g(2x)) = h\left(\frac{1}{2x}\right)$

$= e^{\frac{1}{2x}}$

Hence, associativity verified.

Functions Ex2.2 Q10

We have,

$$h\circ(g\circ f)(x)=h(g\circ f(x))=h(g(f(x)))$$
$$=h(g(2x))=h(3(2x)+4)$$
$$=h(6x+4)=\sin(6x+4)\quad\forall x\in\mathbf{N}$$
$$((h\circ g)\circ f)(x)=(h\circ g)(f(x))=(h\circ g)(2x)$$
$$=h(g(2x))==h(3(2x)+4)$$
$$=h(6x+4)=\sin(6x+4)\quad\forall x\in\mathbf{N}$$

This shows, $h\circ(g\circ f)=(h\circ g)\circ f$

Functions Ex2.2 Q11

Define f: **N** → **N** by, $f(x) = x + 1$

And, g: **N** → **N** by,

$$g(x)=\begin{cases}x-1 \text{ if } x>1\\ 1 \quad\quad \text{if } x=1\end{cases}$$

We first show that f is not onto.

For this, consider element 1 in co-domain **N**. It is clear that this element is not an image of any of the elements in domain **N**.

Therefore, f is not onto.

Now, gof: **N** → **N** is defined by,

Functions Ex2.2 Q12

Define f: **N** → **Z** as $f(x) = x$ and g: **Z** → **Z** as $g(x) = |x|$.

We first show that g is not injective.

It can be observed that:

$g(-1) = |-1| = 1$

$g(1) = |1| = 1$

Therefore, $g(-1) = g(1)$, but $-1 \neq 1$.

Therefore, g is not injective.

Now, gof: **N** → **Z** is defined as $gof(x)=g(f(x))=g(x)=|x|$.

Let x, $y \in$ **N** such that $gof(x) = gof(y)$.

$\Rightarrow\ |x|=|y|$

Since x and $y \in$ **N,** both are positive.

$\therefore |x|=|y| \Rightarrow x=y$

Hence, gof is injective

Functions Ex2.2 Q13

We have, $f: A\rightarrow B$ and $g: B\rightarrow C$ are one-one functions

Now we have to prove : $g\circ f: A\rightarrow C$ in one-one

let $x,y\in A$ such that

$$g\circ f(x)=g\circ f(y)$$

$\Rightarrow\quad g(f(x))=g(f(y))$

$\Rightarrow\quad f(x)=f(y)\quad\quad [\because g \text{ in one-one}]$

$\Rightarrow\quad x=y\quad\quad [\because f \text{ in one-one}]$

$\therefore\quad g\circ f$ is one-one function

Functions Ex2.2 Q14

We have, $f: A \to B$ and $g: B \to C$ are onto functions.

Now, we need to prove: $g \circ f: A \to C$ in onto.

let $y \in C$, then

$$g \circ f(x) = y$$

$$\Rightarrow \quad g(f(x)) = y \text{ .... (i)}$$

Since g is onto, for each element in C, then exists a preimage in B.

$$\therefore \quad g(x) = y \text{ .... (ii)}$$

From (i) & (ii)

$$f(x) = \alpha.$$

Since f is onto, for each element in B there exists a preimage in A

$$\therefore f(x) = \alpha \text{ .... (iii)}$$

From (ii) and (iii) we can conclude that for each $y \in C$, there exists a pre image in A such that $g \circ f(x) = y$

$\therefore g \circ f$ is onto