RD Sharma Solutions Class 12 Maths Chapter 16 Ex 16.2

Tangents and Normals Ex 16.2 Q1

The given equation of the curve is

$$\sqrt{x} + \sqrt{y} = a \qquad \text{---(i)}$$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$$

$$\therefore \quad m = \left(\frac{dy}{dx}\right)_{\left(\frac{a^2}{4}, \frac{a^2}{4}\right)} = -\frac{\frac{a}{2}}{\frac{a}{2}} = -1$$

Thus,

the equation of tangent is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow \quad y - \frac{a^2}{4} = (-1)\left(x - \frac{a^2}{4}\right)$$

$$\Rightarrow \quad x + y = \frac{a^2}{4} + \frac{a^2}{4}$$

$$\Rightarrow \quad x + y = \frac{a^2}{2}$$

Tangents and Normals Ex 16.2 Q2

The equation of the curve is

$$y = 2x^3 - x^2 + 3 \qquad \text{---(i)}$$

$$\text{Slope} = m = \frac{dy}{dx} = 6x^2 - 2x$$

$$\therefore \quad m = \left(\frac{dy}{dx}\right)_{(1,4)} = 4$$

Now,

The equation of normal is (i) is

$$y - y_1 = \frac{-1}{m}(x - x_1)$$

$$\Rightarrow \quad (y - 4) = \frac{-1}{4}(x - 1)$$

$$\Rightarrow \quad x + 4y = 16 + 1$$

$$\Rightarrow \quad x + 4y = 17$$

Tangents and Normals Ex 16.2 Q3(i)

(i) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(0,\ 5)} = -10$$

Thus, the slope of the tangent at (0, 5) is −10. The equation of the tangent is given as:

$$y - 5 = -10(x - 0)$$

$$\Rightarrow y - 5 = -10x$$

$$\Rightarrow 10x + y = 5$$

The slope of the normal at (0, 5) is $\frac{-1}{\text{Slope of the tangent at } (0,\ 5)} = \frac{1}{10}$.

Therefore, the equation of the normal at (0, 5) is given as:

$$y - 5 = \frac{1}{10}(x - 0)$$
$$\Rightarrow 10y - 50 = x$$
$$\Rightarrow x - 10y + 50 = 0$$

Tangents and Normals Ex 16.2 Q3(ii)

(ii) The equation of the curve is $y = x^4 - 6x^3 + 13x^2 - 10x + 5$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 4x^3 - 18x^2 + 26x - 10$$
$$\left.\frac{dy}{dx}\right]_{(1,\ 3)} = 4 - 18 + 26 - 10 = 2$$

Thus, the slope of the tangent at (1, 3) is 2. The equation of the tangent is given as:

$$y - 3 = 2(x - 1)$$
$$\Rightarrow y - 3 = 2x - 2$$
$$\Rightarrow y = 2x + 1$$

The slope of the normal at (1, 3) is $\frac{-1}{\text{Slope of the tangent at } (1,\ 3)} = \frac{-1}{2}$.

Therefore, the equation of the normal at (1, 3) is given as:

$$y - 3 = -\frac{1}{2}(x - 1)$$
$$\Rightarrow 2y - 6 = -x + 1$$
$$\Rightarrow x + 2y - 7 = 0$$

Tangents and Normals Ex 16.2 Q3(iii)

The equation of the curve is $y = x^2$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x$$

$$\left.\frac{dy}{dx}\right]_{(0,\,0)} = 0$$

Thus, the slope of the tangent at (0, 0) is 0 and the equation of the tangent is given as:

$y - 0 = 0\,(x - 0)$

$\Rightarrow y = 0$

The slope of the normal at (0, 0) is $\dfrac{-1}{\text{Slope of the tangent at }(0,\,0)} = -\dfrac{1}{0}$, which is not defined.

Therefore, the equation of the normal at $(x_0, y_0) = (0,\,0)$ is given by

$x = x_0 = 0$.

Tangents and Normals Ex 16.2 Q3(iv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \qquad \text{Normal}$$

Where m is the slope

We have,

$$y = 2x^2 - 3x - 1 \qquad P = (1, -2)$$

Slope $m = \frac{dy}{dx} = 4x - 3$

$$m = \left(\frac{dy}{dx}\right)_P = 1$$

$\therefore$ equation of tangent from (A)

$$(y + 2) = 1(x - 1)$$

$\Rightarrow \quad x - y = 3$

And equation of normal from (B)

$$(y + 2) = -1(x - 1)$$

$\Rightarrow \quad x + y + 1 = 0$

Tangents and Normals Ex 16.2 Q3(v)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \quad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = \frac{x^3}{4 - x} \qquad P - (2, -2)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = \frac{3x^2(4 - x) + x^3}{(4 - x)^2}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{3x^2(4 - x) + x^3}{2y(4 - x)^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{3 \times 4(4 - 2) + 8}{-2 \times 2(4 - 2)^2}$$

$$= \frac{32}{-16} = -2$$

From (A)

Equation of tangent is

$$(y + 2) = -2(x - 2)$$

$$\Rightarrow \quad 2x + y = 2$$

From (B)

Equation of Normal is

$$(y + 2) = \frac{1}{2}(x - 2)$$

$$\Rightarrow \quad x - 2y = 6$$

Tangents and Normals Ex 16.2 Q3(vi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$y = x^2 + 4x + 1 \qquad \text{and} \qquad P = (x = 3)$$

$$\text{Slope} = \frac{dy}{dx} = 2x + 4$$

$$\therefore \qquad m = \left(\frac{dy}{dx}\right)_P = 10$$

From (A)

Equation of tangent is

$$(y - 22) = 10(x - 3)$$

$$\Rightarrow \qquad 10x - y = 8$$

From (B)

Equation of normal is

$$(y - 22) = \frac{-1}{10}(x - 3)$$

$$\Rightarrow \qquad x + 10y = 223$$

Tangents and Normals Ex 16.2 Q3(vii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{and} \quad P = (a\cos\theta, b\sin\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{-a\cos\theta b^2}{b\sin\theta a^2} = \frac{-b}{a}\cot\theta$$

From (A)

Equation of tangent is,

$$(y - b\sin\theta) = \frac{-b}{a}\cot\theta(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{b}{a}x\cot\theta + y = b\sin\theta + b\cot\theta \times \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cot\theta + \frac{y}{b} = \frac{1}{\sin\theta}$$

$$\Rightarrow \quad \frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$$

From (B)

Equation of normal is

$$(y - b\sin\theta) = \frac{a}{b}\frac{\sin\theta}{\cos\theta}(x - a\cos\theta)$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2}{b}\sin\theta - b\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\tan\theta - y = \frac{a^2 - b^2}{b}\sin\theta$$

$$\Rightarrow \quad \frac{a}{b}x\sec\theta - y\,\mathrm{cosec}\theta = \frac{a^2 - b^2}{b}$$

$$\Rightarrow \quad ax\sec\theta - by\,\mathrm{cosec}\theta = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(viii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad (A) \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad (B) \qquad \text{Normal}$$

Where m is the slope

We have,

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad P = (a\sec\theta, b\tan\theta)$$

Differentiating with respect to x, we get

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{a\sec\theta b^2}{b\tan\theta a^2}$$

$$= \frac{b}{a\sin\theta}$$

From (A)

Equation of tangent is,

$$(y - b\tan\theta) = \frac{b}{a\sin\theta}(x - a\sec\theta)$$

$$\Rightarrow \quad \frac{b}{a}\frac{x}{\sin\theta} - y = \frac{b\sec\theta}{\sin\theta} - b\tan\theta$$

$$\Rightarrow \quad \frac{bx}{a\sin\theta} - y = \frac{b\sec\theta}{\sin\theta}\left(1 - \sin^2\theta\right)$$

$$\Rightarrow \quad \frac{x}{a} - \frac{y}{b}\sin\theta = \cos\theta$$

$$\Rightarrow \quad \frac{x}{a}\sec\theta - \frac{y}{b}\tan\theta = 1$$

From (B)

Equation of normal is

$$y - b\tan\theta = \frac{-a\sin\theta}{b}(x - a\sec\theta)$$

$$\Rightarrow \quad ax\sin\theta + by = b^2\tan\theta + a^2\tan\theta$$

$$\Rightarrow \quad ax\cos\theta + by\cot\theta = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q3(ix)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \qquad \text{Normal}$$

Where m is the slope

We have,

$$y^2 = 4ax \qquad P\left(\frac{a}{m^2}, \frac{2a}{m}\right)$$

Differentiating with respect to x, we get

$$2y\frac{dy}{dx} = 4a$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2a}{y}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = m$$

From (A)

Equation of tangent is

$$\left(y - \frac{2a}{m}\right) = m\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad m^2x - my = 2a - a$$

$$\Rightarrow \quad m^2x - my = a$$

From (B)

Equation of normal is

$$\left(y - \frac{2a}{m}\right) = \frac{-1}{m}\left(x - \frac{a}{m^2}\right)$$

$$\Rightarrow \quad (my - 2a) = \frac{-m^2x + a}{m^2}$$

$$\Rightarrow \quad m^2x + m^3y = 2am^2 + a$$

$$\Rightarrow \quad m^2x + m^3y - 2am^2 - a = 0$$

Tangents and Normals Ex 16.2 Q3(x)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \qquad \text{Normal}$$

Where m is the slope

We have,

$$c^2\left(x^2 + y^2\right) = x^2y^2 \qquad P = \left(\frac{c}{\cos\theta}, \frac{c}{\sin\theta}\right)$$

Differentiating with respect to x, we get

$$c^2\left(2x + 2y\frac{dy}{dx}\right) = 2xy^2 + 2x^2y\frac{dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx}\left(2yc^2 - 2x^2y\right) = 2xy^2 - 2xc^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{x\left(y^2 - c^2\right)}{y\left(c^2 - x^2\right)}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{c}{\cos\theta}\left(\frac{c^2}{\sin^2\theta} - c^2\right)}{\frac{c}{\sin\theta}\left(c^2 - \frac{c^2}{\cos^2\theta}\right)}$$

$$= \frac{c^2\tan\theta\left(1 - \sin^2\theta\right)}{c^2\tan^2\theta\left(\cos^2\theta - 1\right)}$$

$$= \frac{1}{-\tan\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{-\cos^3\theta}{\sin^3\theta}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{-\cos^3\theta}{\sin^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c\sin^2\theta + c\cos^2\theta$$

$$\Rightarrow \quad x\cos^3\theta + y\sin^3\theta = c$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{\sin\theta}\right) = \frac{\sin^3\theta}{\cos^3\theta}\left(x - \frac{c}{\cos\theta}\right)$$

$$\Rightarrow \quad x\sin^3\theta - y\cos^3\theta = \frac{c\sin^3\theta}{\cos\theta} - \frac{c\cos^3\theta}{\sin\theta}$$

$$\Rightarrow \quad x\sin^3\theta - y\cos^3\theta = \frac{c\left(\sin^4\theta - \cos^4\theta\right)}{\cos\theta \times \sin\theta}$$

$$= \frac{c\left(\sin^2\theta - \cos^2\theta\right)\left(\sin^2\theta + \cos^2\theta\right)}{\frac{1}{2}\sin 2\theta}$$

$$= \frac{-2c\cos 2\theta}{\sin 2\theta} = -2c\cot 2\theta$$

$$\therefore \quad x\sin^3\theta - y\cos^3\theta + 2c\cot 2\theta = 0$$

Tangents and Normals Ex 16.2 Q3(xi)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

We have,

$$xy = c^2 \qquad P = \left(ct, \frac{c}{t}\right)$$

Differentiating with respect to x, we get

$$y + x\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{-c}{t}}{ct} = \frac{-1}{t^2}$$

From (A)

Equation of tangent is

$$\left(y - \frac{c}{t}\right) = \frac{-1}{t^2}(x - ct)$$

$$\Rightarrow \quad x + t^2 y = tc + ct$$

$$\Rightarrow \quad x + t^2 y = 2ct$$

From (B)

Equation of normal is

$$\left(y - \frac{c}{t}\right) = t^2(x - ct)$$

$$\Rightarrow \quad xt^3 - ty = ct^3 \times t - c$$

$$\Rightarrow \quad xt^3 - ty = ct^4 - c$$

Tangents and Normals Ex 16.2 Q3(xii)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \quad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \quad \text{Normal}$$

Where m is the slope

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{---(i)} \qquad P = (x_1, y_1)$$

Differentiating with resect to x, we get

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{xb^2}{ya^2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = -\frac{x_1 b^2}{y_1 a^2}$$

From (A)

Equation of tangent is

$$(y - y_1) = -\frac{x_1 b^2}{y_1 a^2}(x - x_1)$$

$$\Rightarrow \quad xx_1 b^2 + yy_1 a^2 = x_1^2 b^2 + y_1^2 a^2$$

Divide by $a^2 b^2$ both side

$$\Rightarrow \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2}$$

$$= 1 \qquad [\because (x_1, y_1) \text{ lies on (i)}]$$

$$\therefore \quad \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

From (B)

Equation of normal is

$$(y - y_1) = \frac{y_1 a^2}{x_1 b^2}(x - x_1)$$

$$xy_1 a^2 - yx_1 b^2 = x_1 y_1 a^2 - y_1 x_1 b^2$$

Dividing by $x_1 y_1$ both side

$$\frac{xa^2}{x_1} - \frac{yb^2}{y_1} = a^2 - b^2$$

Tangents and Normals Ex 16.2 Q3(xiii)

Differentiating $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with respect to x, we have:

$$\frac{2x}{a^2}-\frac{2y}{b^2}\frac{dy}{dx}=0$$
$$\Rightarrow \frac{2y}{b^2}\frac{dy}{dx}=\frac{2x}{a^2}$$
$$\Rightarrow \frac{dy}{dx}=\frac{b^2x}{a^2y}$$

Therefore, the slope of the tangent at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}=\frac{b^2x_0}{a^2y_0}$.

Then, the equation of the tangent at (x_0, y_0) is given by,

$$y-y_0=\frac{b^2x_0}{a^2y_0}(x-x_0)$$
$$\Rightarrow a^2yy_0-a^2y_0^2=b^2xx_0-b^2x_0^2$$
$$\Rightarrow b^2xx_0-a^2yy_0-b^2x_0^2+a^2y_0^2=0$$

Tangents and Normals Ex 16.2 Q3(xiv)

Differentiating $x^{\frac{2}{3}}+y^{\frac{2}{3}}=2$ with respect to x, we get

$$\frac{2}{3}x^{\frac{-1}{3}}+\frac{2}{3}y^{\frac{-1}{3}}\frac{dy}{dx}=0$$
$$\Rightarrow \quad \frac{dy}{dx}=-\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

Therefore, the slope of the tangent at $(1,1)$ is $\left.\frac{dy}{dx}\right]_{(1,1)}=-1$

So, the equation of the tangent at $(1,1)$ is

$$y-1=-1(x-1)$$
$$\Rightarrow \quad y+x-2=0$$

Also, the slope of the normal at $(1,1)$ is given by $\frac{-1}{\text{slope of tangent at }(1,1)}=1$

$\therefore$ the equation of the normal at $(1,1)$ is

$$y-1=1(x-1)$$
$$\Rightarrow \quad y-x=0$$

Tangents and Normals Ex 16.2 Q3(xv)

We know that the equation of tangent and the normal to any curve is given by

$$y - y_1 = m(x - x_1) \qquad \text{(A)} \qquad \text{Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{(B)} \qquad \text{Normal}$$

Where m is the slope

We have,

$$x^2 = 4y \qquad P = (2, 1)$$

$$\therefore \quad 2x = \frac{4dy}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x}{2}$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = 1$$

From (A)

Equation of tangent is

$$y - 1 = 1(x - 2)$$

$$\Rightarrow \quad x - y = 1$$

From (B)

Equation of normal is

$$(y - 1) = -1(x - 2)$$

$$\Rightarrow \quad x + y = 3$$

Tangents and Normals Ex 16.2 Q3(vi)

The equation of the given curve is $y^2 = 4x$.

Differentiating with respect to x, we have:

$$2y\frac{dy}{dx} = 4$$

$$\Rightarrow \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y}$$

$$\therefore \left.\frac{dy}{dx}\right]_{(1,2)} = \frac{2}{2} = 1$$

Now, the slope at point (1, 2) is $\frac{-1}{\left.\frac{dy}{dx}\right]_{(1,2)}} = \frac{-1}{1} = -1.$

$\therefore$Equation of the tangent at (1, 2) is $y - 2 = -1(x - 1)$.

$$\Rightarrow y - 2 = -x + 1$$

$$\Rightarrow x + y - 3 = 0$$

Equation of the normal is ,

$$y - 2 = -(-1)(x - 1)$$

$$y - 2 = x - 1$$

$$x - y + 1 = 0$$

Tangents and Normals Ex 16.2 Q3(xix)

Let $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be the equation of the curve.

Rewriting the above equation as,

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$$\Rightarrow y^2 = \frac{b^2}{a^2}x^2 - b^2$$

Differentiating the above function w.r.t. x, we get,

$$2y\frac{dy}{dx} = \frac{b^2}{a^2}2x$$

$$\Rightarrow \frac{dy}{dx} = \frac{b^2}{a^2}\frac{x}{y}$$

$$\Rightarrow \left[\frac{dy}{dx}\right]_{(\sqrt{2}a, b)} = \frac{b^2}{a^2}\frac{\sqrt{2}a}{b} = \frac{\sqrt{2}b}{a}$$

Slope of the tangent $m = \frac{\sqrt{2}b}{a}$

Equation of the tangent is

$$(y - y_1) = m(x - x_1)$$

$$\Rightarrow (y - b) = \frac{\sqrt{2}b}{a}(x - \sqrt{2}a)$$

$$\Rightarrow a(y - b) = \sqrt{2}b(x - \sqrt{2}a)$$

$$\Rightarrow \sqrt{2}bx - ay + ab - 2ab = 0$$

$$\Rightarrow \sqrt{2}bx - ay - ab = 0$$

Slope of the normal is $-\frac{1}{\frac{\sqrt{2}b}{a}} = -\frac{a}{b\sqrt{2}}$

Equation of the normal is

$$(y - y_1) = m(x - x_1)$$

$$\Rightarrow (y - b) = \frac{-a}{\sqrt{2}b}(x - \sqrt{2}a)$$

$$\Rightarrow \sqrt{2}b(y - b) = -a(x - \sqrt{2}a)$$

$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}b^2 - \sqrt{2}a^2 = 0$$

$$\Rightarrow ax + \sqrt{2}by - \sqrt{2}(a^2 + b^2) = 0$$

Tangents and Normals Ex 16.2 Q4

The given equations are,

$$x=\theta+\sin\theta \quad , \quad y=1+\cos\theta$$

$$\frac{dx}{d\theta}=1+\cos\theta \quad , \quad \frac{dy}{d\theta}=-\sin\theta$$

$$\therefore \quad \frac{dx}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{-\sin\theta}{1+\cos\theta}$$

Slope,

$$m=\left(\frac{dy}{dx}\right)_{\theta=\frac{\pi}{4}}=\frac{-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$

$$=-1+\frac{1}{\sqrt{2}}$$

Thus, equation of tangent is,

$$y-y_1=m(x-x_1)$$

$$y-\left(1+\frac{1}{\sqrt{2}}\right)=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\left(\frac{\pi}{4}+\frac{1}{\sqrt{2}}\right)\right)$$

$$y-1-\frac{1}{\sqrt{2}}=\left(-1+\frac{1}{\sqrt{2}}\right)\left(x-\frac{\pi}{4}-\frac{1}{\sqrt{2}}\right)$$

Tangents and Normals Ex 16.2 Q5(i)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y-y_1=m(x-x_1) \qquad \text{---(A) Tangent}$$

$$y-y_1=\frac{-1}{m}(x-x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x=\theta+\sin\theta, \quad y=1+\cos\theta, \quad \theta=\frac{\pi}{2}$$

$$\therefore \quad P=\left[\left(\frac{\pi}{2}+1\right),1\right]$$

and $\dfrac{dx}{d\theta}=1+\cos\theta, \dfrac{dy}{d\theta}=-\sin\theta$

$$\therefore \quad \text{Slope } m=\left(\frac{dy}{dx}\right)_P=\left(\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right)=\frac{-1}{+1}=-1$$

Equation of tangent from (A)

$$(y-1)=-1\left(x-\left(\frac{\pi}{2}+1\right)\right)$$

$$\Rightarrow \quad x+y=\frac{\pi}{2}+1+1$$

$$\Rightarrow \quad 2(x+y)=\pi+4$$

From (B)

Equation of normal is

$$(y-1)=1\left(x-\left(\frac{\pi}{2}+1\right)\right)$$

$$\Rightarrow \quad 2(x-y)=\pi$$

Tangents and Normals Ex 16.2 Q5(ii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = \frac{2at^2}{1+t^2}, \qquad y = \frac{2at^3}{1+t^2}, \qquad t = \frac{1}{2}$$

$$\therefore \qquad P = \left(x = \frac{a}{2+\frac{1}{2}} = \frac{2a}{5}, y = \frac{a}{4+1} = \frac{a}{5}\right)$$

Now,

$$\frac{dx}{dt} = \frac{4a + \left(1+t^2\right) - 2at^2(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{4at}{\left(1+t^2\right)^2}$$

$$\frac{dy}{dt} = \frac{6at^2\left(1+t^2\right) - \left(2at^3\right)(2t)}{\left(1+t^2\right)^2}$$

$$= \frac{6at^2 - 2at^4}{\left(1+t^2\right)^2}$$

$$\therefore \qquad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6at^2 + 2at^4}{4at}$$

$$\therefore \qquad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{3a}{2} + \frac{a}{8}}{2a} = \frac{13}{16}$$

From (A)

Equation of tangent is,

$$\left(y - \frac{a}{5}\right) = \frac{13}{16}\left(x - \frac{2a}{5}\right)$$

$$16y - \frac{16a}{5} = 13x - \frac{26a}{5}$$

$$13x - 16y - 2a = 0$$

Equation of normal is,

$$\left(y - \frac{a}{5}\right) = -\frac{16}{13}\left(x - \frac{2a}{5}\right)$$

$$13y - \frac{13a}{5} = -16x + \frac{32a}{5}$$

$$16x + 13y - 9a = 0$$

Tangents and Normals Ex 16.2 Q5(iii)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \quad \text{---(B) Normal}$$

Where m is slope.

$x = at^2$, $y = 2at$, $t = 1$

$\therefore \quad P = (a, 2a)$

and

$$\frac{dx}{dt} = 2at, \quad \frac{dy}{dt} = 2a$$

$$\therefore \quad \text{Slope } m = \left(\frac{dy}{dx}\right)_P = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2a} = 1$$

From (A)

Equation of tangent is

$$(y - 2a) = 1(x - a)$$

$$\Rightarrow \quad x - y + a = 0$$

From (B)

Equation of normaol is

$$(y - 2a) = -1(x - a)$$

$$\Rightarrow \quad x + y = 3a$$

Tangents and Normals Ex 16.2 Q5(iv)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$x = a\sec t, \quad y = b\tan t, \quad t = t$

$$\therefore \quad \frac{dx}{dt} = a\sec t \times \tan t$$

and

$$\frac{dy}{dt} = b\sec^2 t$$

$$\therefore \quad \text{Slope } m = \frac{dy}{dx} = \frac{b\sec^2 t}{a\sec t \times \tan t}$$

$$= \frac{b}{a}\operatorname{cosec} t$$

From (A)

Equatin of tangent

$$(y - b\tan t) = \frac{b}{a}\operatorname{cosec} t\,(x - a\sec t)$$

$$\Rightarrow \quad bx\operatorname{cosec} t - ay = ab\operatorname{cosec} t \times \sec t - ab\tan t$$

$$= \frac{ab\left[1 - \sin^2 t\right]}{\sin t \times \cos t}$$

$$= \frac{ab\cos t}{\sin t}$$

$$\Rightarrow \quad bx\sec t - ay\tan t = ab$$

From (B)

Equation of normal is

$$(y - b\tan t) = \frac{-a\sin t}{b}(x - a\sec t)$$

$$\Rightarrow \quad ax\sin t + by = a^2\tan t + b^2\tan t$$

$$\Rightarrow \quad ax\cos t + by\cot t = a^2 + b^2$$

Tangents and Normals Ex 16.2 Q5(v)

We know that the equation of tangent and normal to any curve at the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad \text{---(A) Tangent}$$

$$y - y_1 = \frac{-1}{m}(x - x_1) \qquad \text{---(B) Normal}$$

Where m is slope.

$$x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$$

$$\frac{dx}{d\theta} = a(1 + \cos\theta), \frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \qquad \text{Slope } m = \frac{dy}{dx} = \frac{\sin\theta}{1 + \cos\theta} = \frac{\frac{2\sin\theta}{2} \times \frac{\cos\theta}{2}}{\frac{2\cos^2\theta}{2}}$$

$$= \frac{\tan\theta}{2}$$

Now,

From (A)

Equation of tangent

$$y - a(1 - \cos\theta) = \frac{\tan\theta}{2}(x - a(\theta + \sin\theta))$$

$$\Rightarrow \qquad \frac{x\tan\theta}{2} - y = a(\theta + \sin\theta)\frac{\tan\theta}{2} - a(1 - \cos\theta)$$

From (B)

Equation of normal is

$$y - a(1 - \cos\theta) = \frac{-\cot\theta}{2}(x - a(\theta + \sin\theta))$$

$$\Rightarrow \qquad (y - 2a)\frac{\tan\theta}{2} + x - a\theta = 0$$

Tangents and Normals Ex 16.2 Q5(vi)

$$x = 3\cos\theta - \cos^3\theta, \; y = 3\sin\theta - \sin^3\theta$$

$$\Rightarrow \frac{dx}{d\theta} = -3\sin\theta + 3\cos^2\theta\sin\theta \text{ and } \frac{dy}{d\theta} = 3\cos\theta - 3\sin^2\theta\cos\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{3\cos\theta - 3\sin^2\theta\cos\theta}{-3\sin\theta + 3\cos^2\theta\sin\theta} = \frac{\cos\theta(1 - \sin^2\theta)}{-\sin\theta(1 - \cos^2\theta)} = \frac{\cos^3\theta}{-\sin^3\theta} = -\tan^3\theta$$

So equation of the tangent at θ is

$$y - 3\sin\theta + \sin^3\theta = -\tan^3\theta(x - 3\cos\theta + \cos^3\theta)$$

$$\Rightarrow 4(y\cos^3\theta - x\sin^3\theta) = 3\sin 4\theta$$

So equation of normal at θ is

$$y - 3\sin\theta + \sin^3\theta = \frac{1}{\tan^3\theta}(x - 3\cos\theta + \cos^3\theta)$$

$$\Rightarrow y\cos^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

$$\Rightarrow y\sin^3\theta - x\cos^3\theta = 3\sin^4\theta - \sin^6\theta - 3\cos^4\theta + \cos^6\theta$$

Tangents and Normals Ex 16.2 Q6

The given equation of curve is

$$x^2 + 2y^2 - 4x - 6y + 8 = 0 \qquad \text{---(i) at } x = 2$$

Differentiating with respect to x, we get

$$2x + 4y\frac{dy}{dx} - 4 - 6\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx}[4y - 6] = 4 - 2x$$

$$\therefore \quad \frac{dy}{dx} = \frac{2 - x}{2y - 3}$$

Now,

From (i) at $x = 2$

$$4 + 2y^2 - 8 - 6y + 8 = 0$$

$$\Rightarrow \quad 2y^2 - 6y + 4 = 0$$

$$\Rightarrow \quad y^2 - 3y + 2 = 0$$

$$\Rightarrow \quad (y - 2)(y - 1) = 0$$

$$\Rightarrow \quad y = 2, 1$$

Thus,

$$\text{Slope } m_1 = \left(\frac{dy}{dx}\right)_{(2,2)} = 0$$

$$m_2 = \left(\frac{dy}{dx}\right)_{(2,1)} = 0$$

Thus, the equation of normal is

$$(y - y_1) = \frac{-1}{0}(x - 2)$$

$$\Rightarrow \quad x = 2$$

Tangents and Normals Ex 16.2 Q7

The equation of the given curve is $ay^2 = x^3$.

On differentiating with respect to x, we have:

$$2ay\frac{dy}{dx} = 3x^2$$
$$\Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$

The slope of a tangent to the curve at (x_0, y_0) is $\left.\frac{dy}{dx}\right]_{(x_0, y_0)}$.

$\Rightarrow$ The slope of the tangent to the given curve at (am^2, am^3) is

$$\left.\frac{dy}{dx}\right]_{(am^2,\ am^3)} = \frac{3\left(am^2\right)^2}{2a\left(am^3\right)} = \frac{3a^2m^4}{2a^2m^3} = \frac{3m}{2}.$$

$\therefore$ Slope of normal at (am^2, am^3)

$$= \frac{-1}{\text{slope of the tangent at }\left(am^2,\ am^3\right)} = \frac{-2}{3m}$$

Hence, the equation of the normal at (am^2, am^3) is given by,

$$y - am^3 = \frac{-2}{3m}\left(x - am^2\right)$$

$$\Rightarrow 3my - 3am^4 = -2x + 2am^2$$
$$\Rightarrow 2x + 3my - am^2\left(2 + 3m^2\right) = 0$$

Tangents and Normals Ex 16.2 Q8

The given equations are

$$y^2 = ax^3 + b \qquad \text{---(i)}$$

$$y = 4x - 5 \qquad \text{---(ii)} \qquad P = (2,3)$$

Differentiating (i) with respect to x, we get

$$2y\frac{dy}{dx} = 3ax^2$$

$$\therefore \quad \frac{dy}{dx} = \frac{3ax^2}{2y}$$

$$\therefore \quad m_1 = \left(\frac{dy}{dx}\right)_P = \frac{12a}{6} = 2a$$

$$m_2 = \text{slope of (ii)} = 4$$

According to the question

$$m_1 = m_2 \Rightarrow 2a = 4 \Rightarrow a = 2$$

From (i)

$$y^2 = 2 \times 2^3 + b$$

$$\Rightarrow \quad 9 = 16 + b$$

$$\Rightarrow \quad b = -7$$

Thus,

$$a = 2, b = -7$$

Tangents and Normals Ex 16.2 Q9

The given equatioins are,

$$y = x^2 + 4x - 16 \quad \text{---(i)}$$

$$3x - y + 1 = 0 \quad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 2x + 4$$

Slope m_2 of (ii)

$$m_2 = 3$$

As per question

$$m_1 = m_2$$

$$\Rightarrow \quad 2x + 4 = 3$$

$$\Rightarrow \quad x = \frac{-1}{2}$$

From (i)

$$y = \frac{1}{4} - 2 - 16 = -\frac{71}{4}$$

$$\therefore \quad P = \left(\frac{-1}{2}, \frac{-71}{4}\right)$$

Thus, the equation of tangent

$$\left(y + \frac{71}{4}\right) = 3\left(x + \frac{1}{2}\right)$$

$$\Rightarrow \quad 3x - y = \frac{71}{4} - \frac{3}{2}$$

$$\Rightarrow \quad 3x - y = \frac{65}{4}$$

$$\Rightarrow \quad 12x - 4y - 65 = 0$$

Tangents and Normals Ex 16.2 Q10

The given equation is

$$y = x^3 + 2x + 6 \qquad \text{---(i)}$$

$$x + 14y + 4 = 0 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 3x^2 + 2$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{14}$$

$\therefore$ Slope of normal to (i) is

$$\frac{-1}{m_1} = \frac{-1}{3x^2 + 2}$$

According to the question

$$\frac{-1}{3x^2 + 2} = \frac{-1}{14}$$

$\Rightarrow \quad 3x^2 2 = 14$

$\Rightarrow \quad x^2 = 4$

$\Rightarrow \quad x = \pm 2$

From (i)

$$y = 8 + 4 + 6 \quad \text{or} \quad -8 - 4 + 6$$

$$= 18 \quad \text{or} \quad -6$$

so, $\quad P = (2, 18)$ and $Q = (-2, -6)$

Thus, the equation of normal is

$$(y - 18) = \frac{-1}{14}(x - 2) \quad \Rightarrow \quad x + 14y + 86 = 0$$

or $\quad (y + 6) = \frac{-1}{14}(x + 2) \quad \Rightarrow \quad x + 14y - 254 = 0$

Tangents and Normals Ex 16.2 Q11

The given equations are,

$$y = 4x^3 - 3x + 5 \qquad \text{---(i)}$$

$$9y + x + 3 = 0 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = 12x^2 - 3$$

Slope m_2 of (ii)

$$m_2 = \frac{-1}{9}$$

According to the question

$$m_1 \times m_2 = -1$$

$$\Rightarrow \quad \left(12x^2 - 3\right)\left(-\frac{1}{9}\right) = -1$$

$$\Rightarrow \quad 4x^2 - 1 = 3$$

$$\Rightarrow \quad x^2 = 1$$

$$\Rightarrow \quad x = \pm 1$$

From (i)

$$y = 4 - 3 + 5 \qquad \text{or} \qquad -4 + 3 + 5$$

$$= 6 \quad \text{or} \quad 4$$

$$\therefore \quad P = (1, 6) \text{ or } Q = (-, 1, 4)$$

Thus, the equation of tangent is

$$(y - 6) = 9(x - 1) \quad \Rightarrow \quad 9x - y - 3 = 0$$

$$(y - 4) = 9(x + 1) \quad \Rightarrow \quad 9x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q12

The given equations are,

$$y = x \log_e x \qquad \text{---(i)}$$

$$2x - 2y + 3 = 0 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = \log_e x + 1$$

slope m_2 of (ii)

$$m_2 = 1$$

Tangents and Normals Ex 16.2 Q13

The equation of the given curve is $y = x^2 - 2x + 7$.

On differentiating with respect to x, we get:

$$\frac{dy}{dx} = 2x - 2$$

(a) The equation of the line is $2x - y + 9 = 0$.

$$2x - y + 9 = 0 \Rightarrow y = 2x + 9$$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 2

If a tangent is parallel to the line $2x - y + 9 = 0$, then the slope of the tangent is equal to the slope of the line.

Therefore, we have:

$$2 = 2x - 2$$

$$\Rightarrow 2x = 4$$
$$\Rightarrow x = 2$$

Now, $x = 2$

$\Rightarrow y = 4 - 4 + 7 = 7$

Thus, the equation of the tangent passing through (2, 7) is given by,

$$y - 7 = 2(x - 2)$$
$$\Rightarrow y - 2x - 3 = 0$$

Hence, the equation of the tangent line to the given curve (which is parallel to line $2x - y + 9 = 0$) is $y - 2x - 3 = 0$.

(b) The equation of the line is $5y - 15x = 13$.

$$5y - 15x = 13 \Rightarrow y = 3x + \frac{13}{5}$$

This is of the form $y = mx + c$.

$\therefore$Slope of the line = 3

If a tangent is perpendicular to the line $5y - 15x = 13$, then the slope of the tangent is $\frac{-1}{\text{slope of the line}} = \frac{-1}{3}$.

$$\Rightarrow 2x - 2 = \frac{-1}{3}$$

$$\Rightarrow 2x = \frac{-1}{3} + 2$$

$$\Rightarrow 2x = \frac{5}{3}$$

$$\Rightarrow x = \frac{5}{6}$$

Now, $x = \frac{5}{6}$

$$\Rightarrow y = \frac{25}{36} - \frac{10}{6} + 7 = \frac{25 - 60 + 252}{36} = \frac{217}{36}$$

Thus, the equation of the tangent passing through $\left(\frac{5}{6}, \frac{217}{36}\right)$ is given by,

$$y - \frac{217}{36} = -\frac{1}{3}\left(x - \frac{5}{6}\right)$$

$$\Rightarrow \frac{36y - 217}{36} = \frac{-1}{18}(6x - 5)$$

$$\Rightarrow 36y - 217 = -2(6x - 5)$$

$$\Rightarrow 36y - 217 = -12x + 10$$

$$\Rightarrow 36y + 12x - 227 = 0$$

Hence, the equation of the tangent line to the given curve (which is perpendicular to line $5y - 15x = 13$) is $36y + 12x - 227 = 0$.

Tangents and Normals Ex 16.2 Q14

The equation of the given curve is $y = \frac{1}{x-3}$, $x \neq 3$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx} = \frac{-1}{(x-3)^2}$$

If the slope of the tangent is 2, then we have:

$$\frac{-1}{(x-3)^2} = 2$$

$$\Rightarrow 2(x-3)^2 = -1$$

$$\Rightarrow (x-3)^2 = \frac{-1}{2}$$

This is not possible since the L.H.S. is positive while the R.H.S. is negative.

Hence, there is no tangent to the given curve having slope 2.

Tangents and Normals Ex 16.2 Q15

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx}=\frac{-(2x-2)}{(x^2-2x+3)^2}=\frac{-2(x-1)}{(x^2-2x+3)^2}$$

If the slope of the tangent is 0, then we have:

$$\frac{-2(x-1)}{(x^2-2x+3)^2}=0$$

$$\Rightarrow -2(x-1)=0$$
$$\Rightarrow x=1$$

When $x=1$, $y=\frac{1}{1-2+3}=\frac{1}{2}$.

$\therefore$The equation of the tangent through $\left(1,\ \frac{1}{2}\right)$ is given by,

$$y-\frac{1}{2}=0(x-1)$$
$$\Rightarrow y-\frac{1}{2}=0$$
$$\Rightarrow y=\frac{1}{2}$$

Hence, the equation of the required line is $y = \frac{1}{2}$.

Tangents and Normals Ex 16.2 Q16

The equation of the given curve is $y=\sqrt{3x-2}$.

The slope of the tangent to the given curve at any point (x, y) is given by,

$$\frac{dy}{dx}=\frac{3}{2\sqrt{3x-2}}$$

The equation of the given line is $4x-2y+5=0$.

$4x-2y+5=0 \Rightarrow y=2x+\frac{5}{2}$ (which is of the form $y=mx+c$)

$\therefore$Slope of the line $=2$

Now, the tangent to the given curve is parallel to the line $4x-2y-5=0$ if the slope of the tangent is equal to the slope of the line.

$$\frac{3}{2\sqrt{3x-2}}=2$$
$$\Rightarrow \sqrt{3x-2}=\frac{3}{4}$$
$$\Rightarrow 3x-2=\frac{9}{16}$$
$$\Rightarrow 3x=\frac{9}{16}+2=\frac{41}{16}$$
$$\Rightarrow x=\frac{41}{48}$$

When $x=\frac{41}{48}$, $y=\sqrt{3\left(\frac{41}{48}\right)-2}=\sqrt{\frac{41}{16}-2}=\sqrt{\frac{41-32}{16}}=\sqrt{\frac{9}{16}}=\frac{3}{4}$.

$\therefore$Equation of the tangent passing through the point $\left(\frac{41}{48}, \frac{3}{4}\right)$ is given by,

$$y-\frac{3}{4}=2\left(x-\frac{41}{48}\right)$$

$$\Rightarrow \frac{4y-3}{4}=2\left(\frac{48x-41}{48}\right)$$
$$\Rightarrow 4y-3=\frac{48x-41}{6}$$
$$\Rightarrow 24y-18=48x-41$$
$$\Rightarrow 48x-24y=23$$

Hence, the equation of the required tangent is $48x-24y=23$

Tangents and Normals Ex 16.2 Q17

The given equations are,

$$x^2 + 3y - 3 = 0 \qquad \text{---(i)}$$

$$y = 4x - 5 \qquad \text{---(ii)}$$

Slope m_1 of (i)

$$m_1 = \frac{dy}{dx} = -\frac{2x}{3}$$

Slope m_2 of (ii)

$$m_2 = 4$$

According to the question

$$m_1 = m_2$$

$$\Rightarrow \quad -\frac{2x}{3} = 4$$

$$\Rightarrow \quad x = -6$$

From (i)

$$36 + 3y - 3 = 0$$

$$\Rightarrow \quad 3y = -33$$

$$\therefore \quad y = -11$$

So, $P = (-6, -11)$

Thus, the equation of tangent is

$$(y + 11) = 4(x + 6)$$

$$\Rightarrow \quad 4x - y + 13 = 0$$

Tangents and Normals Ex 16.2 Q18

The equations are

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2 \qquad \text{---(i)}$$

$$\frac{x}{a} + \frac{y}{b} = 2 \qquad \text{---(ii)}$$

$$P = (a, b)$$

We need to prove (ii) is the tangent to (i)

Differentiating (i) with respect to x, we get

$$n\left(\frac{x}{a}\right)^n \times \frac{1}{a} + n\left(\frac{y}{b}\right)^{n-1} \times \frac{1}{b} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{x^{n-1}}{a^n} + \frac{y^{n-1}}{b^n} \times \frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$

$$\therefore \text{ Slope } m = \left(\frac{dy}{dx}\right)_P = -\left(\frac{a}{b}\right)^{n-1} \times \left(\frac{b}{a}\right)^n$$

$$= -\frac{b}{a}$$

Thus, the equation of tangent is

$$(y - b) = -\frac{b}{a}(x - a)$$

$$\Rightarrow \quad bx + ay = ab + ab$$

$$\Rightarrow \quad bx + ay = 2ab$$

$$\Rightarrow \quad \frac{x}{a} + \frac{y}{b} = 2$$

Tangents and Normals Ex 16.2 Q19

We have,

$$x = \sin 3t, \qquad y = \cos 2t, \qquad t = \frac{\pi}{4}$$

$$\therefore \quad P = \left(x = \frac{1}{\sqrt{2}}, y = 0\right)$$

Now,

$$\frac{dx}{dt} = 3\cos 3t, \quad \frac{dy}{dt} = -2\sin 2t$$

$$\therefore \text{ Slope } m = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$$

$$= \frac{-2}{-3 \times \frac{1}{\sqrt{2}}}$$

$$= \frac{+2\sqrt{2}}{3}$$

Thus, equation of tangent is

$$(y - 0) = \frac{+2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$2\sqrt{2}x - 3y = 2$$