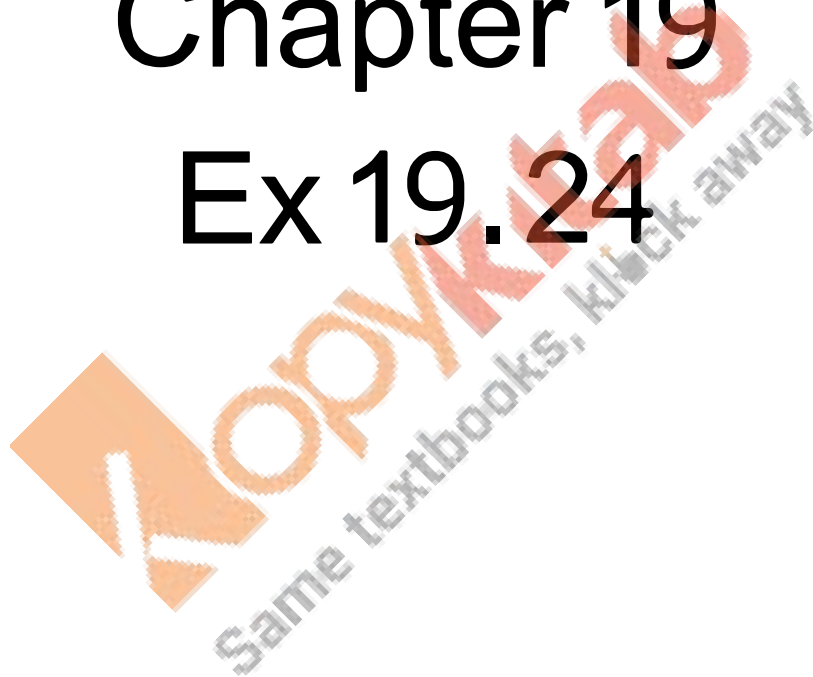


RD Sharma
Solutions Class
12 Maths
Chapter 19
Ex 19.24



Indefinite Integrals Ex 19.24 Q1

$$\begin{aligned}\text{Let } I &= \int \frac{1}{1 - \cot x} dx \\ &= \int \frac{1}{1 - \frac{\cos x}{\sin x}} dx \\ &= \int \frac{\sin x}{\sin x - \cos x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } \sin x &= \lambda \frac{d}{dx}(\sin x - \cos x) + \mu(\sin x - \cos x) + \nu \\ \sin x &= \lambda \frac{d}{dx}(\cos x + \sin x) + \mu(\sin x - \cos x) + \nu \\ \sin x &= \cos(\lambda - \mu) + \sin x(\lambda + \mu) + \nu\end{aligned}$$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$$\lambda + \mu = 1 \quad \dots (1)$$

$$\lambda - \mu = 1 \quad \dots (2)$$

$$\nu = 0 \quad \dots (3)$$

Equation (1), (2), (3) gives

$$\lambda = \frac{1}{2}, \mu = \frac{1}{2}, \nu = 0$$

$$I = \int \frac{\frac{1}{2}(\cos x + \sin x) + \frac{1}{2}(\sin x - \cos x)}{(\sin x - \cos x)} dx$$

$$= \frac{1}{2} \int \frac{(\cos x + \sin x)}{(\sin x - \cos x)} dx + \frac{1}{2} \int dx$$

$$I = \frac{1}{2} \log |\sin x - \cos x| + \frac{1}{2} x + c$$

Indefinite Integrals Ex 19.24 Q2

$$\begin{aligned} \text{Let } I &= \int \frac{1}{1 - \tan x} dx \\ &= \int \frac{1}{1 - \frac{\sin x}{\cos x}} dx \\ &= \int \frac{\cos x}{\cos x - \sin x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \cos x &= \lambda \frac{d}{dx} (\cos x - \sin x) + \mu (\cos x - \sin x) + v \\ &= \lambda \frac{d}{dx} (-\sin x - \cos x) + \mu (\cos x - \sin x) + v \\ \cos x &= \sin x (-\lambda - \mu) + \cos x (-\lambda + \mu) + v \end{aligned}$$

Comparing the coefficients of $\cos x$ & $\sin x$ on the both the sides,

$$-\lambda - \mu = 0 \text{ ----- (1)}$$

$$-\lambda + \mu = 1 \text{ ----- (2)}$$

$$v = 0 \text{ ----- (3)}$$

Equation (1), (2), (3) gives

$$\lambda = -\frac{1}{2}, \mu = \frac{1}{2}, v = 0$$

$$I = \int \frac{-\frac{1}{2}(-\sin x - \cos x) + \frac{1}{2}(\cos x - \sin x)}{(\cos x - \sin x)} dx$$

$$= \frac{1}{2} \int \frac{(\sin x + \cos x)}{(\cos x - \sin x)} dx + \frac{1}{2} \int dx$$

$$= -\frac{1}{2} \log |\cos x - \sin x| + \frac{1}{2} x + c$$

$$I = \frac{1}{2} x - \frac{1}{2} \log |\cos x - \sin x| + c$$

$$\text{Let } I = \int \frac{3 + 2 \cos x + 4 \sin x}{2 \sin x + \cos x + 3} dx$$

$$\text{Let } 3 + 2 \cos x + 4 \sin x = \lambda \frac{d}{dx} (2 \sin x + \cos x + 3) + \mu (2 \sin x + \cos x + 3) + \nu$$

$$3 + 2 \cos x + 4 \sin x = \lambda (2 \cos x - \sin x) + \mu (2 \sin x + \cos x + 3) + \nu$$

$$3 + 2 \cos x + 4 \sin x = (-\lambda + 2\mu) \sin x + (2\lambda + \mu) \cos x + 3\mu + \nu$$

Comparing the coefficients of $\sin x$ & $\cos x$ on the both the sides,

$$-\lambda + 2\mu = 4 \text{ ----- (1)}$$

$$2\lambda + \mu = 2 \text{ ----- (2)}$$

$$2\mu + \nu = 3 \text{ ----- (3)}$$

Solving equation (1), (2) and (3), we get

$$\lambda = 0, \mu = 2, \nu = -3$$

$$I = \int \frac{2(2 \sin x + \cos x + 3) - 3}{(2 \sin x + \cos x + 3)} dx$$

$$= 2 \int dx - 3 \int \frac{1}{2 \sin x + \cos x + 3} dx$$

$$I = 2x - 3I_1 + C_1 \text{ ----- (4)}$$

$$\text{Let } I_1 = \int \frac{1}{2 \sin x + \cos x + 3} dx$$

$$\text{Put } \sin x = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}, \quad \cos x = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$$

$$I_1 = \int \frac{1}{2 \left(\frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + \left(\frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}} \right) + 3} dx$$

$$= \int \frac{(1 + \tan^2 \frac{x}{2})}{4 \tan \frac{x}{2} + 1 - \tan^2 \frac{x}{2} + 3(1 + \tan^2 \frac{x}{2})} dx$$

$$= \int \frac{\sec^2 \frac{x}{2}}{2 \tan^2 \frac{x}{2} + 4 \tan \frac{x}{2} + 4} dx$$

Let $\tan \frac{x}{2} = t$

$$\frac{1}{2} \sec^2 \frac{x}{2} = dt$$

$$I_1 = \int \frac{2dt}{2t^2 + 4t + 4}$$

$$= \frac{2}{2} \int \frac{dt}{t^2 + 2t + 2}$$

$$= \int \frac{dt}{t^2 + 2t + 1 - 1 + 2}$$

$$= \int \frac{dt}{(t+1)^2 + 1}$$

$$= \tan^{-1}(t+1) + C_2$$

$$= \tan^{-1}\left(\tan \frac{x}{2} + 1\right) + C_2$$

Now, using equation (1),

$$I = 2x - 3 \tan^{-1}\left(\tan \frac{x}{2} + 1\right) + C$$

$$\begin{aligned} \text{Let } I &= \int \frac{1}{p+q \tan x} dx \\ &= \int \frac{1}{p+q \left(\frac{\sin x}{\cos x}\right)} dx \\ &= \int \frac{\cos x}{p \cos x + q \sin x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } \cos x &= \lambda \frac{d}{dx} (p \cos x + q \sin x) + \mu (p \cos x + q \sin x) + v \\ \cos x &= \lambda (-p \sin x + q \cos x) + \mu (p \cos x + q \sin x) + v \\ \cos x &= (-p\lambda + q\mu) \sin x + (q\lambda + p\mu) \cos x + v \end{aligned}$$

Comparing the coefficients of $\sin x$, $\cos x$ on the both the sides,

$$-p\lambda + q\mu = 0 \text{ ----- (1)}$$

$$q\lambda + p\mu = 1 \text{ ----- (2)}$$

$$v = 0 \text{ ----- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = \frac{q}{(p^2 + q^2)}$$

$$\mu = \frac{p}{(p^2 + q^2)}$$

$$v = 0$$

Now,

$$I = \int \frac{q}{(p^2 + q^2)} \frac{(-p \sin x + q \cos x)}{(p \cos x + q \sin x)} dx + \int \frac{p}{(p^2 + q^2)} \frac{(p \cos x + q \sin x)}{(p \cos x + q \sin x)} dx$$

$$I = \frac{q}{(p^2 + q^2)} (\log |p \cos x + q \sin x|) + \frac{p}{(p^2 + q^2)} x + c$$

$$\text{Let } I = \int \frac{5 \cos x + 6}{2 \cos x + \sin x + 3} dx$$

$$\text{Let } (5 \cos x + 6) = \lambda \frac{d}{dx} (2 \cos x + \sin x + 3) + \mu (2 \cos x + \sin x + 3) + \nu$$

$$(5 \cos x + 6) = \lambda (-2 \sin x + \cos x) + \mu (2 \cos x + \sin x + 3) + \nu$$

$$(5 \cos x + 6) = (-2\lambda + \mu) \sin x + (\lambda + 2\mu) \cos x + (3\mu + \nu)$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-2\lambda + \mu = 0 \text{ ----- (1)}$$

$$\lambda + 2\mu = 5 \text{ ----- (2)}$$

$$3\mu + \nu = 6 \text{ ----- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = 1$$

$$\mu = 2$$

$$\nu = 0$$

Now,

$$I = \int \frac{(-2 \sin x + \cos x)}{(2 \cos x + \sin x + 3)} dx + 2 \int dx$$

$$I = \log|2 \cos x + \sin x + 3| + 2x + c$$

$$\text{Let } I = \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx$$

$$\text{Let } (2 \sin x + 3 \cos x) = \lambda \frac{d}{dx} (3 \sin x + 4 \cos x) + \mu (3 \sin x + 4 \cos x) + \nu$$

$$(2 \sin x + 3 \cos x) = \lambda (3 \cos x - 4 \sin x) + \mu (3 \sin x + 4 \cos x) + \nu$$

$$(2 \sin x + 3 \cos x) = (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + \nu$$

Comparing the coefficients of $\sin x$, $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 \text{ ----- (1)}$$

$$-4\lambda + 3\mu = 2 \text{ ----- (2)}$$

$$\nu = 0 \text{ ----- (3)}$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{1}{25}$$

$$\mu = \frac{18}{25}$$

$$\nu = 0$$

$$I = \frac{1}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{18}{25} \int dx$$

$$I = \frac{1}{25} \log |3 \sin x + 4 \cos x| + \frac{18}{25} x + c$$

$$\begin{aligned}\text{Let } I &= \int \frac{1}{3 + 4 \cot x} dx \\ &= \int \frac{\sin x}{3 \sin x + 4 \cos x} dx\end{aligned}$$

$$\begin{aligned}\text{Let } \sin x &= \lambda \frac{d}{dx} (3 \sin x + 4 \cos x) + \mu (3 \sin x + 4 \cos x) + v \\ \sin x &= \lambda (3 \cos x - 4 \sin x) + \mu (3 \sin x + 4 \cos x) + v \\ \sin x &= (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + v\end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 0 \text{ ----- (1)}$$

$$-4\lambda + 3\mu = 1 \text{ ----- (2)}$$

$$v = 0 \text{ ----- (3)}$$

Solving the equation (1), (2) and (3), we get

$$\lambda = -\frac{4}{25}$$

$$\mu = \frac{3}{25}$$

$$v = 0$$

$$I = -\frac{4}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{3}{25} \int dx$$

$$I = -\frac{4}{25} \log |3 \sin x + 4 \cos x| + \frac{3}{25} x + c$$

$$\begin{aligned} \text{Let } I &= \int \frac{2 \tan x + 3}{3 \tan x + 4} dx \\ &= \int \frac{2 \sin x + 3 \cos x}{3 \sin x + 4 \cos x} dx \end{aligned}$$

$$\begin{aligned} \text{Let } 2 \sin x + 3 \cos x &= \lambda \frac{d}{dx} (3 \sin x + 4 \cos x) + \mu (3 \sin x + 4 \cos x) + \nu \\ 2 \sin x + 3 \cos x &= \lambda (3 \cos x - 4 \sin x) + \mu (3 \sin x + 4 \cos x) + \nu \\ 2 \sin x + 3 \cos x &= (3\lambda + 4\mu) \cos x + (-4\lambda + 3\mu) \sin x + \nu \end{aligned}$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$3\lambda + 4\mu = 3 \text{ ----- (1)}$$

$$-4\lambda + 3\mu = 2 \text{ ----- (2)}$$

$$\nu = 0$$

Solving the equation (1), (2) and (3),

$$\mu = \frac{18}{25}$$

$$\lambda = \frac{1}{25}$$

$$\nu = 0$$

$$I = \frac{1}{25} \int \frac{(3 \cos x - 4 \sin x)}{(3 \sin x + 4 \cos x)} dx + \frac{18}{25} \int dx$$

$$I = \frac{18}{25} x + \frac{1}{25} \log |3 \sin x + 4 \cos x| + c$$

$$\text{Let } I = \int \frac{1}{4 + 3 \tan x} dx$$

$$I = \int \frac{\cos x}{4 \cos x + 3 \sin x} dx$$

$$\text{Let } \cos x = \lambda \frac{d}{dx} (4 \cos x + 3 \sin x) + \mu (4 \cos x + 3 \sin x) + \nu$$

$$\cos x = \lambda (-4 \sin x + 3 \cos x) + \mu (4 \cos x + 3 \sin x) + \nu$$

$$\cos x = (-4\lambda + 3\mu) \sin x + (3\lambda + 4\mu) \cos x + \nu$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-4\lambda + 3\mu = 0 \text{ ----- (1)}$$

$$3\lambda + 4\mu = 1 \text{ ----- (2)}$$

$$\nu = 0 \text{ ----- (3)}$$

Solving the equation (1), (2) and (3),

$$\lambda = \frac{3}{25}$$

$$\mu = \frac{4}{25}$$

$$\nu = 0$$

$$I = \int \frac{3}{25} \frac{(-4 \sin x + 3 \cos x)}{(4 \cos x + 3 \sin x)} dx + \frac{4}{25} \int dx$$

$$I = \frac{3}{25} \log |4 \cos x + 3 \sin x| + \frac{4}{25} x + c$$

$$\text{Let } I = \int \frac{8 \cot x + 1}{3 \cot x + 2} dx$$

$$I = \int \frac{8 \cos x + \sin x}{3 \cos x + 2 \sin x} dx$$

$$\text{Let } 8 \cos x + \sin x = \lambda \frac{d}{dx} (3 \cos x + 2 \sin x) + \mu (3 \cos x + 2 \sin x) + \nu$$

$$8 \cos x + \sin x = \lambda (-3 \sin x + 2 \cos x) + \mu (3 \cos x + 2 \sin x) + \nu$$

$$8 \cos x + \sin x = (-3\lambda + 2\mu) \sin x + (2\lambda + 3\mu) \cos x + \nu$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$2\lambda + 3\mu = 8 \text{ ----- (1)}$$

$$-3\lambda + 2\mu = 1 \text{ ----- (2)}$$

$$\nu = 0 \text{ ----- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = 1$$

$$\mu = 2$$

$$\nu = 0$$

$$I = \int \frac{(-3 \sin x + 2 \cos x)}{(3 \cos x + 2 \sin x)} dx + 2 \int dx$$

$$I = \log |3 \cos x + 2 \sin x| + 2x + c$$

$$\text{Let } I = \int \frac{4 \sin x + 5 \cos x}{5 \sin x + 4 \cos x} dx$$

$$\text{Let } 4 \sin x + 5 \cos x = \lambda \frac{d}{dx} (5 \sin x + 4 \cos x) + \mu (5 \sin x + 4 \cos x) + \nu$$

$$4 \sin x + 5 \cos x = \lambda (5 \cos x - 4 \sin x) + \mu (5 \sin x + 4 \cos x) + \nu$$

$$4 \sin x + 5 \cos x = (5\lambda + 4\mu) \cos x + (-4\lambda + 5\mu) \sin x + \nu$$

Comparing the coefficients of $\sin x$ and $\cos x$ on the both the sides,

$$-4\lambda + 5\mu = 4 \text{ ----- (1)}$$

$$5\lambda + 4\mu = 5 \text{ ----- (2)}$$

$$\nu = 0 \text{ ----- (3)}$$

Solving equation (1), (2) and (3),

$$\lambda = \frac{9}{41}$$

$$\mu = \frac{40}{41}$$

$$\nu = 0$$

Now,

$$I = \frac{40}{41} \int dx + \frac{9}{41} \int \frac{(5 \cos x - 4 \sin x)}{(5 \sin x + 4 \cos x)} dx$$

$$I = \frac{40}{41} x + \frac{9}{41} \log |5 \sin x + 4 \cos x| + c$$