# RD Sharma Solutions Class 12 Maths Chapter 11 Ex 11.6

**Chapter: Differentiation**

**Exercise: 11.6**

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**Q1.**

**Answer :**

$$\text{We have, } y=\sqrt{x+\sqrt{x+\sqrt{x+\dots \text{ to } \infty}}}$$
$$\Rightarrow y=\sqrt{x+y}$$
*Squaring both sides, we get,*
$$y^2=x+y$$
$$\Rightarrow 2y\frac{dy}{dx}=1+\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}(2y-1)=1$$
$$\Rightarrow \frac{dy}{dx}=\frac{1}{2y-1}$$

**Q2.**

**Answer :**

$$\text{We have, } y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+\dots \text{ to } \infty}}}$$
$$\Rightarrow y=\sqrt{\cos x+y}$$
*Squaring both sides, we get,*
$$y^2=\cos x+y$$
$$\Rightarrow 2y\frac{dy}{dx}=-\sin x+\frac{dy}{dx}$$
$$\Rightarrow \frac{dy}{dx}(2y-1)=-\sin x$$
$$\Rightarrow \frac{dy}{dx}=\frac{-\sin x}{2y-1}$$
$$\Rightarrow \frac{dy}{dx}=\frac{\sin x}{1-2y}$$

**Q3.**

**Answer :**

$$We\ have,\ y=\sqrt{\log x+\sqrt{\log x+\sqrt{\log x+\dots\ to\ \infty}}}$$

$$\Rightarrow y=\sqrt{\log x+y}$$

*Squaring both sides, we get,*

$$y^2=\log x+y$$

$$\Rightarrow 2y\frac{dy}{dx}=\frac{1}{x}+\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y-1)=\frac{1}{x}$$

**Q4.**

**Answer :**

$$We\ have,\ y=\sqrt{\tan x+\sqrt{\tan x+\sqrt{\tan x+\dots\ to\ \infty}}}$$

$$\Rightarrow y=\sqrt{\tan x+y}$$

*Squaring both sides, we get,*

$$y^2=\tan x+y$$

$$\Rightarrow 2y\frac{dy}{dx}=\sec^2 x+\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y-1)=\sec^2 x$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sec^2 x}{2y-1}$$

**Q5.**

**Answer :**

$$We\ have,\ y=(\sin x)^{(\sin x)^{(\sin x)^{\dots x}}}$$

$$\Rightarrow y=(\sin x)^y$$

Taking log on both sides,

$$\log y=\log(\sin x)^y$$

$$\Rightarrow \log y=y\log(\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\{\log(\sin x)\} + \log\ \sin x\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\sin x}\right)\frac{d}{dx}(\sin x) + \log\ \sin x\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log\ \sin x\right) = \frac{y}{\sin x}(\cos x)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log\ \sin x}{y}\right) = y\cot x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y^2\cot x}{(1 - y\log\ \sin x)}$$

**Q6.**

**Answer :**

$$We\, have, y = (\tan x)^{(\tan x)^{(\tan x)^{\cdots\infty}}}$$

$$\Rightarrow y = (\tan x)^y$$

Taking log on both sides,

$$\log y = \log(\tan x)^y$$

$$\Rightarrow \log y = y\ \log \tan x$$

Differentiating with respect to x using chain rule ,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\{\log\ \tan x\} + \log\ \tan\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{y}{\tan x}\frac{d}{dx}(\tan x) + \log\ \tan\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log\ \tan x\right) = \frac{y}{\tan x}\sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{\tan x}\sec^2 x \times \left(\frac{y}{1 - y\log\ \tan x}\right)$$

$$Now, \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y\sec^2\left(\frac{\pi}{4}\right)}{\tan\left(\frac{\pi}{4}\right)} \times \frac{y}{1 - y\log\ \tan\left(\frac{\pi}{4}\right)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{y^2\left(\sqrt{2}\right)^2}{1(1 - y\log\ \tan 1)}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{2(1)^2}{(1-0)} \qquad \left[\because (y)_{\frac{\pi}{4}} = \left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\left(\tan\frac{\pi}{4}\right)^{\dots\infty}}} = 1\right]$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = 2$$

**Q7.**

**Answer :**

$$We\ have,\ y = e^{x^{e^x}} + x^{e^{e^x}} + e^{x^{x^e}}$$

$$\Rightarrow y = u + v + w$$

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} \quad \dots(i)$$

$$where\ \ u = e^{x^{e^x}},\ v = x^{e^{e^x}}\ and\ w = e^{x^{x^e}}$$

$$Now, u = e^{x^{e^x}} \qquad \dots(ii)$$

Taking log on both sides,

$$\log u = \log e^{x^{e^x}}$$

$$\Rightarrow \log u = x^{e^x}\ \log e$$

$$\Rightarrow \log u = x^{e^x} \qquad \dots(iii)$$

$$\log \log u = \log x^{e^x} \Rightarrow \log \log u = e^x \ \log x$$

Differentiating with respect to x,

$$\Rightarrow \frac{1}{\log u} \frac{d}{dx}(\log u) = e^x \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(e^x)$$

$$\Rightarrow \frac{1}{\log u} \frac{1}{u} \frac{du}{dx} = \frac{e^x}{x} + e^x \ \log x$$

$$\Rightarrow \frac{du}{dx} = u \log u \left[ \frac{e^x}{x} + e^x \ \log x \right]$$

$$\Rightarrow \frac{du}{dx} = e^{x^{e^x}} \times x^{e^x} \left[ \frac{e^x}{x} + e^x \ \log x \right] \qquad ...(A)$$

$$Now, \ v = x^{e^{e^x}} \qquad ...(iv)$$

Taking log on both sides,

$$\log v = \log x^{e^{e^x}}$$

$$\Rightarrow \log v = e^{e^x} \ \log x$$

Taking log on both sides,

$$\Rightarrow \frac{1}{\log w} \frac{d}{dx}(\log w) = x^e \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^e)$$

$$\Rightarrow \frac{1}{\log w}\left(\frac{1}{w}\right)\frac{dw}{dx} = x^e \left(\frac{1}{x}\right) + \log x e x^{e-1}$$

$$\Rightarrow \frac{dw}{dx} = w \log w \left[ x^{e-1} + e \log x x^{e-1} \right]$$

$$\Rightarrow \frac{dw}{dx} = e^{x^{x^e}} x^{x^e} x^{e-1} (1 + e \log x) \qquad ----(C)$$

$$Using \ equation \ (A), (B) \ and \ (C) \ in \ equation \ (i), \ we \ get$$

$$\frac{dy}{dx} = e^{x^{e^x}} x^{e^x} \left[ \frac{e^x}{x} + e^x \log x \right] + x^{e^{e^x}} \times e^{e^x} \left[ \frac{1}{x} + e^x \ \log x \right] + e^{x^{x^e}} x^{x^e} x^{e-1} (1 + e \log x)$$

**Q8.**

**Answer :**

$$We \, have, y = (\cos x)^{(\cos x)^{(\cos x)^{\dots \infty}}}$$

$$\Rightarrow y = (\cos x)^y$$

Taking log on both sides,

$$\log y = \log(\cos x)^y$$

$$\Rightarrow \log y = y \log(\cos x)$$

Differentiating with respect to x using chain rule,

$$\frac{1}{y}\frac{dy}{dx} = y\frac{d}{dx}\{\log \cos x\} + \log \cos x \frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = y\left(\frac{1}{\cos x}\right)\frac{d}{dx}(\cos x) + \log \cos x \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} - \log \cos x\right) = \frac{y}{\cos x}(-\sin x)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{1 - y\log \cos x}{y}\right) = -y \tan x$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^2 \tan x}{(1 - y\log \cos x)}$$