RD Sharma Solutions Class 12 Maths Chapter 11 Ex 11.7

Chapter: Differentiation

Exercise: 11.7

Page Number: 11.112

Q1.

Solution :

$$We\ have,\ \ x = at^2\ \ and\ \ y = 2at$$

$$\Rightarrow \frac{dx}{dt} = 2at\ \ and\ \ \frac{dy}{dt} = 2a$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2a}{2at} = \frac{1}{t}$$

Q2.

Solution :

$$We\,have, x = a(\theta + \sin\theta)\,and\ y = a(1 - \cos\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = a(1 + \cos\theta)\,and\ \frac{dy}{d\theta} = a\ \sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\ \sin\theta}{a(1 + \cos\theta)} = \frac{2\ \sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\theta} = \tan\frac{\theta}{2}$$

Q3.

Solution :

$$We\ have,\ x = a\ \cos\theta\ and\ y = b\sin\theta$$

$$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta\ and\ \frac{dy}{d\theta} = b\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\cos\theta}{-a\sin\theta} = -\frac{b}{a}\cot\theta$$

Q4.

Solution :

$$We\ have,\ x = ae^{\theta}\left(\sin\theta - \cos\theta\right) and\ y = ae^{\theta}\left(\sin\theta + \cos\theta\right)$$

$$\Rightarrow \frac{dx}{d\theta} = a\left[e^{\theta}\frac{d}{d\theta}\left(\sin\theta - \cos\theta\right) + \left(\sin\theta - \cos\theta\right)\frac{d}{d\theta}\left(e^{\theta}\right)\right]\ \ and\ \frac{dy}{d\theta}$$

$$= a\left[e^{\theta}\frac{d}{d\theta}\left(\sin\theta + \cos\theta\right) + \left(\sin\theta + \cos\theta\right)\frac{d}{d\theta}\left(e^{\theta}\right)\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a\left[e^{\theta}\left(\cos\theta + \sin\theta\right) + \left(\sin\theta - \cos\theta\right)e^{\theta}\right]\ and\ \frac{dy}{d\theta} = a\left[e^{\theta}\left(\cos\theta - \sin\theta\right) + \left(\sin\theta + \cos\theta\right)e^{\theta}\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a\left[2e^{\theta}\sin\theta\right]\ and\ \frac{dy}{d\theta} = a\left[2e^{\theta}\cos\theta\right]$$

$$\therefore \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\left(2e^{\theta}\cos\theta\right)}{a\left(2e^{\theta}\sin\theta\right)} = \cot\theta$$

Q5.

Solution :

$$We\ have,\ x = b\sin^2\theta\ and\ y = a\cos^2\theta$$

$$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}\left(b\sin^2\theta\right) = 2b\sin\theta\cos\theta$$

and,

$$\frac{dy}{d\theta} = \frac{d}{d\theta}\left(a\cos^2\theta\right) = -2a\cos\theta\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-2a\cos\theta\sin\theta}{2b\sin\theta\cos\theta} = -\frac{a}{b}$$

Q6.

Solution :

$$We\ have,\ x=a(1-\cos\theta)\ \ and\ \ y=a(\theta+\sin\theta)$$

$$\therefore\ \frac{dx}{d\theta}=\frac{d}{d\theta}\left[a(1-\cos\theta)\right]=a(\sin\theta)$$

and

$$\frac{dy}{d\theta}=\frac{d}{d\theta}\left[a(\theta+\sin\theta)\right]=a(1+\cos\theta)$$

$$\therefore\ \left[\frac{dy}{dx}\right]_{\theta=\frac{\pi}{2}}=\left[\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}\right]_{\theta=\frac{\pi}{2}}=\left[\frac{a(1+\cos\theta)}{a(\sin\ \theta)}\right]_{\theta=\frac{\pi}{2}}=\frac{a(1+0)}{a}=1$$

Q7.

Solution :

$$We\,have, x=\frac{e^t+e^{-t}}{2}\ and\ y=\frac{e^t-e^{-t}}{2}$$

$$\Rightarrow\frac{dx}{dt}=\frac{1}{2}\left[\frac{d}{dt}(e^t)+\frac{d}{dt}(e^{-t})\right]\ and\ \frac{dy}{dt}=\frac{1}{2}\left[\frac{d}{dt}(e^t)-\frac{d}{dt}e^{-t}\right]$$

$$\Rightarrow\frac{dx}{dt}=\frac{1}{2}\left[e^t+e^{-t}\frac{d}{dt}(-t)\right]\ and\ \frac{dy}{dt}=\frac{1}{2}\left[e^t-e^{-t}\frac{d}{dt}(e^{-t})\right]$$

$$\Rightarrow\frac{dx}{dt}=\frac{1}{2}\left(e^t-e^{-t}\right)=y\ and\ \frac{dy}{dt}=\frac{1}{2}\left(e^t+e^{-t}\right)=x$$

$$\therefore\ \frac{dy}{dt}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{x}{y}$$

Q8.

Solution :

$$We\,have, x = \frac{3at}{1+t^2}$$

Differentiating with respect to t,

$$\frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at\right) - 3at\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right] \qquad \left[using\ quotient\ rule\right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{\left(1+t^2\right)\left(3a\right) - 3at\left(2t\right)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{3a + 3at^2 - 6at^2}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt} = \left[\frac{3a - 3at^2}{\left(1-t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{3a\left(1-t^2\right)}{\left(1+t^2\right)^2} \qquad ...(i)$$

$$and,\ y = \frac{3at^2}{1+t^2}$$

Q9.

Solution :

$$\frac{dx}{dt}=\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(3at^2\right)-3at^2\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right] \qquad \left[\text{using quotient rule}\right]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{\left(1+t^2\right)\left(6at\right)-3at^2\left(2t\right)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{6at+6at^3-6at^3}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt}=\frac{6at}{\left(1+t^2\right)^2} \qquad \ldots(ii)$$

Dividing equation (ii) *by* (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{6at}{\left(1+t^2\right)^2}\times\frac{\left(1+t^2\right)^2}{3a\left(1-t^2\right)}=\frac{2t}{1-t^2}$$

We have, $x=a\left(\cos\theta+\theta\sin\theta\right)$ *and* $y=a\left(\sin\theta-\theta\cos\theta\right)$

$$\Rightarrow \frac{dx}{d\theta}=a\left[\frac{d}{d\theta}\cos\theta+\frac{d}{d\theta}\left(\theta\sin\theta\right)\right] \text{ and } \frac{dy}{d\theta}=a\left[\frac{d}{d\theta}\left(\sin\theta\right)-\frac{d}{d\theta}\left(\theta\cos\theta\right)\right]$$

$$\Rightarrow \frac{dx}{d\theta}=a\left[-\sin\theta+\theta\frac{d}{d\theta}\left(\sin\theta\right)+\sin\theta\frac{d}{d\theta}\left(\theta\right)\right] \text{ and } \frac{dy}{d\theta}=a\left[\cos\theta-\left\{\theta\frac{d}{d\theta}\left(\cos\theta\right)+\cos\theta\frac{d}{d\theta}\left(\theta\right)\right\}\right]$$

$$\Rightarrow \frac{dx}{d\theta}=a\left[-\sin\theta+\theta\cos\theta\right] \text{ and } \frac{dy}{d\theta}=a\left[\cos\theta+\theta\sin\theta-\cos\theta\right]$$

$$\Rightarrow \frac{dx}{d\theta}=a\theta\cos\theta \text{ and } \frac{dy}{d\theta}=a\theta\sin\theta$$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{a\theta\sin\theta}{a\theta\cos\theta}=\tan\theta$$

Q10.

Solution :

We have, $x=e^{\theta}\left(\theta+\frac{1}{\theta}\right)$

$$\frac{dx}{d\theta} = e^{\theta}\frac{d}{d\theta}\left(\theta + \frac{1}{\theta}\right) + \left(\theta + \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{\theta}\right) \qquad [\textit{using product rule}]$$

$$\Rightarrow \frac{dx}{d\theta} = e^{\theta}\left(1 - \frac{1}{\theta^2}\right) + \left(\frac{\theta^2 + 1}{\theta}\right)e^{\theta}$$

$$\Rightarrow \frac{dx}{d\theta} = e^{\theta}\left(1 - \frac{1}{\theta^2} + \frac{\theta^2 + 1}{\theta}\right)$$

$$\Rightarrow \frac{dx}{d\theta} = e^{\theta}\left(\frac{\theta^2 - 1 + \theta^3 + \theta}{\theta^2}\right)$$

$$\Rightarrow \frac{dx}{d\theta} = \frac{e^{\theta}\left(\theta^3 + \theta^2 + \theta - 1\right)}{\theta^2} \qquad \ldots(i)$$

and,

$$y = e^{\theta}\left(\theta - \frac{1}{\theta}\right)$$

$$\frac{dy}{d\theta} = e^{-\theta}\frac{d}{d\theta}\left(\theta - \frac{1}{\theta}\right) + \left(\theta - \frac{1}{\theta}\right)\frac{d}{d\theta}\left(e^{-\theta}\right) \qquad [\textit{using product rule}]$$

$$\Rightarrow \frac{dy}{d\theta} = e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{\theta}\frac{d}{d\theta}(-\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = e^{-\theta}\left(1 + \frac{1}{\theta^2}\right) + \left(\theta - \frac{1}{\theta}\right)e^{-\theta}(-1)$$

$$\Rightarrow \frac{dy}{d\theta} = e^{-\theta}\left(1 + \frac{1}{\theta^2} - \theta + \frac{1}{\theta}\right)$$

$$\Rightarrow \frac{dy}{d\theta} = e^{-\theta}\left(\frac{\theta^2 + 1 - \theta^3 + \theta}{\theta^2}\right)$$

$$\Rightarrow \frac{dy}{d\theta} = \frac{e^{-\theta}\left(-\theta^3 + \theta^2 + \theta + 1\right)}{\theta^2} \qquad \ldots(ii)$$

Dividing equation (ii) *by* (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = e^{-\theta}\left(\frac{\theta^2 - \theta^3 + \theta + 1}{\theta^2}\right) \times \frac{\theta^2}{e^{\theta}\left(\theta^3 + \theta^2 + \theta - 1\right)}$$

$$= e^{-2\theta}\left(\frac{\theta^2 - \theta^3 + \theta + 1}{\theta^3 + \theta^2 + \theta - 1}\right)$$

Q11.

Solution :

$$\textit{We have}, x = \frac{2t}{1 + t^2}$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t)-2t\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right] \qquad [using\ quotient\ rule]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{\left(1+t^2\right)(2)-2t(2t)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{2-2t^2}{\left(1+t^2\right)^2}\right] \qquad ...(i)$$

and,

$$y=\frac{1-t^2}{1+t^2}$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)-\left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{\left(1+t^2\right)(-2t)-\left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{-4t}{\left(1+t^2\right)^2}\right] \qquad ...(ii)$$

Dividing equation (ii) by (i), we get,

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-4t}{\left(1+t^2\right)^2}\times\frac{\left(1+t^2\right)^2}{2\left(1-t^2\right)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-2t}{1-t^2}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{x}{y} \qquad \left[\because \frac{x}{y}=\frac{2t}{1+t^2}\times\frac{1+t^2}{1-t^2}=\frac{2t}{1-t^2}\right]$$

Q12.

Solution :

$$We\,have, x=\cos^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dx}{dt}=\frac{-1}{\sqrt{1-\left(\frac{1}{\sqrt{1+t^2}}\right)^2}}\frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dx}{dt}=\frac{-1}{\sqrt{1-\frac{1}{\left(1+t^2\right)}}}\left\{\frac{-1}{2\left(1+t^2\right)^{\frac{3}{2}}}\right\}\frac{d}{dt}\left(1+t^2\right)$$

$$\Rightarrow \frac{dx}{dt}=\frac{\left(1+t^2\right)^{\frac{1}{2}}}{\sqrt{1+t^2-1}}\times\frac{1}{2\left(1+t^2\right)^{\frac{3}{2}}}\left(2t\right)$$

$$\Rightarrow \frac{dx}{dt}=\frac{t}{\sqrt{t^2}\times\left(1+t^2\right)}$$

$$\Rightarrow \frac{dx}{dt}=\frac{1}{1+t^2} \qquad \ldots(i)$$

$$Now,\ y=\sin^{-1}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{1-\left(\frac{1}{\sqrt{1+t^2}}\right)^2}} \frac{d}{dt}\left(\frac{1}{\sqrt{1+t^2}}\right)$$

$$\Rightarrow \frac{dy}{dt} = \frac{1}{\sqrt{1-\frac{1}{(1+t^2)}}} \left\{\frac{-1}{2(1+t^2)^{\frac{3}{2}}}\right\} \frac{d}{dt}(1+t^2)$$

$$\Rightarrow \frac{dx}{dt} = \frac{(1+t^2)^{\frac{1}{2}}}{\sqrt{1+t^2-1}} \times \frac{-1}{2(1+t^2)^{\frac{3}{2}}}(2t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-1}{2\sqrt{t^2} \times (1+t^2)}(2t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{-1}{1+t^2} \quad ...(ii)$$

Dividing equation (ii) *by* (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1}{(1+t^2)} \times \frac{(1+t^2)}{-1}$$

$$\Rightarrow \frac{dy}{dx} = -1$$

Q13.

Solution :

$$We\, have, y = \frac{2t}{1+t^2}$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{\left(1+t^2\right)\frac{d}{dt}(2t)-2t\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right] \quad [using\ quotient\ rule]$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{\left(1+t^2\right)(2)-2t(2t)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{2+2t^2-4t^2}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dy}{dt}=\left[\frac{2-2t^2}{\left(1+t^2\right)^2}\right] \quad ...(i)$$

and,

$$x=\frac{1-t^2}{1+t^2}$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{\left(1+t^2\right)\frac{d}{dt}\left(1-t^2\right)-\left(1-t^2\right)\frac{d}{dt}\left(1+t^2\right)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{\left(1+t^2\right)(-2t)-\left(1-t^2\right)(2t)}{\left(1+t^2\right)^2}\right]$$

$$\Rightarrow \frac{dx}{dt}=\left[\frac{-4t}{\left(1+t^2\right)^2}\right] \quad ...(ii)$$

Dividing equation (i) *by* (ii)*, we get,*

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{2\left(1-t^2\right)}{\left(1+t^2\right)^2}\times\frac{\left(1+t^2\right)^2}{-4t}$$

$$\Rightarrow \frac{dy}{dx}=\frac{2\left(1-t^2\right)}{-4t}$$

$$\Rightarrow \frac{dy}{dx}=\frac{t^2-1}{2t}$$

Q14.

Solution :

$We\ have, x = 2\cos\theta - \cos 2\theta$

$$\Rightarrow \frac{dx}{d\theta} = 2(-\sin\theta) - (-\sin 2\theta)\frac{d}{d\theta}(2\theta)$$

$$\Rightarrow \frac{dx}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\Rightarrow \frac{dx}{d\theta} = 2(\sin 2\theta - \sin\theta) \qquad \ldots(i)$$

and,

$y = 2\sin\theta - \sin 2\theta$

$$\Rightarrow \frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta \frac{d}{d\theta}(2\theta)$$

$$\Rightarrow \frac{dy}{d\theta} = 2\cos\theta - \cos 2\theta(2)$$

$$\Rightarrow \frac{dy}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\Rightarrow \frac{dy}{d\theta} = 2(\cos\theta - \cos 2\theta) \qquad \ldots(ii)$$

Dividing equation (ii) by equation (i),

$$\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{2(\cos\theta - \cos 2\theta)}{2(\sin 2\theta - \sin\theta)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos\theta - \cos 2\theta}{\sin 2\theta - \sin\theta}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-2\sin\left(\frac{\theta + 2\theta}{2}\right)\sin\left(\frac{\theta - 2\theta}{2}\right)}{2\cos\left(\frac{2\theta + \theta}{2}\right)\sin\left(\frac{2\theta - \theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin\left(\frac{3\theta}{2}\right)\sin\left(\frac{-\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sin\left(\frac{3\theta}{2}\right)\left(-\sin\frac{\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)\sin\left(\frac{\theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin\left(\frac{3\theta}{2}\right)}{\cos\left(\frac{3\theta}{2}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \tan\left(\frac{3\theta}{2}\right)$$

Q15.

Solution :

We have, $x = e^{\cos 2t}$ *and* $y = e^{\sin 2t}$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left(e^{\cos 2t}\right) \text{ and } \frac{dy}{dt} = \frac{d}{dt}\left(e^{\sin 2t}\right)$$

$$\Rightarrow \frac{dx}{dt} = e^{\cos 2t}\frac{d}{dt}(\cos 2t) \text{ and } \frac{dy}{dt} = e^{\sin 2t}\frac{d}{dt}(\sin 2t)$$

$$\Rightarrow \frac{dx}{dt} = e^{\cos 2t}(-\sin 2t)\frac{d}{dt}(2t) \text{ and } \frac{dy}{dt} = e^{\sin 2t}(\cos 2t)\frac{d}{dt}(2t)$$

$$\Rightarrow \frac{dx}{dt} = -2\sin 2te^{\cos 2t} \text{ and } \frac{dy}{dt} = 2\cos 2te^{\sin 2t}$$

$$\because \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\cos 2te^{\sin 2t}}{-2\sin 2te^{\cos 2t}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y\log x}{x\log y} \qquad \left[\begin{array}{l}\because x = e^{\cos 2t} \Rightarrow \log x = \cos 2t \\ y = e^{\sin 2t} \Rightarrow \log y = \sin 2t\end{array}\right]$$

Q16.

Solution :

We have, $x = \cos t$ *and* $y = \sin t$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}(\cos t) \text{ and } \frac{dy}{dt} = \frac{d}{dt}(\sin t)$$

$$\Rightarrow \frac{dx}{dt} = -\sin t \text{ and } \frac{dy}{dt} = \cos t$$

$$\therefore \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\cos t}{-\sin t} = -\cot t$$

$$Now, \left(\frac{dy}{dx}\right)_{t=\frac{2\pi}{3}} = -\cot\left(\frac{2\pi}{3}\right) = \frac{1}{\sqrt{3}}$$

Q17.

Solution :

We have, $x = a\left(t + \frac{1}{t}\right)$ *and* $y = a\left(t - \frac{1}{t}\right)$

$$\Rightarrow \frac{dx}{dt} = a\frac{d}{dt}\left(t+\frac{1}{t}\right) \text{ and } \frac{dy}{dt} = a\frac{d}{dt}\left(t-\frac{1}{t}\right)$$

$$\Rightarrow \frac{dx}{dt} = a\left(1-\frac{1}{t^2}\right) \text{ and } \frac{dy}{dt} = a\left(1+\frac{1}{t^2}\right)$$

$$\Rightarrow \frac{dx}{dt} = a\left(\frac{t^2-1}{t^2}\right) \text{ and } \frac{dy}{dt} = a\left(\frac{t^2+1}{t^2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\left(t^2+1\right)}{t^2} \times \frac{t^2}{a\left(t^2-1\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{a\left(t^2+1\right)}{t} \times \frac{t}{a\left(t^2-1\right)}$$

$$\Rightarrow \frac{dy}{dx} = a\left(t+\frac{1}{t}\right) \times \frac{1}{a\left(t-\frac{1}{t}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y}$$

Q18.

Solution :

We have, $x = \sin^{-1}\left(\frac{2t}{1+t^2}\right)$

Put $t = \tan\theta$

$\Rightarrow -1 < \tan\theta < 1$

$$\Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$$

$$\Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}$$

$$\therefore x = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$$\Rightarrow x = \sin^{-1}\left(\sin 2\theta\right)$$

$$\Rightarrow x = 2\theta \qquad \left[\because -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}\right]$$

$$\Rightarrow x = 2\left(\tan^{-1} t\right) \qquad \left[\because t = \sin\theta\right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{2}{1+t^2} \qquad ...(ii)$$

Dividing equation (ii) *by* (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2}{1+t^2} \times \frac{1+t^2}{2}$$

$$\Rightarrow \frac{dy}{dx} = 1$$

$$\Rightarrow \frac{dx}{dt} = \frac{2}{1+t^2} \qquad ...(i)$$

$$Now,\ y = \tan^{-1}\left(\frac{2t}{1-t^2}\right)$$

$$put\ t = \tan\theta$$

$$\Rightarrow \quad y = \tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)$$

$$\Rightarrow \quad y = \tan^{-1}(\tan 2\theta)$$

$$\Rightarrow y = 2\theta \qquad \left[\because -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}\right]$$

$$\Rightarrow y = 2\tan^{-1} t \qquad [\because\ t = \tan\theta]$$

Q19.

Solution :

We have, $x=\frac{\sin^3 t}{\sqrt{\cos 2t}}$ and $y=\frac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\Rightarrow \frac{dx}{dt}=\frac{d}{dt}\left[\frac{\sin^3 t}{\sqrt{\cos 2t}}\right]$$

$$\Rightarrow \frac{dx}{dt}=\frac{\sqrt{\cos 2t}\frac{d}{dt}\left(\sin^3 t\right)-\sin^3 t\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t} \qquad \text{[Using quotient rule]}$$

$$\Rightarrow \frac{dx}{dt}=\frac{\sqrt{\cos 2t}\left(3\sin^2 t\right)\frac{d}{dt}\left(\sin t\right)-\sin^3 t\times\frac{1}{2\sqrt{\cos 2t}}\frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$\Rightarrow \frac{dx}{dt}=\frac{3\sqrt{\cos 2t}\left(\sin^2 t\cos t\right)-\frac{\sin^3 t}{2\sqrt{\cos 2t}}\left(-2\sin 2t\right)}{\cos 2t}$$

$$\Rightarrow \frac{dx}{dt}=\frac{3\cos 2t\sin^2 t\cos t+\sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

Now, $\frac{dy}{dt}=\frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right]$

$$\Rightarrow \frac{dy}{dt}=\frac{\sqrt{\cos 2t}\frac{d}{dt}\left(\cos^3 t\right)-\cos^3 t\frac{d}{dt}\sqrt{\cos 2t}}{\cos 2t} \qquad \text{[Using quotient rule]}$$

$$\Rightarrow \frac{dy}{dt}=\frac{\sqrt{\cos 2t}\left(3\cos^2 t\right)\frac{d}{dt}\left(\cos t\right)-\cos^3 t\times\frac{1}{2\sqrt{\cos 2t}}\frac{d}{dt}\left(\cos 2t\right)}{\cos 2t}$$

$$\Rightarrow \frac{dy}{dt}=\frac{3\sqrt{\cos 2t}\cos^2 t\left(-\sin t\right)-\frac{\cos^3 t}{2\sqrt{\cos 2t}}\left(-2\sin 2t\right)}{\cos 2t}$$

$$\Rightarrow \frac{dy}{dt}=\frac{-3\cos 2t\cos^2 t\sin t+\cos^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}$$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{-3\cos 2t\cos^2 t\sin t+\cos^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}\times\frac{\cos 2t\sqrt{\cos 2t}}{3\cos 2t\sin^2 t\cos t+\sin^3 t\sin 2t}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sin t\cos t\left[-3\cos 2t\cos t+2\cos^3 t\right]}{\sin t\cos t\left[3\cos 2t\sin t+2\sin^3 t\right]}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\left[-3\left(2\cos^2 t-1\right)\cos t+2\cos^3 t\right]}{\left[3\left(1-2\sin^2 t\right)\sin t+2\sin^3 t\right]} \qquad \left[\begin{matrix}\cos 2t=2\cos^2 t-1\\ \cos 2t=1-2\sin^2 t\end{matrix}\right]$$

$$\Rightarrow \frac{dy}{dx}=\frac{-4\cos^3 t+3\cos t}{3\sin t-4\sin^3 t}$$

$$\Rightarrow \frac{dy}{dx}=\frac{-\cos 3t}{\sin 3t} \qquad \left[\begin{matrix}\cos 3t=4\cos^3 t-3\cos t\\ \sin 3t=3\sin t-4\sin^3 t\end{matrix}\right]$$

$$\therefore \frac{dy}{dx}=-\cot 3t$$

Q20.

Solution :

$$We\,have, x=\left(t+\frac{1}{t}\right)^{a}$$

$$\Rightarrow \frac{dx}{dt}=\frac{d}{dt}\left[\left(t+\frac{1}{t}\right)^{a}\right]$$

$$\Rightarrow \frac{dx}{dt}=a\left(t+\frac{1}{t}\right)^{a-1}\frac{d}{dt}\left(t+\frac{1}{t}\right)$$

$$\Rightarrow \frac{dx}{dt}=a\left(t+\frac{1}{t}\right)^{a-1}\left(1-\frac{1}{t^{2}}\right) \qquad ...(i)$$

$and,$

$$y=a^{\left(t+\frac{1}{t}\right)}$$

$$\Rightarrow \frac{dy}{dt}=\frac{d}{dt}\left[a^{\left(t+\frac{1}{t}\right)}\right]$$

$$\Rightarrow \frac{dy}{dt}=a^{\left(t+\frac{1}{t}\right)}\times \log a\frac{d}{dt}\left(t+\frac{t}{t}\right)$$

$$\Rightarrow \frac{dy}{dt}=a^{\left(t+\frac{1}{t}\right)}\times \log a\left(1-\frac{1}{t^{2}}\right) \qquad ...(ii)$$

$Dividing\ equation\ (ii)\ by\ (i),$

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{a^{\left(t+\frac{1}{t}\right)}\times \log a\left(1-\frac{1}{t^{2}}\right)}{a\left(t+\frac{1}{t}\right)^{a-1}\left(1-\frac{1}{t^{2}}\right)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{a^{\left(t+\frac{1}{t}\right)}\times \log a}{a\left(t+\frac{1}{t}\right)^{a-1}}$$

Q21.

Solution :

$$We\,have, x=a\left(\frac{1+t^{2}}{1-t^{2}}\right)$$

$$\Rightarrow \frac{dx}{dt} = a\left[\frac{(1-t^2)\frac{d}{dt}(1+t^2)-(1+t^2)\frac{d}{dt}(1-t^2)}{(1-t^2)^2}\right] \quad [Using\ quotient\ rule]$$

$$\Rightarrow \frac{dx}{dt} = a\left[\frac{(1-t^2)(2t)-(1+t^2)(-2t)}{(1-t^2)^2}\right]$$

$$\Rightarrow \frac{dx}{dt} = a\left[\frac{2t-2t^3+2t+2t^3}{(1-t^2)^2}\right]$$

$$\Rightarrow \frac{dx}{dt} = \frac{4at}{(1-t^2)^2} \quad ...(i)$$

and,

$$y = \frac{2t}{1-t^2}$$

$$\Rightarrow \frac{dy}{dt} = 2\left[\frac{(1-t^2)\frac{d}{dt}(t)-t\frac{d}{dt}(1-t^2)}{(1-t^2)^2}\right] \quad [Using\ quotient\ rule]$$

$$\Rightarrow \frac{dy}{dt} = 2\left[\frac{(1-t^2)(1)-t(-2t)}{(1-t^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dt} = 2\left[\frac{1-t^2+2t^2}{(1-t^2)^2}\right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{2(1+t^2)}{(1-t^2)^2} \quad ...(ii)$$

Dividing equation (ii) *by* (i),

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2(1+t^2)}{(1-t^2)^2} \times \frac{(1-t^2)^2}{4at}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(1+t^2)}{2at}$$

Q22.

Solution :

We have, $x = 10(t - \sin t)$ *and* $y = 12(1 - \cos t)$

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left[10(t - \sin t)\right] \text{ and } \frac{dy}{dt} = \frac{d}{dt}\left[12(1 - \cos t)\right]$$

$$\Rightarrow \frac{dx}{dt} = 10\frac{d}{dt}(t - \sin t) \text{ and } \frac{dy}{dt} = 12\frac{d}{dt}(1 - \cos t)$$

$$\Rightarrow \frac{dx}{dt} = 10(1 - \cos t) \text{ and } \frac{dy}{dt} = 12\left[0 - (-\sin t)\right] = 12 \sin t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{12 \sin t}{10(1 - \cos t)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{12 \times 2 \sin \frac{t}{2} \cos \frac{t}{2}}{10 \times 2 \sin^2 \frac{t}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6}{5} \cot \frac{t}{2}$$

Q23.

Solution :

We have, $x = a(\theta - \sin \theta)$ *and* $y = a(1 + \cos \theta)$

$$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta}\left[a(\theta - \sin \theta)\right] \text{ and } \frac{dy}{d\theta} = \frac{d}{d\theta}\left[a(1 + \cos \theta)\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a(1 - \cos \theta) \text{ and } \frac{dy}{d\theta} = a(-\sin \theta)$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a \sin \theta}{a(1 - \cos \theta)}$$

$$\text{Now, } \left[\frac{dy}{dx}\right]_{\theta = \frac{\pi}{3}} = -\frac{\sin \frac{\pi}{3}}{1 - \cos \frac{\pi}{3}} = -\frac{\frac{\sqrt{3}}{2}}{1 - \frac{1}{2}} = -\sqrt{3}$$