RD Sharma Solutions Class 12 Maths Chapter 18 Ex 18.3

Kopykitab
Same textbooks, klick away

Maxima and Minima 18.3 Q1(i)

$f(x) = x^4 - 62x^2 + 120x + 9$

$\therefore \quad f'(x) = 4x^3 - 124x + 120 = 4(x^3 - 31x + 30)$

$f''(x) = 12x^2 - 124 = 4(3x^2 - 31)$

For maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 4(x^3 - 31x + 30) = 0$

$\Rightarrow \quad 4(x^3 - 31x + 30) = 0$

$\Rightarrow \quad x = 5, 1, -6$

Now,

$f''(5) = 176 > 0$

$\Rightarrow \quad x = 5$ is point of local minima

$f''(1) = -112 < 0$

$\Rightarrow \quad x = 1$ is point of local maxima

$f''(-6) = 308 > 0$

$\Rightarrow \quad x = -6$ is point of local minima

$\therefore \quad$ local max value $= f(1) = 68$

local min value $= f(5) = -316$

and $= f(-6) = -1647.$

Maxima and Minima 18.3 Q1(ii)

We have,

$$f(x) = x^3 - 6x^2 + 9x + 15$$

$$\therefore \quad f'(x) = 3x^2 - 12x + 9$$

$$= 3(x^2 - 4x + 3)$$

$$f''(x) = 6x - 12$$

$$= 6(x - 2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 3(x^2 - 4x + 3) = 0$$

$$\Rightarrow \quad 3(x - 3)(x - 1) = 0$$

$$\Rightarrow \quad x = 3, 1$$

Now,

$$f''(3) = 6 > 0$$

$\therefore$ $x = 3$ is point of local minima

$$f''(1) = -6 < 0$$

$\therefore$ $x = 1$ is point of local maxima

$\therefore$ local max value $= f(1) = 19$

local min value $= f(3) = 15$.

Maxima and Minima 18.3 Q1(iii)

We have,

$$f(x) = (x - 1)(x + 2)^2$$

$$\therefore \quad f'(x) = (x + 2)^2 + 2(x - 1)(x + 2)$$

$$= (x + 2)(x + 2 + 2x - 2)$$

$$= (x + 2)(3x)$$

and, $f''(x) = 3(x + 2) + 3x$

$$= 6x + 6$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 3x(x + 2) = 0$$

$$\Rightarrow \quad x = 0, -2$$

Now,

$$f''(0) = 6 > 0$$

$\therefore$ $x = 0$ is point of local minima

$$f''(-2) = -6 < 0$$

$\therefore$ $x = -2$ is point of local maxima

$\therefore$ local max value $= f(-2) = 0$

local min value $= f(0) = -4$.

Maxima and Minima 18.3 Q1(iv)

Kopykitab same textbooks, klick away

We have,

$$f(x) = \frac{2}{x} - \frac{2}{x^2},\ x > 0$$

$$\therefore \quad f'(x) = \frac{-2}{x^2} + \frac{4}{x^3}$$

and, $$f''(x) = \frac{+4}{x^3} - \frac{12}{x^4}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{-2}{x^2} + \frac{4}{x^3} = 0$$

$$\Rightarrow \quad \frac{-2(x-2)}{x^3} = 0$$

$$\Rightarrow \quad x = 2$$

Now,

$$f''(2) = \frac{4}{8} - \frac{12}{6} = \frac{1}{2} - \frac{3}{4} = \frac{-1}{4} < 0$$

$\therefore$ $x = 2$ is point of local maxima

local max value $= f(2) = \frac{1}{2}$.

Maxima and Minima 18.3 Q1(v)

We have,

$$f(x) = xe^x$$

$$\therefore \quad f'(x) = e^x + xe^x = e^x(x+1)$$

$$f''(x) = e^x(x+1) + e^x$$

$$= e^x(x+2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad e^x(x+1) = 0$$

$$\Rightarrow \quad x = -1$$

Now,

$$f''(-1) = e^{-1} = \frac{1}{e} > 0$$

$\therefore$ $x = -1$ is point of local minima

Hence,

local min value $= f(-1) = \frac{-1}{e}$.

Maxima and Minima 18.3 Q1(vi)

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x},\ x > 0$$

$$\therefore \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

$$\text{and,} \quad f''(x) = \frac{4}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \quad \frac{x^2 - 4}{2x^2} = 0$$

$$\Rightarrow \quad x = 2, -2$$

Now,

$$f''(2) = \frac{1}{2} > 0$$

$\therefore$ $x = 2$ is point of minima

We will not consider $x = -2$ as $x > 0$

$\therefore$ local min value $= f(2) = 2.$

Maxima and Minima 18.3 Q1(vii)

We have,

$$f(x) = (x+1)(x+2)^{\frac{1}{3}},\ x \geq -2$$

$$\therefore \quad f'(x) = (x+2)^{\frac{1}{3}} + \frac{1}{3}(x+1)(x+2)^{\frac{-2}{3}}$$

$$= (x+2)^{\frac{-2}{3}}\left(x + 2 + \frac{1}{3}(x+1)\right)$$

$$= \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7)$$

$$\text{and,} \quad f''(x) = -\frac{2}{9}(x+2)^{\frac{-5}{3}}(4x+7) + \frac{1}{3}(x+2)^{\frac{-2}{3}} \times 4$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1}{3}(x+2)^{\frac{-2}{3}}(4x+7) = 0$$

$$\Rightarrow \quad x = -\frac{7}{4}$$

Now,

$$f''\left(-\frac{7}{4}\right) = \frac{4}{3}\left(-\frac{7}{4} + 2\right)^{\frac{-2}{3}}$$

$\therefore$ $x = \frac{-7}{4}$ is point of minima

$\therefore$ local min value $= f\left(\frac{-7}{4}\right) = \frac{-3}{4^{\frac{4}{3}}}.$

Maxima and Minima 18.3 Q1(viii)

We have,

$$f(x) = x\sqrt{32-x^2}, -5 \le x \le 5$$

$$\therefore \quad f'(x) = \sqrt{32-x^2} + \frac{x}{2\sqrt{32-x^2}} \times (-2x)$$

$$= \frac{2\left(32-x^2\right) - 2x^2}{2\sqrt{32-x^2}}$$

$$= \frac{64-4x^2}{2\sqrt{32-x^2}}$$

and, $$f''(x) = \frac{2\sqrt{32-x^2} \times (-8x) \frac{-2\left(64-4x^2\right)}{2\sqrt{32-x^2}} \times (-2x)}{4\left(32-x^2\right)}$$

$$= \frac{-4\left(32-x^2\right) \times 8x + 4x\left(64-x^2\right)}{8\left(32-x^2\right)^{\frac{3}{2}}}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{4\left(16-x^2\right)}{2\sqrt{32-x^2}} = 0$$

$$\Rightarrow \quad x = \pm 4$$

Now,

$$f''(4) = \frac{4 \times 4(64 - 16 - 8 \times 32 + 8 \times 16)}{8(32-16)^{\frac{3}{2}}} < 0$$

$\therefore \quad x = 4$ is point of maxima

Maxima and Minima 18.3 Q1(ix)

Local Maximum value $= f(4)$

$$= 4\sqrt{32-4^2}$$

$$= 4\sqrt{32-16}$$

$$= 4\sqrt{16}$$

$$= 16$$

Local minimum at $x = -4$;

Local Minimum value $= f(-4)$

$$= -4\sqrt{32-(-4)^2}$$

$$= -4\sqrt{32-16}$$

$$= -4\sqrt{16}$$

$$= -16$$

Maxima and Minima 18.3 Q1(x)

$$f(x) = x + \frac{a^2}{x}$$

$$\therefore \quad f'(x) = 1 - \frac{a^2}{x^2}$$

$$f''(x) = \frac{2a^2}{x^3}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 1 - \frac{a^2}{x^2} = 0$$

$$\Rightarrow \quad x^2 - a^2 = 0$$

$$\Rightarrow \quad x = \pm a$$

Now,

$$f''(a) = \frac{2}{a} > 0 \text{ as } a > 0$$

$\therefore \quad x = a$ is point of minima

$$f''(-a) = \frac{-2}{a} < 0 \text{ as } a > 0$$

$\therefore \quad x = -a$ is point of maxima

Hence,

local max value $= f(-a) = -2a$

local min value $= f(a) = 2a$.

Maxima and Minima 18.3 Q1(xi)

$$f(x) = x\sqrt{2 - x^2}$$

$$\therefore \quad f'(x) = \sqrt{2 - x^2} - \frac{2x^2}{2\sqrt{2 - x^2}}$$

$$= \frac{2\left(2 - x^2\right) - 2x^2}{2\sqrt{2 - x^2}}$$

$$= \frac{2 - 2x^2}{\sqrt{2 - x^2}}$$

$$f''(x) = \frac{\sqrt{2 - x^2}\,(-4x) + \frac{\left(2 - 2x^2\right)2x}{\sqrt{2 - x^2}}}{\left(\sqrt{2 - x^2}\right)^2}$$

$$= \frac{-\left(2 - x^2\right)4x + 4x - 4x^3}{\left(2 - x^2\right)^{\frac{3}{2}}}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2\left(1 - x^2\right)}{\sqrt{2 - x^2}} = 0$$

$$\Rightarrow \quad x = \pm 1$$

Now,

$$f''(1) < 0$$

$\Rightarrow$ $x = 1$ is point of local maxima

$$f''(-1) > 0$$

$\Rightarrow$ $x = -1$ is point of local minima

Hence,

local max value $= f(1) = 1$

local min value $= f(-1) = -1$.

Maxima and Minima 18.3 Q1(xii)

Kopykitab same textbooks, klick away

$$f(x) = x + \sqrt{1-x}$$

$$\therefore \quad f'(x) = 1 - \frac{1}{2\sqrt{1-x}} = \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}}$$

$$\therefore \quad f'(x) = \frac{2\sqrt{1-x}\left(\frac{-1}{\sqrt{1-x}}\right) + \frac{\left(2\sqrt{1-x} - 1\right)}{\sqrt{1-x}}}{4(1-x)}$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2\sqrt{1-x} - 1}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \quad \sqrt{1-x} = \frac{1}{2}$$

$$\Rightarrow \quad x = 1 - \frac{1}{4} = \frac{3}{4}$$

Now,

$$f''\left(\frac{3}{4}\right) < 0$$

$$\Rightarrow \quad x = \frac{3}{4} \text{ is point of local maxima}$$

Hence,

$$\text{local max value} = f\left(\frac{3}{4}\right) = \frac{5}{4}.$$

Maxima and Minima 18.3 Q2(i)

$$f(x) = (x-1)(x-2)^2$$

$$\therefore \quad f'(x) = (x-2)^2 + 2(x-1)(x-2)$$
$$= (x-2)(x-2+2x-2)$$
$$= (x-2)(3x-4)$$
$$f''(x) = (3x-4) + 3(x-2)$$

For maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad (x-2)(3x-4) = 0$$

$$\Rightarrow \quad x = 2, \frac{4}{3}$$

Now,

$$f''(2) > 0$$

$$\therefore \quad x = 2 \text{ is local minima}$$

$$f''\left(\frac{4}{3}\right) = -2 < 0$$

$$\therefore \quad x = \frac{4}{3} \text{ is point of local maxima}$$

$$\therefore \quad \text{local max value} = f\left(\frac{4}{3}\right) = \frac{4}{27}$$

$$\text{local min value} = f(2) = 0.$$

Maxima and Minima 18.3 Q2(ii)

Zopykitab
Same textbooks, klick away

$$f(x) = x\sqrt{1-x}$$

$$\therefore \quad f'(x) = \sqrt{1-x} + \frac{x}{2\sqrt{1-x}}(-1)$$

$$= \frac{2(1-x) - x}{2\sqrt{1-x}}$$

$$= \frac{2-3x}{2\sqrt{1-x}}$$

$$f''(x) = \frac{2\sqrt{1-x}(-3) + \frac{(2-3x)}{\sqrt{1-x}}}{4(1-x)}$$

For maximum and minimum,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{2-3x}{2\sqrt{1-x}} = 0$$

$$\Rightarrow \quad x = \frac{2}{3}$$

Now,

$$f''\left(\frac{2}{3}\right) < 0$$

$\therefore \quad x = \frac{2}{3}$ is point ofmaxima

$\therefore \quad$ local max value $= f\left(\frac{2}{3}\right) = \frac{2}{3\sqrt{3}}$.

Maxima and Minima 18.3 Q2(iii)

Koppykitab Same textbooks, klick away

$$f(x) = -(x-1)^3(x+1)^2$$

$$\therefore \quad f'(x) = -3(x-1)^2(x+1)^2 - 2(x-1)^3(x+1)$$

$$= -(x-1)^2(x+1)(3x+3+2x-2)$$

$$= -(x-1)^2(x+1)(5x+1)$$

$$\therefore \quad f''(x) = -2(x-1)(x+1)(5x+1) - (x-1)^2(5x+1) - 5(x-1)^2(x+1)$$

For maximum and minimum value,

$$f'(x) = 0$$

$$\Rightarrow \quad -(x-1)^2(x+1)(5x+1) = 0$$

$$\Rightarrow \quad x = 1, -1, -\frac{1}{5}$$

Now,

$$f''(1) = 0$$

$\therefore$ $x = 1$ is inflection point

$$f''(-1) = -4 \times -4 = 16 > 0$$

$\therefore$ $x = -1$ is point of minima

$$f''\left(\frac{-1}{5}\right) = -5\left(\frac{36}{25}\right) \times \frac{4}{5} = \frac{-144}{25} < 0$$

$\therefore$ $x = \frac{-1}{5}$ is point of maxima

Hence,

$$\text{local max value} = f\left(-\frac{1}{5}\right) = \frac{3456}{3125}$$

$$\text{local min value} = f(-1) = 0.$$

Maxima and Minima 18.3 Q3

We have,

$$y = a\log x + bx^2 + x$$

$$\therefore \quad \frac{dy}{dx} = \frac{a}{x} + 2bx + 1$$

and $$\frac{d^2y}{dx^2} = \frac{-a}{x^2} + 2b$$

For maximum and minimum value,

$$\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{a}{x} + 2bx + 1 = 0$$

Given that extreme value exist at $x = 1, 2$

$$\Rightarrow \quad a + 2b = -1 \qquad \text{---(i)}$$

$$\frac{a}{2} + 4b = -1$$

$$\Rightarrow \quad a + 8b = -2 \qquad \text{---(ii)}$$

Solving (i) and (ii), we get

$$a = \frac{-2}{3}, \ b = \frac{-1}{6}.$$

Maxima and Minima 18.3 Q4

The given function is $f(x)=\frac{\log x}{x}$.

$$f'(x)=\frac{x\left(\frac{1}{x}\right)-\log x}{x^2}=\frac{1-\log x}{x^2}$$

Now, $f'(x)=0$

$\Rightarrow 1-\log x=0$

$\Rightarrow \log x=1$

$\Rightarrow \log x=\log e$

$\Rightarrow x=e$

$$\text{Now, } f''(x)=\frac{x^2\left(-\frac{1}{x}\right)-(1-\log x)(2x)}{x^4}$$

$$=\frac{-x-2x(1-\log x)}{x^4}$$

$$=\frac{-3+2\log x}{x^3}$$

$$\text{Now, } f''(e)=\frac{-3+2\log e}{e^3}=\frac{-3+2}{e^3}=\frac{-1}{e^3}<0$$

Therefore, by second derivative test, f is the maximum at $x=e$.

Maxima and Minima 18.3 Q5

$$f(x)=\frac{4}{x+2}+x$$

$$\therefore \quad f'(x)=\frac{-4}{(x+2)^2}+1$$

$$f''(x)=\frac{8}{(x+2)^3}$$

For maximum and minimum value,

$f'(x)=0$

$\Rightarrow \quad \frac{-4}{(x+2)^2}+1=0$

$\Rightarrow \quad (x+2)^2=4$

$\Rightarrow \quad x^2+4x=0$

$\Rightarrow \quad x(x+4)=0$

$x=0,-4$

Now,

$f''(0)=1>0$

$\therefore \quad x=0$ is point of minima

$f''(-4)=-1<0$

$\therefore \quad x=-4$ is point of maxima

$\therefore \quad$ local max value $=f(-4)=-6$

local min value $=f(0)=2$.

Kopykitab
Same textbooks, knock away

Maxima and Minima 18.3 Q6

We have,

$$y = \tan x - 2x$$

$$\therefore \quad y' = \sec^2 x - 2$$

$$y'' = 2\sec^2 x \tan x$$

For maximum and minimum value,

$$y' = 0$$

$$\Rightarrow \quad \sec^2 x = 2$$

$$\Rightarrow \quad \sec x = \pm\sqrt{2}$$

$$\Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$$

$$\therefore \quad y''\left(\frac{\pi}{4}\right) = 4 > 0$$

$$\therefore \quad x = \frac{\pi}{4} \text{ is point of minima}$$

$$y''\left(\frac{3\pi}{4}\right) = -4 < 0$$

$$\therefore \quad x = \frac{3\pi}{4} \text{ is point of maxima}$$

Hence,

$$\text{max value} = f\left(\frac{3\pi}{4}\right) = -1 - \frac{3\pi}{2}$$

$$\text{min value} = f\left(\frac{\pi}{4}\right) = 1 - \frac{\pi}{2}.$$

Maxima and Minima 18.3 Q7

Consider the function

$f(x) = x^3 + ax^2 + bx + c$

Then $f'(x) = 3x^2 + 2ax + b$

It is given that $f(x)$ is maximum at $x = -1$.

$\therefore f'(-1) = 3(-1)^2 + 2a(-1) + b = 0$

$\Rightarrow f'(-1) = 3 - 2a + b = 0 \ldots (1)$

It is given that $f(x)$ is minimum at $x = 3$.

$\therefore f'(3) = 3(3)^2 + 2a(3) + b = 0$

$\Rightarrow f'(3) = 27 + 6a + b = 0 \ldots (2)$

Solving equations (1) and (2), we have,

$a = -3$ and $b = -9$

Since $f'(x)$ is independent of constant c, it can be any real number.

Kopykitab
Same textbooks, click away