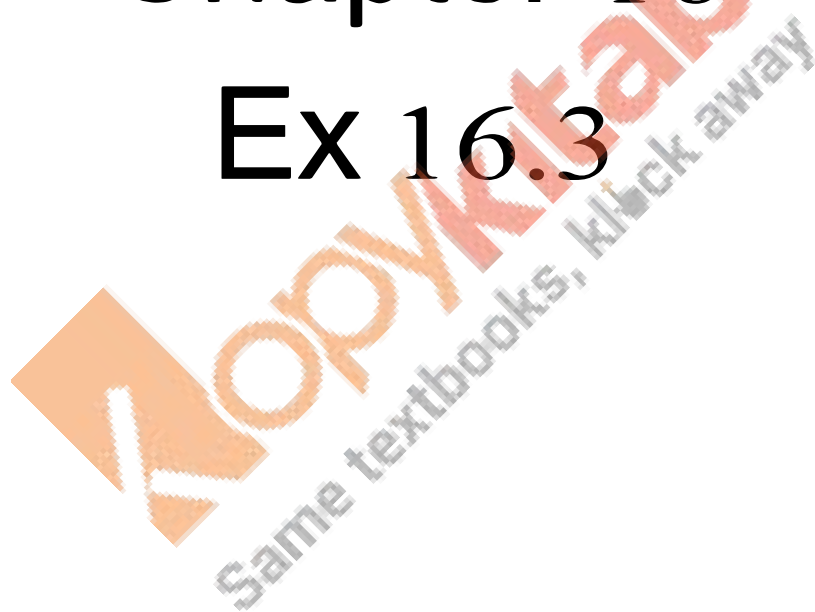


RD Sharma  
Solutions  
Class 12 Maths  
Chapter 16  
Ex 16.3



Solution 1(i)



We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

The given equations are

$$y^2 = x \quad \text{---(i)}$$

$$x^2 = y \quad \text{---(ii)}$$

$$m_1 = \frac{dy}{dx} = \frac{1}{2y}$$

$$m_2 = \frac{dy}{dx} = 2x$$

Solving (i) and (ii)

$$x^4 - x = 0 \quad \Rightarrow \quad x(x^3 - 1) = 0$$

and  $y = 0, 1$

$$\therefore m_1 = \frac{1}{2}, \infty \quad \text{and} \quad m_2 = 0 \text{ or } 2$$

$$\therefore \tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \frac{3}{4}$$

$$\therefore \theta = \tan^{-1} \left( \frac{3}{4} \right)$$

$$\text{and} \quad \tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \infty$$

$$\theta = \frac{7\pi}{2}$$

Solution 1(ii)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

$$y = x^2 \quad \text{---(i)}$$

$$x^2 + y^2 = 20 \quad \text{---(ii)}$$

Solving (i) and (ii)

$$y + y^2 = 20$$

$$\Rightarrow y^2 + y - 20 = 0$$

$$\Rightarrow (y + 5)(y - 4) = 0$$

$$\Rightarrow y = -5, 4$$

$$\therefore x = \sqrt{-5}, \pm 2$$

$$\therefore \text{Points are } P = (2, 4), Q = (-2, 4)$$

Now,

Slope  $m_1$  for (i)

$$m_1 = 2x = 4$$

Slope  $m_2$  for (ii)

$$m_2 = \frac{dy}{dx} = \frac{-x}{y} = \frac{-1}{2}$$

Now,

$$\begin{aligned} \tan \theta &= \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \left| \frac{\frac{-1}{2} - 4}{1 - \frac{1}{2} \times 4} \right| \\ &= \frac{9}{2} \end{aligned}$$

$$\therefore \theta = \tan^{-1} \frac{9}{2}$$

Solution 1(iii)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

$$2y^2 = x^3 \quad \text{---(i)}$$

$$y^2 = 32x \quad \text{---(ii)}$$

Solving (i) and (ii)

$$x^3 = 64x$$

$$\Rightarrow x(x^2 - 64) = 0$$

$$\Rightarrow x(x + 8)(x - 8) = 0$$

$$\Rightarrow x = 0, -8, 8$$

$$\therefore y = 0, -16$$

$$\therefore P = (0, 0), Q = (8, 16)$$

Now,

$$m_1 = \frac{dy}{dx} = \frac{3x^2}{4y} = 0 \text{ or } 3$$

$$m_2 = \frac{dy}{dx} = \frac{32}{2y} = \infty \text{ or } 1$$

From (A)

$$\tan \theta = \left| \frac{\infty - 0}{10} \right| = \infty \Rightarrow \theta = \frac{\pi}{2}$$

$$\text{and } \tan \theta = \left| \frac{3 - 1}{13} = \frac{2}{4} = \frac{1}{2} \right|$$

$$\therefore \theta = \tan^{-1} \left( \frac{1}{2} \right)$$

Thus,

$$\theta = \frac{\pi}{2} \text{ and } \tan^{-1} \left( \frac{1}{2} \right)$$

Solution 1(iv)

We have,

$$x^2 + y^2 - 4x - 1 = 0 \quad \text{---(i)}$$

$$\text{and } x^2 + y^2 - 2y - 9 = 0 \quad \text{---(ii)}$$

Equation (i) can be written as

$$(x - 2)^2 + y^2 - 5 = 0 \quad \text{---(iii)}$$

Subtracting (ii) from (i), we get

$$-4x + 2y + 8 = 0$$

$$\Rightarrow y = 2x - 4$$

Substituting in (iii), we get

$$(x - 2)^2 + (2x - 4)^2 - 5 = 0$$

$$\Rightarrow (x - 2)^2 + 4(x - 2)^2 - 5 = 0$$

$$\Rightarrow (x - 2)^2 = 1$$

$$\Rightarrow x - 2 = 1, x - 2 = -1$$

$$\Rightarrow x = 3 \text{ or } x = 1$$

$$\therefore y = 2(3) - 4 = 2 \text{ or } y = -2$$

$\therefore$  The points of intersection of the two curves are  $(3, 2)$  and  $(-1, -2)$

Differentiation (i) and (ii), w.r.t  $x$  we get

$$2x + 2y \frac{dy}{dx} - 4 = 0 \quad \text{---(iv)}$$

$$\text{and } 2x + 2y \frac{dy}{dx} - 2 \frac{dy}{dx} = 0 \quad \text{---(v)}$$

$\therefore$  At  $(3, 2)$ , from equation (iv) we have,

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{4 - 2(3)}{2(2)} = \frac{-1}{2}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = \frac{-2(3)}{(2 \times 2 - 3)} = \frac{-6}{2} = -3$$

$\therefore$  If  $\phi$  is the angle between the curves

Then,

$$\tan \phi = \frac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1} \left(\frac{dy}{dx}\right)_{C_2}}$$

$$= \frac{\left(\frac{-1}{2} - (-3)\right)}{1 + \left(\frac{-1}{2}\right)(-3)}$$

$$= \frac{\left(\frac{-1}{2} + 3\right)}{1 + \left(\frac{-1}{2}\right)(-3)}$$

$$= \frac{\frac{-1}{2} + 3}{1 + \frac{3}{2}} = \frac{5}{2} \times \frac{2}{5} = 1$$

$$\therefore \phi = \frac{\pi}{4}$$

Solution 1 (v)



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{---(i)}$$

$$x^2 + y^2 = ab \quad \text{---(ii)}$$

From (ii), we get

$$y^2 = ab - x^2$$

∴ From (i), we get

$$\frac{x^2}{a^2} + \frac{ab - x^2}{b^2} = 1$$

$$\Rightarrow b^2 x^2 + a^3 b - a^2 x^2 = a^2 b^2$$

$$\Rightarrow (b^2 - a^2) x^2 = a^2 b^2 - a^3 b$$

$$\begin{aligned} \Rightarrow x^2 &= \frac{a^2 b^2 - a^3 b}{b^2 - a^2} \\ &= \frac{a^2 b (b - a)}{(b - a)(b + a)} \\ &= \frac{a^2 b}{b + a} \end{aligned}$$

$$\therefore x = \pm \sqrt{\frac{a^2 b}{a + b}}$$

$$\begin{aligned} \therefore y^2 &= ab - x^2 = ab - \frac{a^2 b}{a + b} \\ &= \frac{a^2 b + ab^2 - a^2 b}{a + b} = \frac{ab^2}{a + b} \end{aligned}$$

Differentiating (i) and (ii) w.r.t.  $x$  we get

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \left(\frac{dy}{dx}\right)_{C_1} = 0$$

$$\text{and } 2x + 2y \left(\frac{dy}{dx}\right)_{C_2} = 0$$

$$\therefore \left(\frac{dy}{dx}\right)_{C_1} = \frac{-x}{a^2} \times \frac{b^2}{y} = \frac{-b^2 x}{a^2 y}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = \frac{-x}{y}$$

At  $\left(\pm \sqrt{\frac{a^2 b}{a + b}}, \pm \sqrt{\frac{ab^2}{a + b}}\right)$  we get

$$\left(\frac{dy}{dx}\right)_{C_1} = \frac{-b^2}{a^2} \sqrt{\frac{a}{b}} = \frac{-b^2 \sqrt{a}}{a^2 \sqrt{b}}$$

$$\left(\frac{dy}{dx}\right)_{C_2} = -\sqrt{\frac{a}{b}}$$

Let  $\alpha$  be the angle between the two curves then,

$$\begin{aligned} \tan \alpha &= \frac{\left(\frac{dy}{dx}\right)_{C_1} - \left(\frac{dy}{dx}\right)_{C_2}}{1 + \left(\frac{dy}{dx}\right)_{C_1} \left(\frac{dy}{dx}\right)_{C_2}} \\ &= \frac{\frac{-b^2 \sqrt{a}}{a^2 \sqrt{b}} + \sqrt{\frac{a}{b}}}{-2 \frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{a}}{\sqrt{b}}} \end{aligned}$$



$$\begin{aligned}
 &= \frac{a - \sqrt{D}}{1 + \left( \frac{-b^2\sqrt{a}}{a^2\sqrt{b}} \right) \left( -\frac{\sqrt{a}}{\sqrt{b}} \right)} \\
 &= \frac{-b^2\sqrt{a} + a^2\sqrt{a}}{a^2\sqrt{b}} \\
 &= \frac{1 + \frac{b^2a}{a^2b}}{\frac{\sqrt{a}(a^2 - b^2)}{a^2\sqrt{b}}} \times \frac{a}{a+b} \\
 &= \frac{(a-b)}{\sqrt{ab}}
 \end{aligned}$$

$$\therefore \alpha = \tan^{-1} \left( \frac{a-b}{\sqrt{ab}} \right)$$

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Solution 1(vi)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

$$x^2 + 4y^2 = 8 \quad \text{---(i)}$$

$$x^2 - 2y^2 = 2 \quad \text{---(ii)}$$

Solving (i) and (ii)

$$6y^2 = 6 \Rightarrow y = \pm 1$$

$$\therefore x^2 = 2 + 2 \Rightarrow x = \pm 2$$

$\therefore$  Point of intersection are

$$P = (2, 1) \text{ and } (-2, -1)$$

Now,

Slope  $m_1$  for (i)

$$8y \frac{dy}{dx} = -2x \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$

$$\therefore m_1 = \frac{1}{2}$$

Slope  $m_2$  for (ii)

$$4y \frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$

$$\therefore m_2 = 1$$

From (A)

$$\tan \theta = \left| \frac{1 - \frac{1}{2}}{1 + 1 \times \frac{1}{2}} \right| = \frac{1}{3}$$

$$\therefore \theta = \tan^{-1} \left( \frac{1}{3} \right)$$

Solution 1(vii)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

$$x^2 = 27y \quad \text{---(i)}$$

$$y^2 = 8x \quad \text{---(ii)}$$

Solving (i) and (ii) are

$$\frac{y^4}{64} = 27y$$

$$\Rightarrow y(y^3 - 27 \times 64) = 0$$

$$\Rightarrow y = 0 \text{ or } 12$$

$$\therefore x = 0 \text{ or } 18$$

$\therefore$  Points of intersection is (0,0) and (18,12)

Now,

Slope of (i)

$$m_1 = \frac{2x}{27} = \frac{12}{9} = \frac{4}{3}$$

Slope of (ii)

$$m_2 = \frac{8}{2y} = \frac{8}{24} = \frac{1}{3}$$

From (A)

$$\tan \theta = \left| \frac{\frac{4}{3} - \frac{1}{3}}{1 + \frac{4}{3} \times \frac{1}{3}} \right| = \frac{9}{13}$$

$$\therefore \theta = \tan^{-1} \left( \frac{9}{13} \right)$$

Solution 1(viii)

We know that angle of intersection of two curves is given by

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are slopes of curves.

$$x^2 + y^2 = 2x \quad \text{---(i)}$$

$$y^2 = x \quad \text{---(ii)}$$

Solving (i) and (ii)

$$x^2 + x = 2x$$

$$\Rightarrow x^2 - x = 0$$

$$\Rightarrow x(x - 1) = 0$$

$$\Rightarrow x = 0, 1$$

$$\therefore y = 0 \text{ or } 1$$

$\therefore$  The points of intersection is  $P = (0, 0)$ ,  $Q = (1, 1)$

$\therefore$  Slope of (i)

$$2y \frac{dy}{dx} = 2 - 2x$$

$$\therefore \frac{dy}{dx} = \frac{2 - 2x}{2y} = \frac{1 - x}{y}$$

$$\therefore m_1 = 0$$

Slope of (ii)

$$m_2 = \frac{1}{2y} = \frac{1}{2}$$

From (A)

$$\tan \theta = \left| \frac{\frac{1}{2} - 0}{1 + \frac{1}{2} \times 0} \right| = \frac{1}{2}$$

$$\therefore \theta = \tan^{-1} \left( \frac{1}{2} \right)$$

Solution 1 (ix)

$$y = 4 - x^2 \dots\dots (i)$$

$$y = x^2 \dots\dots (ii)$$

Substituting eq (ii) in (i) we get,

$$x^2 = 4 - x^2$$

$$\Rightarrow x^2 = 2$$

$$\Rightarrow x = \pm\sqrt{2}$$

From (i) when  $x = \sqrt{2}$ , we get  $y = 2$  and when  $x = -\sqrt{2}$ , we get  $y =$

Thus the two curves intersect at  $(\sqrt{2}, 2)$  and  $(-\sqrt{2}, 2)$ .

Differentiating (i) wrt  $x$ , we get

$$\frac{dy}{dx} = 0 - 2x = -2x$$

Differentiating (ii) wrt  $x$ , we get

$$\frac{dy}{dx} = 2x$$

Angle of intersection at  $(\sqrt{2}, 2)$

$$m_1 = \left(\frac{dy}{dx}\right)_{(\sqrt{2}, 2)} = -2\sqrt{2}$$

Angle of intersection at  $(-\sqrt{2}, 2)$

$$m_2 = \left(\frac{dy}{dx}\right)_{(-\sqrt{2}, 2)} = 2\sqrt{2}$$

Let  $\theta$  be the angle of intersection of the two curves.

$$\tan \theta = \left| \frac{m_2 - m_1}{1 + m_1 m_2} \right| = \left| \frac{2\sqrt{2} + 2\sqrt{2}}{1 + (2\sqrt{2})(-2\sqrt{2})} \right| = \left| \frac{4\sqrt{2}}{-7} \right| = \frac{4\sqrt{2}}{7}$$

$$\Rightarrow \theta = \tan^{-1} \left( \frac{4\sqrt{2}}{7} \right)$$

We know that two curves intersects orthogonally if

$$m_1 \times m_2 = -1 \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are the slopes of two curves

$$y = x^3 \quad \text{---(i)}$$

$$6y = 7 - x^2 \quad \text{---(ii)}$$

Slope of (i)

$$\frac{dy}{dx} = 3x^2 = m_1$$

Slope of (ii)

$$\frac{dy}{dx} = -\frac{2}{6}x = m_2$$

Point of intersection of (i) and (ii) is

$$6x^3 = 7 - x^2$$

$$\Rightarrow 6x^3 + x^2 - 7 = 0$$

$$\Rightarrow x = 1$$

$$\therefore y = 1$$

$$\therefore P = (1, 1)$$

$$\therefore m_1 = 3 \text{ and } m_2 = -\frac{1}{3}$$

Now,

$$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$$

$\therefore$  (i) and (ii) cuts orthogonally.

Solution 2(ii)

We know that two curves intersect orthogonally if

$$m_1 \times m_2 = -1 \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are the slopes of two curves

$$x^3 - 3xy^2 = -2 \quad \text{---(i)}$$

$$3x^2y - y^3 = 2 \quad \text{---(ii)}$$

Point of intersection of (i) and (ii)

$$(i) + (ii)$$

$$\Rightarrow x^3 - 3xy^2 + 3x^2y - y^3 = 0$$

$$\Rightarrow (x - y)^3 = 0$$

$$\Rightarrow x = y$$

$\therefore$  from (i)

$$x^3 - 3x^2 = -2$$

$$\Rightarrow -2x^3 = -2$$

$$\Rightarrow x = 1$$

$\therefore P = (1, 1)$  is the point of intersection

Now,

Slope of (i)

$$3x^2 - 3y^2 - 6xy \frac{dy}{dx} = 0$$

$$\therefore m_1 = \frac{dy}{dx} = \frac{3(x^2 - y^2)}{6xy}$$

Slope of (ii)

$$6xy + 3x^2 \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-6xy}{3(x^2 - y^2)}$$

$$\therefore m_1 \times m_2 = \frac{(x^2 - y^2)}{2xy} \times \frac{-2xy}{(x^2 - y^2)} = -1$$

Solution 2(iii)

We know that two curves intersect orthogonally if

$$m_1 \times m_2 = -1 \quad \text{---(A)}$$

Where  $m_1$  and  $m_2$  are the slopes of two curves

$$x^2 + 4y^2 = 8 \quad \text{---(i)}$$

$$x^2 - 2y^2 = 4 \quad \text{---(ii)}$$

Point of intersection of (i) and (ii) is (i) - (ii), we get

$$6y^2 = 4$$

$$\Rightarrow y = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\therefore x^2 = 4 + \frac{8}{6}$$

$$x^2 = \frac{32}{6}$$

$$\Rightarrow x = \frac{4}{\sqrt{3}}$$

Now,

Slope of (i)

$$2x + 8y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$$

$$\Rightarrow m_1 = -\frac{1}{4} \times \frac{4}{\sqrt{2}} = -\frac{1}{\sqrt{2}} \quad \left[ \because \frac{x}{y} = \frac{4}{\sqrt{2}} \right]$$

Slope of (ii)

$$2x - 4y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{2y}$$

$$\Rightarrow m_2 = \frac{1}{2} \times \frac{4}{\sqrt{2}} = \sqrt{2}$$

$$\therefore m_1 \times m_2 = -\frac{1}{\sqrt{2}} \times \sqrt{2} = -1$$

$\therefore$  (i) and (ii) cuts orthogonally.

Solution 3(i)



We have,

$$x^2 = 4y \quad \text{---(i)}$$

$$4y + x^2 = 8 \quad \text{---(ii)} \quad P = (2, 1)$$

Slope of (i)

$$2x = 4 \frac{dy}{dx}$$

$$\therefore m_1 = \left( \frac{dy}{dx} \right)_P = \left( \frac{x}{2} \right)_P = 1$$

Slope of (ii)

$$4 \frac{dy}{dx} + 2x = 0$$

$$\therefore m_2 = \left( \frac{dy}{dx} \right)_P = \left( -\frac{x}{2} \right)_P = -1$$

$$\therefore m_1 \times m_2 = 1 \times -1 = -1$$

Hence the result.

### Solution 3(ii)

We have,

$$x^2 = y \quad \text{---(i)}$$

$$x^3 + 6y = 7 \quad \text{---(ii)} \quad P = (1, 1)$$

Slope of (i)

$$2x = \frac{dy}{dx}$$

$$\therefore m_1 = \left( \frac{dy}{dx} \right)_P = 2$$

Slope of (ii)

$$3x^2 + 6 \frac{dy}{dx} = 0$$

$$\therefore m_2 = \left( \frac{dy}{dx} \right)_P = \left( -\frac{x^2}{2} \right)_P = -\frac{1}{2}$$

$$\therefore m_1 \times m_2 = 2 \times \frac{-1}{2} = -1$$

### Solution 3(iii)

We have,

$$y^2 = 8x \quad \text{---(i)}$$

$$2x^2 + y^2 = 10 \quad \text{---(ii)} \quad P(1, 2\sqrt{2})$$

Slope of (i)

$$2y \frac{dy}{dx} = 8$$

$$\therefore m_1 = \left(\frac{dy}{dx}\right)_P = \left(\frac{4}{y}\right)_P = \sqrt{2}$$

Slope of (ii)

$$4x + 2y \frac{dy}{dx} = 0$$

$$\therefore m_2 = \left(\frac{dy}{dx}\right)_P = \left(-\frac{2x}{y}\right)_P = \frac{-1}{\sqrt{2}}$$

$$\therefore m_1 \times m_2 = \sqrt{2} \times \frac{-1}{\sqrt{2}} = -1$$

We have,

$$4x = y^2 \quad \text{--- (i)}$$

$$4xy = k \quad \text{--- (ii)}$$

Slope of (i)

$$4 = 2y \frac{dy}{dx}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{2}{y}$$

Slope of (ii)

$$y + x \frac{dy}{dx} = 0$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow y^3 = k$$

$$k = \frac{2}{4}$$

$\therefore$  (i) and (ii) cuts orthogonally

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \frac{2}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \frac{2}{x} = 1$$

$$\Rightarrow x = 2$$

$$\Rightarrow \frac{k^{\frac{2}{3}}}{4} = 2$$

$$\Rightarrow k^{\frac{2}{3}} = 8$$

$$\therefore k^2 = 512$$

We have,

$$2x = y^2 \quad \text{--- (i)}$$

$$2xy = k \quad \text{--- (ii)}$$

Slope of (i)

$$2 = 2y \frac{dy}{dx}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{1}{y}$$

Slope of (ii)

$$y + x \left( \frac{dy}{dx} \right) = 0$$

$$\therefore m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

Now,

Solving (i) and (ii)

$$\frac{k}{y} = y^2$$

$$\Rightarrow y^3 = k$$

$$\therefore x = \frac{y^2}{2} = \frac{k^{\frac{2}{3}}}{2}$$

$\therefore$  (i) and (ii) cuts orthogonally

$$\therefore m_1 \times m_2 = -1$$

$$\frac{1}{y} \times \frac{-y}{x} = -1$$

$$\Rightarrow \frac{1}{x} = 1$$

$$\Rightarrow x = 1$$

$$\Rightarrow \frac{k^{\frac{2}{3}}}{2} = 1$$

$$\Rightarrow k^{\frac{2}{3}} = 2$$

Closing both side, we get

$$k^2 = 8$$

$$xy = 4$$

$$\Rightarrow x = \frac{4}{y} \dots\dots (i)$$

$$x^2 + y^2 = 8 \dots\dots (ii)$$

Substituting eq (i) in (ii) we get,

$$x^2 + y^2 = 8$$

$$\Rightarrow \left(\frac{4}{y}\right)^2 + y^2 = 8$$

$$\Rightarrow 16 + y^4 = 8y^2$$

$$\Rightarrow y^4 - 8y^2 + 16 = 0$$

$$\Rightarrow (y^2 - 4)^2 = 0$$

$$\Rightarrow y^2 = 4$$

$$\Rightarrow y = \pm 2$$

From (i) when  $y = 2$ , we get  $x = 2$  and when  $y = -2$ , we get  $x = -2$

Thus the two curves intersect at  $(2, 2)$  and  $(-2, 2)$ .

Differentiating (i) wrt  $x$ , we get

$$y + x \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

Differentiating (ii) wrt  $x$ , we get

$$2x + 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

At  $(2, 2)$

$$\left(\frac{dy}{dx}\right)_{C_1} = -1$$

$$\left(\frac{dy}{dx}\right)_{C_2} = -1$$

Clearly  $\left(\frac{dy}{dx}\right)_{C_1} = \left(\frac{dy}{dx}\right)_{C_2}$  at  $(2, 2)$

So given two curves touch each other at  $(2, 2)$ .

Similarly, it can be seen that two curves touch each other at  $(-2, 2)$ .

## Solution 7

$$y^2 = 4x \dots (i)$$

$$x^2 + y^2 - 6x + 1 = 0 \dots (ii)$$

Differentiating (i) wrt  $x$ , we get

$$2y \frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

Differentiating (ii) wrt  $x$ , we get

$$2x + 2y \frac{dy}{dx} - 6 + 0 = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{3-x}{y}$$

At (1, 2)

$$\left(\frac{dy}{dx}\right)_{c_1} = \frac{2}{2} = 1$$

$$\left(\frac{dy}{dx}\right)_{c_2} = \frac{3-1}{2} = \frac{2}{2} = 1$$

Clearly  $\left(\frac{dy}{dx}\right)_{c_1} = \left(\frac{dy}{dx}\right)_{c_2}$  at (1, 2)

So given two curves touch each other at (1, 2).

## Solution 8(i)

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{--- (i)}$$

$$xy = c^2 \quad \text{--- (ii)}$$

Slope of (i)

$$\frac{2x}{a^2} - \frac{2y}{b^2} \frac{dy}{dx} = 0$$

$$\therefore m_1 = \frac{dy}{dx} = \frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$y + x \frac{dy}{dx} = 0$$

$$\therefore m_2 = \frac{dy}{dx} = \frac{-y}{x}$$

$\therefore$  (i) and (ii) cuts orthogonally

$$\therefore m_1 \times m_2 = -1$$

$$\Rightarrow \frac{x}{y} \times \frac{-y}{x} \times \frac{a^2}{b^2} = -1$$

$$\Rightarrow a^2 = b^2$$

Solution 8(ii)

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{--- (i)}$$

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \quad \text{--- (ii)}$$

Slope of (i)

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$\therefore m_1 = \frac{dy}{dx} = -\frac{x}{y} \times \frac{b^2}{a^2}$$

Slope of (ii)

$$\frac{2x}{A^2} - \frac{2y}{B^2} \times \frac{dy}{dx} = 0$$

$$\therefore m_2 = \frac{dy}{dx} = \frac{x}{y} \times \frac{B^2}{A^2}$$

\(\therefore\) (i) and (ii) cuts orthogonally

$$\therefore m_1 \times m_2 = -1$$

$$\therefore \frac{-x}{y} \times \frac{b^2}{a^2} \times \frac{x}{y} \times \frac{B^2}{A^2} = -1$$

$$\Rightarrow \frac{x^2}{y^2} \times \frac{b^2 B^2}{a^2 A^2} = 1$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{a^2 A^2}{b^2 B^2} \quad \text{--- (iii)}$$

Now,

(i) - (ii) gives

$$x^2 \left[ \frac{1}{a^2} - \frac{1}{A^2} \right] + y^2 \left[ \frac{1}{b^2} + \frac{1}{B^2} \right] = 0$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{B^2 + b^2}{b^2 B^2} \times \frac{a^2 A^2}{a^2 - A^2}$$

Put in (iii), we get

$$\frac{(B^2 + b^2)}{b^2 B^2} \times \frac{a^2 A^2}{(a^2 - A^2)} = \frac{a^2 A^2}{b^2 B^2}$$

$$\Rightarrow B^2 + b^2 = a^2 - A^2$$

$$\Rightarrow a^2 - b^2 = A^2 + B^2$$



We have,

$$\frac{x^2}{a^2 + \lambda_1} + \frac{y^2}{b^2 + \lambda_1} = 1 \quad \text{---(i)}$$

$$\frac{x^2}{a^2 + \lambda_2} + \frac{y^2}{b^2 + \lambda_2} = 1 \quad \text{---(ii)}$$

slope of (i)

$$\frac{2x}{a^2 + \lambda_1} + \frac{2y}{b^2 + \lambda_1} \times \frac{dy}{dx} = 0$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-x}{y} \times \frac{b^2 + \lambda_1}{a^2 + \lambda_1}$$

Slope of (ii)

$$\frac{2x}{a^2 + \lambda_2} + \frac{2y}{b^2 + \lambda_2} \times \frac{dy}{dx} = 0$$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y} \times \frac{b^2 + \lambda_2}{a^2 + \lambda_2}$$

Now,

Subtracting (ii) from (i), we get

$$x^2 \left[ \frac{1}{a^2 + \lambda_1} - \frac{1}{a^2 + \lambda_2} \right] + y^2 \left[ \frac{1}{b^2 + \lambda_1} - \frac{1}{b^2 + \lambda_2} \right] = 0$$

$$\Rightarrow \frac{x^2}{y^2} = \frac{\lambda_2 - \lambda_1}{(b^2 + \lambda_1)(b^2 + \lambda_2)} \times \frac{1}{\frac{\lambda_1 - \lambda_2}{(a^2 + \lambda_1)(a^2 + \lambda_2)}}$$

Now,

$$\begin{aligned} m_1 \times m_2 &= \frac{x^2}{y^2} \times \frac{(b^2 + \lambda_1)(b^2 + \lambda_2)}{(a^2 + \lambda_1)(a^2 + \lambda_2)} \\ &= \frac{(\lambda_2 - \lambda_1)}{(b^2 + \lambda_1)(b^2 + \lambda_2)} \times -\frac{(a^2 + \lambda_1)(a^2 + \lambda_2)}{\lambda_2 - \lambda_1} \times \frac{(b^2 + \lambda_1)(b^2 + \lambda_2)}{(a^2 + \lambda_1)(a^2 + \lambda_2)} \\ &= -1 \end{aligned}$$

$\therefore$  (i) and (ii) cuts orthogonally

Suppose the straight line  $x \cos \alpha + y \sin \alpha = p$  touches the curve at  $Q(x_1, y_1)$ .

But equation of tangent to  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  at  $Q(x_1, y_1)$  is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Thus equation  $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$  and  $x \cos \alpha + y \sin \alpha = p$  represent the same l

$$\therefore \frac{x_1/a^2}{\cos \alpha} + \frac{y_1/b^2}{\sin \alpha} = \frac{1}{p}$$

$$\Rightarrow x_1 = \frac{a^2 \cos \alpha}{p}, \quad y_1 = \frac{b^2 \sin \alpha}{p} \dots \dots \dots (i)$$

The point  $Q(x_1, y_1)$  lies on the curve  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$$\therefore \frac{a^4 \cos^2 \alpha}{p^2 a^2} + \frac{b^4 \sin^2 \alpha}{p^2 b^2} = 1$$

$$\Rightarrow a^2 \cos^2 \alpha - b^2 \sin^2 \alpha = p^2$$