RD Sharma Solutions Class 12 Maths Chapter 18 Ex 18.2

Maxima and Minima Ex 18.2 Q1

$f(x) = (x-5)^4$

$\therefore \quad f'(x) = 4(x-5)^3$

For local maxima and minima

$f'(x) = 0$

$\Rightarrow \quad 4(x-5)^3 = 0$

$\Rightarrow \quad x - 5 = 0$

$\Rightarrow \quad x = 5$

$f'(x)$ changes from $-$ve to $+$ve as passes through 5.

So, $x = 5$ is the point of local minima

Thus, local minimum value is $f(5) = 0$

Maxima and Minima Ex 18.2 Q2

$g(x) = x^3 - 3x$

$\therefore g'(x) = 3x^2 - 3$

Now,

$g'(x) = 0 \Rightarrow 3x^2 = 3 \Rightarrow x = \pm 1$

$g''(x) = 6x$

$g''(1) = 6 > 0$

$g''(-1) = -6 < 0$

By second derivative test, $x = 1$ is a point of local minima and local minimum value of g at $x = 1$ is $g(1) = 1^3 - 3 = 1 - 3 = -2$. However,

$x = -1$ is a point of local maxima and local maximum value of g at

$x = -1$ is $g(1) = (-1)^3 - 3(-1) = -1 + 3 = 2$.

Maxima and Minima Ex 18.2 Q3

$f(x) = x^3 (x-1)^2$

$$\begin{aligned}\therefore \quad f'(x) &= 3x^2 (x-1)^2 + 2x^3 (x-1)\\ &= (x-1)\left(3x^2 (x-1) + 2x^3\right)\\ &= (x-1)\left(3x^3 - 3x^2 + 2x^3\right)\\ &= (x-1)\left(5x^3 - 3x^2\right)\\ &= x^2 (x-1)(5x-3)\end{aligned}$$

For all maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad x^2 (x-1)(5x-3) = 0$

$\Rightarrow \quad x = 0, 1, \frac{3}{5}$

At $x = \frac{3}{5}$ $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = \frac{3}{5}$ is point of minima.

At $x = 1$ $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 1$ is point of maxima

Maxima and Minima Ex 18.2 Q4

$f(x) = (x-1)(x+2)^2$

$\therefore \quad f'(x) = (x+2)^2 + 2(x-1)(x+2)$

$= (x+2)(x+2+2x-2)$

$= (x+2)(3x)$

For point of maxima and minima

$f'(x) = 0$

$\Rightarrow \quad (x+2) \times 3x = 0$

$\Rightarrow \quad x = 0, -2$

At $x = -2$ $f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = -2$ is point of local maxima

At $x = 0$ $f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 0$ is point of local minima

Thus, local min value $= f(0) = -4$

local max value $= f(-2) = 0.$

Maxima and Minima Ex 18.2 Q5

$f(x) = (x-1)^3 (x+1)^2$

$\therefore \quad f'(x) = 3(x-1)^2 (x+1)^2 + 2(x-1)^3 (x+1)$

$= (x-1)^2 (x+1)\{3(x+1) + 2(x-1)\}$

$= (x-1)^2 (x+1)(5x+1)$

For the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad (x-1)^2 (x+1)(5x+1) = 0$

$\Rightarrow \quad x = 1, -1, -\frac{1}{5}$

Here,

At $x = -1$ $f'(x)$ changes from $+$ve to $-$ve so $x = -1$ is point of maxima.

At $x = -\frac{1}{5}$, $f'(x)$ changes from $-$ve to $+$ve so $x = -\frac{1}{5}$ is point of minima

Hence, local max value $= 0$

local min value $= -\frac{3456}{3125}.$

Maxima and Minima Ex 18.2 Q6

$f(x) = x^3 - 6x^2 + 9x + 15$

$\therefore \quad f'(x) = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x - 3)(x - 1)$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 3(x - 3)(x - 1) = 0$

$\Rightarrow \quad x = 3, 1$

At $\quad x = -1,\ f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = 1$ is point of local maxima

At $\quad x = 3,\ f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = 3$ is point of local manima

Hence, local max value $= f(1) = 19$

local min value $= f(3) = 15.$

Maxima and Minima Ex 18.2 Q7

$f(x) = \sin 2x,\ 0 < x, \pi$

$\therefore \quad f'(x) = 2\cos 2x$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 2x = \frac{\pi}{2}, \frac{3\pi}{2}$

$\Rightarrow \quad x = \frac{\pi}{4}, \frac{3\pi}{4}$

At $\quad x = \frac{\pi}{4},\ f'(x)$ changes from $+$ve to $-$ve

$\therefore \quad x = \frac{\pi}{4}$ is point of local maxima

At $\quad x = \frac{3\pi}{4},\ f'(x)$ changes from $-$ve to $+$ve

$\therefore \quad x = \frac{3\pi}{4}$ is point of local minima,

Hence, local max value $= f\left(\frac{\pi}{4}\right) = 1$

local min value $= f\left(\frac{3\pi}{4}\right) = -1.$

Maxima and Minima Ex 18.2 Q8

$f(x)=\sin x-\cos x,\ 0<x<2\pi$

$\therefore f'(x)=\cos x+\sin x$

$f'(x)=0\Rightarrow \cos x=-\sin x\Rightarrow \tan x=-1\Rightarrow x=\frac{3\pi}{4},\frac{7\pi}{4}\in(0,2\pi)$

$f''(x)=-\sin x+\cos x$

$f''\left(\frac{3\pi}{4}\right)=-\sin\frac{3\pi}{4}+\cos\frac{3\pi}{4}=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\sqrt{2}<0$

$f''\left(\frac{7\pi}{4}\right)=-\sin\frac{7\pi}{4}+\cos\frac{7\pi}{4}=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}>0$

Therefore, by second derivative test, $x=\frac{3\pi}{4}$ is a point of local maxima and the local maximum value of f at $x=\frac{3\pi}{4}$ is

$f\left(\frac{3\pi}{4}\right)=\sin\frac{3\pi}{4}-\cos\frac{3\pi}{4}=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\sqrt{2}$. However, $x=\frac{7\pi}{4}$ is a point of local minima and the local minimum value of f at $x=\frac{7\pi}{4}$ is $f\left(\frac{7\pi}{4}\right)=\sin\frac{7\pi}{4}-\cos\frac{7\pi}{4}=-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}=-\sqrt{2}$.

Maxima and Minima Ex 18.2 Q9

$f(x)=\cos x,\ 0<x<\pi$

$\therefore \quad f'(x)=-\sin x$

For, the point of local maxima and minima,

$f'(x)=0$

$\Rightarrow \quad -\sin x=0$

$\Rightarrow \quad x=0$, and π

But, these two points lies outside the interval $(0,\pi)$

So, no local maxima and minima will exist in the interval $(0,\pi)$.

Maxima and Minima Ex 18.2 Q10

$\therefore \quad f'(x) = 2\cos 2x - 1$

For, the point of local maxima and minima,

$f'(x) = 0$

$\Rightarrow \quad 2\cos 2x - 1 = 0$

$\Rightarrow \quad \cos 2x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow \quad 2x = \frac{\pi}{3}, -\frac{\pi}{3}$

$\Rightarrow \quad x = \frac{\pi}{6}, -\frac{\pi}{6}$

At $\quad x = -\frac{\pi}{6}$, $f'(x)$ changes from $-$ve to $+$ ve

$\therefore \quad x = -\frac{\pi}{6}$ is point of local manima

At $\quad x = \frac{\pi}{6}$, $f'(x)$ changes from $+$ve to $-$ ve

$\therefore \quad x = \frac{\pi}{6}$ is point of local maxima

Hence, local max value $= f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$

local min value $= f\left(-\frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2} + \frac{\pi}{6}$.

Maxima and Minima Ex 18.2 Q11

$f(x) = 2\sin x - x, -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

For checking the minima and maxima, we have

$f'(x) = 2\cos x - 1 = 0$

$\Rightarrow \cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

$\Rightarrow x = -\frac{\pi}{3}, \frac{\pi}{3}$

At $x = -\frac{\pi}{3}$, $f(x)$ *changes from* $-$*ve to* $+$ *ve*

$\Rightarrow x = -\frac{\pi}{3}$ *is point of local minima with value* $= -\sqrt{3} - \frac{\pi}{3}$

At $x = \frac{\pi}{3}$, $f(x)$ *changes from* $+$*ve to* $+$ *ve*

$\Rightarrow x = \frac{\pi}{3}$ *is point of local maxima with value* $= \sqrt{3} - \frac{\pi}{3}$

Maxima and Minima Ex 18.2 Q12

$$\therefore f'(x)=\sqrt{1-x}+x\cdot\frac{1}{2\sqrt{1-x}}(-1)=\sqrt{1-x}-\frac{x}{2\sqrt{1-x}}$$

$$=\frac{2(1-x)-x}{2\sqrt{1-x}}=\frac{2-3x}{2\sqrt{1-x}}$$

$$f'(x)=0\Rightarrow\frac{2-3x}{2\sqrt{1-x}}=0\Rightarrow 2-3x=0\Rightarrow x=\frac{2}{3}$$

$$f''(x)=\frac{1}{2}\left[\frac{\sqrt{1-x}(-3)-(2-3x)\left(\frac{-1}{2\sqrt{1-x}}\right)}{1-x}\right]$$

$$=\frac{\sqrt{1-x}(-3)+(2-3x)\left(\frac{1}{2\sqrt{1-x}}\right)}{2(1-x)}$$

$$=\frac{-6(1-x)+(2-3x)}{4(1-x)^{\frac{3}{2}}}$$

$$=\frac{3x-4}{4(1-x)^{\frac{3}{2}}}$$

$$f''\left(\frac{2}{3}\right)=\frac{3\left(\frac{2}{3}\right)-4}{4\left(1-\frac{2}{3}\right)^{\frac{3}{2}}}=\frac{2-4}{4\left(\frac{1}{3}\right)^{\frac{3}{2}}}=\frac{-1}{2\left(\frac{1}{3}\right)^{\frac{3}{2}}}<0$$

Therefore, by second derivative test, $x=\frac{2}{3}$ is a point of local maxima and the local maximum value of f at $x=\frac{2}{3}$ is

$$f\left(\frac{2}{3}\right)=\frac{2}{3}\sqrt{1-\frac{2}{3}}=\frac{2}{3}\sqrt{\frac{1}{3}}=\frac{2}{3\sqrt{3}}=\frac{2\sqrt{3}}{9}.$$

Maxima and Minima Ex 18.2 Q13

We have,

$$f(x) = x^3(2x-1)^3$$

$$\therefore \quad f'(x) = 3x^2(2x-1)^3 + 3x^3(2x-1)^2 \times 2$$

$$= 3x^2(2x-1)^2(2x-1+2x)$$

$$= 3x^2(4x-1)$$

For, the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad 3x^2(4x-1) = 0$$

$$\Rightarrow \quad x = 0, \frac{1}{4}$$

At $x = \frac{1}{4}$, $f'(x)$ changes from $-$ve to $+$ ve

$\therefore$ $x = \frac{1}{4}$ is the point of local minima,

$$\therefore \quad \text{local min value} = f\left(\frac{1}{4}\right) = \frac{-1}{512}.$$

Maxima and Minima Ex 18.2 Q14

We have,

$$f(x) = \frac{x}{2} + \frac{2}{x}, \; x > 0$$

$$\therefore \quad f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

For the point of local maxima and minima,

$$f'(x) = 0$$

$$\Rightarrow \quad \frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow \quad x^2 - 4 = 0$$

$$\Rightarrow \quad x = \sqrt{4}, -\sqrt{4}$$

$$\Rightarrow \quad x = 2, -2$$

At $x = 2$, $f'(x)$ changes from $-$ve to $+$ ve

$\therefore$ $x = 2$ is point of local minima.

$$\therefore \quad \text{local min value} = f(2) = 2.$$

Maxima and Minima Ex 18.2 Q15

$$g(x)=\frac{1}{x^2+2}$$

$$\therefore g'(x)=\frac{-(2x)}{(x^2+2)^2}$$

$$g'(x)=0\Rightarrow \frac{-2x}{(x^2+2)^2}=0\Rightarrow x=0$$

Now, for values close to $x=0$ and to the left of 0, $g'(x)>0$. Also, for values close to $x=0$ and to the right of 0, $g'(x)<0$.

Therefore, by first derivative test, $x=0$ is a point of local maxima and the local maximum value of $g(0)$ is $\frac{1}{0+2}=\frac{1}{2}$..