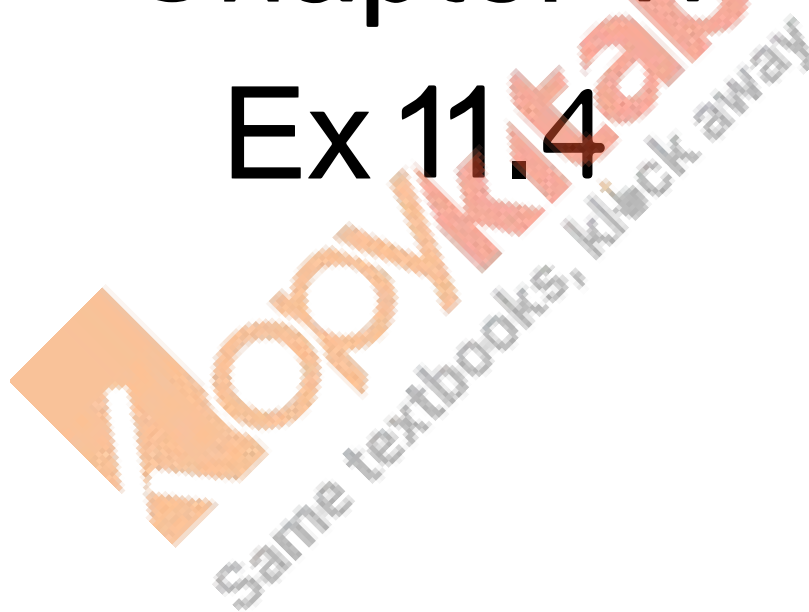


RD Sharma
Solutions
Class 12 Maths
Chapter 11
Ex 11.4



Exercise: 11.4**Page Number: 11.74**

1.

Solution :

We have, $xy = c^2$ *Differentiating with respect to x, we get*

$$\frac{d}{dx}(xy) = \frac{d}{dx}(c^2)$$

$$\Rightarrow x \frac{dy}{dx} + y \frac{d}{dx}(x) = 0 \quad \text{[Using product rule]}$$

$$\Rightarrow x \frac{dy}{dx} + y = 0$$

$$\Rightarrow x \frac{dy}{dx} = -y$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

2.

Solution :

We have, $y^3 - 3xy^2 = x^3 + 3x^2y$ *Differentiating with respect to x, we get*

$$\Rightarrow \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y)$$

$$\Rightarrow 3y^2 \frac{dy}{dx} - 3 \left[x \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x) \right] = 3x^2 + 3 \left[x^2 \frac{d}{dx}(y) + y \frac{d}{dx}(x^2) \right] \quad \text{[Using product rule]}$$

$$\Rightarrow 3y^2 \frac{dy}{dx} - 3 \left[x(2y) \frac{dy}{dx} + y^2 \right] = 3x^2 + 3 \left[x^2 \frac{dy}{dx} + y(2x) \right]$$

$$\Rightarrow 3y^2 \frac{dy}{dx} - 6xy \frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2 \frac{dy}{dx} + 6xy$$

$$\Rightarrow 3y^2 \frac{dy}{dx} - 6xy \frac{dy}{dx} - 3x^2 \frac{dy}{dx} = 3x^2 + 6xy + 3y^2$$

$$\Rightarrow 3 \frac{dy}{dx} (y^2 - 2xy - x^2) = 3(x^2 + 2xy + y^2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{3(x+y)^2}{3(y^2 - 2xy - x^2)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$$

3.

Solution :

$$\text{We have, } x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

Differentiating with respect to x , we get

$$\frac{d}{dx}\left(x^{\frac{2}{3}}\right) + \frac{d}{dx}\left(y^{\frac{2}{3}}\right) = \frac{d}{dx}\left(a^{\frac{2}{3}}\right)$$

$$\Rightarrow \frac{2}{3}(x)^{\frac{2}{3}-1} + \frac{2}{3}(y)^{\frac{2}{3}-1} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3}(x)^{-\frac{1}{3}} + \frac{2}{3}(y)^{-\frac{1}{3}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{2}{3}(y)^{-\frac{1}{3}} \frac{dy}{dx} = -\frac{2}{3}(x)^{-\frac{1}{3}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2}{3}(x)^{-\frac{1}{3}} \times \frac{3}{2y^{\frac{1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^{-\frac{1}{3}}}{y^{\frac{1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$$

4.

Solution :

$$\text{We have, } 4x + 3y = \log(4x - 3y)$$

Differentiating with respect to x , we get

$$\begin{aligned}
& \frac{d}{dx}(4x) + \frac{d}{dx}(3y) = \frac{d}{dx}\{\log(4x-3y)\} \\
\Rightarrow 4 + 3 \frac{dy}{dx} &= \frac{1}{(4x-3y)} \frac{d}{dx}(4x-3y) \\
\Rightarrow 4 + 3 \frac{dy}{dx} &= \frac{1}{(4x-3y)} \left(4 - 3 \frac{dy}{dx}\right) \\
\Rightarrow 3 \frac{dy}{dx} + \frac{3}{(4x-3y)} \frac{dy}{dx} &= \frac{4}{(4x-3y)} - 4 \\
\Rightarrow 3 \frac{dy}{dx} \left\{1 + \frac{1}{(4x-3y)}\right\} &= 4 \left\{\frac{1}{(4x-3y)} - 1\right\} \\
\Rightarrow 3 \frac{dy}{dx} \left\{\frac{4x-3y+1}{(4x-3y)}\right\} &= 4 \left\{\frac{1-4x+3y}{(4x-3y)}\right\} \\
\Rightarrow \frac{dy}{dx} &= \frac{4}{3} \left\{\frac{1-4x+3y}{(4x-3y)}\right\} \left(\frac{4x-3y}{4x-3y+1}\right) \\
\Rightarrow \frac{dy}{dx} &= \frac{4}{3} \left(\frac{1-4x+3y}{4x-3y+1}\right)
\end{aligned}$$

5.

Solution :

We have, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Differentiating with respect to x, we get

$$\begin{aligned}
& \frac{d}{dx} \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right) = \frac{d}{dx}(1) \\
\Rightarrow \frac{d}{dx} \left(\frac{x^2}{a^2} \right) + \frac{d}{dx} \left(\frac{y^2}{b^2} \right) &= 0 \\
\Rightarrow \frac{1}{a^2} (2x) + \frac{1}{b^2} (2y) \frac{dy}{dx} &= 0 \\
\Rightarrow \frac{2y}{b^2} \frac{dy}{dx} &= -\frac{2x}{a^2} \\
\Rightarrow \frac{dy}{dx} &= - \left(\frac{2x}{a^2} \right) \left(\frac{b^2}{2y} \right) \\
\Rightarrow \frac{dy}{dx} &= -\frac{b^2 x}{a^2 y}
\end{aligned}$$

6.

Solution :

We have, $x^5 + y^5 = 5xy$

Differentiating with respect to x, we get

$$\begin{aligned}
& \frac{d}{dx}(x^5) + \frac{d}{dx}(y^5) = \frac{d}{dx}(5xy) \\
\Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5 \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] \\
\Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5 \left[x \frac{dy}{dx} + y(1) \right] \\
\Rightarrow 5x^4 + 5y^4 \frac{dy}{dx} &= 5x \frac{dy}{dx} + 5y \\
\Rightarrow 5y^4 \frac{dy}{dx} - 5x \frac{dy}{dx} &= 5y - 5x^4 \\
\Rightarrow 5 \frac{dy}{dx}(y^4 - x) &= 5(y - x^4) \\
\Rightarrow \frac{dy}{dx} &= \frac{5(y - x^4)}{5(y^4 - x)} \\
\Rightarrow \frac{dy}{dx} &= \frac{y - x^4}{y^4 - x}
\end{aligned}$$

7.

Solution :

We have, $(x + y)^2 = 2axy$

Differentiating with respect to x, we get

$$\begin{aligned}
\Rightarrow \frac{d}{dx}(x + y)^2 &= \frac{d}{dx}(2axy) \\
\Rightarrow 2(x + y) \frac{d}{dx}(x + y) &= 2a \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] \\
\Rightarrow 2(x + y) \left[1 + \frac{dy}{dx} \right] &= 2a \left[x \frac{dy}{dx} + y(1) \right] \\
\Rightarrow 2(x + y) + 2(x + y) \frac{dy}{dx} &= 2ax \frac{dy}{dx} + 2ay \\
\Rightarrow \frac{dy}{dx} [2(x + y) - 2ax] &= 2ay - 2(x + y) \\
\Rightarrow \frac{dy}{dx} &= \frac{2[ay - x - y]}{2[x + y - ax]} \\
\Rightarrow \frac{dy}{dx} &= \left(\frac{ay - x - y}{x + y - ax} \right)
\end{aligned}$$

8.

Solution :

We have, $(x^2 + y^2) = xy$

Differentiating with respect to x, we get

$$\begin{aligned}\Rightarrow \frac{d}{dx} \left[(x^2 + y^2)^2 \right] &= \frac{d}{dx} (xy) \\ \Rightarrow 2(x^2 + y^2) \frac{d}{dx} (x^2 + y^2) &= x \frac{dy}{dx} + y \frac{d}{dx} (x) \\ \Rightarrow 2(x^2 + y^2) \left(2x + 2y \frac{dy}{dx} \right) &= x \frac{dy}{dx} + y(1) \\ \Rightarrow 4x(x^2 + y^2) + 4y(x^2 + y^2) \frac{dy}{dx} &= x \frac{dy}{dx} + y \\ \Rightarrow 4y(x^2 + y^2) \frac{dy}{dx} - x \frac{dy}{dx} &= y - 4x(x^2 + y^2) \\ \Rightarrow \frac{dy}{dx} [4y(x^2 + y^2) - x] &= y - 4x(x^2 + y^2) \\ \Rightarrow \frac{dy}{dx} &= \frac{y - 4x(x^2 + y^2)}{4y(x^2 + y^2) - x} \\ \Rightarrow \frac{dy}{dx} &= \frac{4x(x^2 + y^2) - y}{x - 4y(x^2 + y^2)}\end{aligned}$$

9.

Solution :

We have, $\tan^{-1}(x^2 + y^2) = a$

Differentiating with respect to x, we get

$$\begin{aligned}\frac{d}{dx} [\tan^{-1}(x^2 + y^2)] &= \frac{d}{dx} (a) \\ \Rightarrow \frac{1}{1 + (x^2 + y^2)^2} \times \frac{d}{dx} (x^2 + y^2) &= 0 \\ \Rightarrow \left[\frac{1}{1 + (x^2 + y^2)^2} \right] \left(2x + 2y \frac{dy}{dx} \right) &= 0 \\ \Rightarrow 2x + 2y \frac{dy}{dx} &= 0 \\ \Rightarrow x + y \frac{dy}{dx} &= 0 \\ \Rightarrow \frac{dy}{dx} &= -\frac{x}{y}\end{aligned}$$

10.

Solution :

We have, $e^{x-y} = \log\left(\frac{x}{y}\right)$

Differentiating with respect to x, we get

$$\begin{aligned}\frac{d}{dx}(e^{x-y}) &= \frac{d}{dx}\left\{\log\left(\frac{x}{y}\right)\right\} \\ \Rightarrow e^{(x-y)} \frac{d}{dx}(x-y) &= \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right) \\ \Rightarrow e^{(x-y)} \left(1 - \frac{dy}{dx}\right) &= \frac{y}{x} \left[\frac{y \frac{d}{dx}(x) - x \frac{dy}{dx}}{y^2} \right] \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{xy} \left[y(1) - x \frac{dy}{dx} \right] \\ \Rightarrow e^{(x-y)} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{x} - \frac{1}{y} \frac{dy}{dx} \\ \Rightarrow \frac{1}{y} \frac{dy}{dx} - e^{(x-y)} \frac{dy}{dx} &= \frac{1}{x} - e^{(x-y)} \\ \Rightarrow \frac{dy}{dx} \left[\frac{1}{y} - \frac{e^{(x-y)}}{1} \right] &= \frac{1}{x} - \frac{e^{(x-y)}}{1} \\ \Rightarrow \frac{dy}{dx} \left[\frac{1 - ye^{(x-y)}}{y} \right] &= \frac{1 - xe^{(x-y)}}{x} \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} \left[\frac{1 - xe^{(x-y)}}{1 - ye^{(x-y)}} \right] \\ \Rightarrow \frac{dy}{dx} &= \frac{-y}{-x} \left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1} \right] \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} \left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1} \right]\end{aligned}$$

11.

Solution :

We have, $\sin xy + \cos(x+y) = 1$

Differentiating with respect to x, we get

$$\begin{aligned}
& \frac{d}{dx}(\sin xy) + \frac{d}{dx} \cos(x+y) = \frac{d}{dx}(1) \\
\Rightarrow \cos xy \frac{d}{dx}(xy) - \sin(x+y) \frac{d}{dx}(x+y) &= 0 \\
\Rightarrow \cos xy \left[x \frac{dy}{dx} + y \frac{d}{dx}(x) \right] - \sin(x+y) \left[1 + \frac{dy}{dx} \right] &= 0 \\
\Rightarrow \cos xy \left[x \frac{dy}{dx} + y(1) \right] - \sin(x+y) - \sin(x+y) \frac{dy}{dx} &= 0 \\
\Rightarrow x \cos xy \frac{dy}{dx} + y \cos xy - \sin(x+y) - \sin(x+y) \frac{dy}{dx} &= 0 \\
\Rightarrow [x \cos xy - \sin(x+y)] \frac{dy}{dx} = [\sin(x+y) - y \cos xy] \\
\Rightarrow \frac{dy}{dx} = \left[\frac{\sin(x+y) - y \cos xy}{x \cos xy - \sin(x+y)} \right]
\end{aligned}$$

12.

Solution :

We have, $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$

Let $x = \sin A$, $y = \sin B$

$$\Rightarrow \sqrt{1-\sin^2 A} + \sqrt{1-\sin^2 B} = a(\sin A - \sin B)$$

$$\Rightarrow \cos A + \cos B = a(\sin A - \sin B)$$

$$\Rightarrow a = \frac{\cos A + \cos B}{\sin A - \sin B}$$

$$\Rightarrow a = \frac{2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}}{2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}} \left[\begin{array}{l} \because \sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2} \\ \because \cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2} \end{array} \right]$$

$$\Rightarrow a = \cot \left(\frac{A-B}{2} \right)$$

$$\Rightarrow \cot^{-1} a = \frac{A-B}{2}$$

$$\Rightarrow 2 \cot^{-1} a = A-B$$

$$\Rightarrow 2 \cot^{-1} a = \sin^{-1} x - \sin^{-1} y \quad [\because x = \sin A, y = \sin B]$$

Differentiating with respect to x, we get

$$\begin{aligned}\frac{d}{dx}(2\cot^{-1}a) &= \frac{d}{dx}(\sin^{-1}x) - \frac{d}{dx}(\sin^{-1}y) \\ \Rightarrow 0 &= \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} \\ \Rightarrow \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} &= \frac{1}{\sqrt{1-x^2}} \\ \Rightarrow \frac{dy}{dx} &= \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}} \\ \Rightarrow \frac{dy}{dx} &= \sqrt{\frac{1-y^2}{1-x^2}}\end{aligned}$$

13.

Solution :

We have, $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$

Let $x = \sin A$, $y = \sin B$

$$\Rightarrow \sin B \sqrt{1-\sin^2 A} + \sin A \sqrt{1-\sin^2 B} = 1$$

$$\Rightarrow \sin B \cos A + \sin A \cos B = 1 \quad \left[\because \sin(x+y) = \sin x \cos y + \cos x \sin y \right]$$

$$\Rightarrow \sin(A+B) = 1$$

$$\Rightarrow A+B = \sin^{-1}(1)$$

$$\Rightarrow \sin^{-1}x + \sin^{-1}y = \frac{\pi}{2} \quad \left[\because x = \sin A, y = \sin B \right]$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx}(\sin^{-1}x) + \frac{d}{dx}(\sin^{-1}y) = \frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

14.

Solution :

We have, $xy = 1$

Differentiating with respect to x, we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

$$\Rightarrow x \frac{dy}{dx} + y \frac{d}{dx}(x) = 0 \quad \left[\text{Using product rule} \right]$$

$$\Rightarrow x \frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{\frac{1}{y}} \quad \left[\because x = \frac{1}{y} \right]$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} + y^2 = 0$$

15.

Solution :

$$\text{We have, } xy^2 = 1 \quad \dots(i)$$

Differentiating with respect to x, we get

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$\Rightarrow x \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x) = 0$$

$$\Rightarrow x(2y) \frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x}$$

$$\text{put } x = \frac{1}{y^2} \text{ from equation (i)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow 2 \frac{dy}{dx} = -y^3$$

$$\Rightarrow 2 \frac{dy}{dx} + y^3 = 0$$

16.

Solution :

$$\text{We have, } x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$$

Squaring both sides, we get,

$$\Rightarrow \left(x\sqrt{1+y}\right)^2 = \left(-y\sqrt{1+x}\right)^2$$

$$\Rightarrow x^2(1+y) = y^2(1+x)$$

$$\Rightarrow x^2 + x^2y = y^2 + y^2x$$

$$\Rightarrow x^2 - y^2 = y^2x - x^2y$$

$$\Rightarrow (x-y)(x+y) = xy(y-x)$$

$$\Rightarrow (x+y) = -xy$$

$$\Rightarrow y + xy = -x$$

$$\Rightarrow y(1+x) = -x$$

$$\Rightarrow y = \frac{-x}{(1+x)}$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{dy}{dx} = \left[\frac{-(1+x) \frac{d}{dx}(x) - (-x) \frac{d}{dx}(x+1)}{(1+x)^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{-(1+x)(1) + x(1)}{(1+x)^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{-1 - x + x}{(1+x)^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1+x)^2}$$

$$\Rightarrow (1+x)^2 \frac{dy}{dx} = -1$$

$$\Rightarrow (1+x)^2 \frac{dy}{dx} + 1 = 0$$

17.

Solution :

$$\text{We have, } \log \sqrt{x^2 + y^2} = \tan^{-1} \left(\frac{x}{y} \right)$$

$$\Rightarrow \log(x^2 + y^2)^{\frac{1}{2}} = \tan^{-1} \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{1}{2} \log(x^2 + y^2) = \tan^{-1} \left(\frac{y}{x} \right)$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{1}{2} \frac{d}{dx} \log(x^2 + y^2) = \frac{d}{dx} \tan^{-1} \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{1}{2} \left(\frac{1}{x^2 + y^2} \right) \frac{d}{dx} (x^2 + y^2) = \frac{1}{1 + \left(\frac{y}{x} \right)^2} \frac{d}{dx} \left(\frac{y}{x} \right)$$

$$\Rightarrow \frac{1}{2} \left(\frac{1}{x^2 + y^2} \right) \left[2x + 2y \frac{dy}{dx} \right] = \frac{x^2}{(x^2 + y^2)^2} \left[\frac{x \frac{dy}{dx} - y \frac{d}{dx} (x)}{x^2} \right]$$

$$\Rightarrow \left(\frac{1}{x^2 + y^2} \right) \left(x + y \frac{dy}{dx} \right) = \frac{x^2}{(x^2 + y^2)^2} \left[\frac{x \frac{dy}{dx} - y \frac{d}{dx} (x)}{x^2} \right]$$

$$\Rightarrow \left(\frac{1}{x^2 + y^2} \right) \left(x + y \frac{dy}{dx} \right) = \frac{x^2}{(x^2 + y^2)^2} \left[\frac{x \frac{dy}{dx} - y(1)}{x^2} \right]$$

$$\Rightarrow x + y \frac{dy}{dx} = x \frac{dy}{dx} - y$$

$$\Rightarrow y \frac{dy}{dx} - x \frac{dy}{dx} = -y - x$$

$$\Rightarrow \frac{dy}{dx} (y - x) = -(y + x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{-(y + x)}{y - x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x + y}{x - y}$$

18.

Solution :

$$\text{We have, } \sec \left(\frac{x + y}{x - y} \right) = a$$

$$\Rightarrow \frac{x + y}{x - y} = \sec^{-1} (a)$$

Differentiating with respect to x, we get

$$\Rightarrow \left[\frac{(x-y) \frac{d}{dx}(x+y) - (x+y) \frac{d}{dx}(x-y)}{(x-y)^2} \right] = 0$$

$$\Rightarrow (x-y) \left(1 + \frac{dy}{dx} \right) - (x+y) \left(1 - \frac{dy}{dx} \right) = 0$$

$$\Rightarrow (x-y) + (x-y) \frac{dy}{dx} - (x+y) + (x+y) \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} [x-y+x+y] = x+y-x+y$$

$$\Rightarrow \frac{dy}{dx} (2x) = 2y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

19.

Solution :

$$\text{We have, } \tan^{-1} \left(\frac{x^2 - y^2}{x^2 + y^2} \right) = a$$

$$\Rightarrow \frac{x^2 - y^2}{x^2 + y^2} = \tan a$$

$$\Rightarrow x^2 - y^2 = \tan a (x^2 + y^2)$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx} (x^2 - y^2) = \tan a \frac{d}{dx} (x^2 + y^2)$$

$$\Rightarrow \left(2x - 2y \frac{dy}{dx} \right) = \tan a \left(2x + 2y \frac{dy}{dx} \right)$$

$$\Rightarrow 2x - 2y \frac{dy}{dx} = 2x \tan a + 2y \tan a \frac{dy}{dx}$$

$$\Rightarrow 2y \tan a \frac{dy}{dx} + 2y \frac{dy}{dx} = 2x - 2x \tan a$$

$$\Rightarrow 2y(1 + \tan a) \frac{dy}{dx} = 2x(1 - \tan a)$$

$$\Rightarrow \frac{dy}{dx} = \frac{x}{y} \left(\frac{1 - \tan a}{1 + \tan a} \right)$$

20.

Solution :

$$\text{We have, } xy \log(x+y) = 1$$

Differentiating with respect to x, we get

$$\begin{aligned}
&\Rightarrow \frac{d}{dx} [xy \log(x+y)] = \frac{d}{dx} (1) \\
&\Rightarrow xy \frac{d}{dx} \log(x+y) + x \log(x+y) \frac{dy}{dx} + y \log(x+y) \frac{d}{dx} (x) = 0 \quad [\text{using chain rule and product rule}] \\
&\Rightarrow xy \left(\frac{1}{x+y} \right) \frac{d}{dx} (x+y) + x \log(x+y) \frac{dy}{dx} + y \log(x+y) (1) = 0 \\
&\Rightarrow \left(\frac{xy}{x+y} \right) \left(1 + \frac{dy}{dx} \right) + x \log(x+y) \frac{dy}{dx} + y \log(x+y) = 0 \\
&\Rightarrow \left(\frac{xy}{x+y} \right) \frac{dy}{dx} + \left(\frac{xy}{x+y} \right) + x \left(\frac{1}{xy} \right) \frac{dy}{dx} + y \left(\frac{1}{xy} \right) = 0 \quad [\because xy \log(x+y) = 1] \\
&\Rightarrow \frac{dy}{dx} \left[\frac{xy}{x+y} + \frac{1}{y} \right] = - \left[\frac{1}{x} + \frac{xy}{x+y} \right] \\
&\Rightarrow \frac{dy}{dx} \left[\frac{xy^2 + x + y}{(x+y)y} \right] = - \left[\frac{x + y + x^2 y}{x(x+y)} \right] \\
&\Rightarrow \frac{dy}{dx} = - \left[\frac{x + y + x^2 y}{x(x+y)} \right] \left[\frac{y(x+y)}{xy^2 + x + y} \right] \\
&\Rightarrow \frac{dy}{dx} = - \frac{y}{x} \left(\frac{x + y + x^2 y}{x + y + xy^2} \right)
\end{aligned}$$

Hence, proved.

21.

Solution :

We have, $y = x \sin(a+y)$

Differentiating with respect to x , we get

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx} [x \sin(a+y)] \\
&\Rightarrow \frac{dy}{dx} = x \frac{d}{dx} \{ \sin(a+y) \} + \sin(a+y) \frac{d}{dx} (x) \quad [\text{using product rule and chain rule}] \\
&\Rightarrow \frac{dy}{dx} = x \cos(a+y) \frac{d}{dx} (a+y) + \sin(a+y) (1) \\
&\Rightarrow \frac{dy}{dx} \{ 1 - x \cos(a+y) \} = \sin(a+y) \\
&\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{1 - x \cos(a+y)} \\
&\Rightarrow \frac{dy}{dx} = \frac{\sin(a+y)}{1 - \frac{y}{\sin(a+y)} \cos(a+y)} \quad [\because y = x \sin(a+y)] \\
&\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a+y) - y \cos(a+y)}
\end{aligned}$$

Hence, proved.

22.

Solution :

We have, $x \sin(a + y) + \sin a \cos(a + y) = 0$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx} [x \sin(a + y)] + \frac{d}{dx} [\sin a \cos(a + y)] = 0$$

$$\Rightarrow \left[x \frac{d}{dx} \sin(a + y) + \sin(a + y) \frac{d}{dx} (x) \right] + \sin a \frac{d}{dx} \cos(a + y) = 0$$

$$\Rightarrow \left[x \cos(a + y) \frac{d}{dx} (a + y) + \sin(a + y)(1) \right] + \sin a \left[-\sin(a + y) \frac{d}{dx} (a + y) \right] = 0$$

$$\Rightarrow x \cos(a + y) \frac{dy}{dx} + \sin(a + y) - \sin a \sin(a + y) \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} [x \cos(a + y) - \sin a \sin(a + y)] = -\sin(a + y)$$

$$\Rightarrow \frac{dy}{dx} \left[-\sin a \frac{\cos^2(a + y)}{\sin(a + y)} - \sin a \sin(a + y) \right] = -\sin(a + y) \left[\because x = -\sin a \frac{\cos(a + y)}{\sin(a + y)} \right]$$

$$\Rightarrow -\frac{dy}{dx} \left[\frac{\sin a \cos^2(a + y) + \sin a \sin^2(a + y)}{\sin(a + y)} \right] = -\sin(a + y)$$

$$\Rightarrow \frac{dy}{dx} = \sin(a + y) \left[\frac{\sin(a + y)}{\sin a \{ \cos^2(a + y) + \sin^2(a + y) \}} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin^2(a + y)}{\sin a}$$

23.

Solution :

We have, $y = x \sin y$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx} (x \sin y)$$

$$\Rightarrow \frac{dy}{dx} = x \frac{d}{dx} (\sin y) + \sin y \frac{d}{dx} (x)$$

$$\Rightarrow \frac{dy}{dx} = x \cos y \frac{dy}{dx} + \sin y$$

$$\Rightarrow \frac{dy}{dx} (1 - x \cos y) = \sin y$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin y}{1 - x \cos y}$$

24.

Solution :

We have, $y\sqrt{x^2 + 1} = \log(\sqrt{x^2 + 1} - x)$

Differentiating with respect to x, we get

$$\begin{aligned}
&\Rightarrow \frac{d}{dx} \left(y\sqrt{x^2+1} \right) = \frac{d}{dx} \log \left(\sqrt{x^2+1} - x \right) \quad \left[\text{using product rule and chain rule} \right] \\
&\Rightarrow y \frac{d}{dx} \left(\sqrt{x^2+1} \right) + \sqrt{x^2+1} \frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1} - x \right)} \times \frac{d}{dx} \left(\sqrt{x^2+1} - x \right) \\
&\Rightarrow \frac{y}{2\sqrt{x^2+1}} \times \frac{d}{dx} \left(x^2+1 \right) + \sqrt{x^2+1} \frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1} - x \right)} \times \left[\frac{1}{2\sqrt{x^2+1}} \frac{d}{dx} \left(x^2+1 \right) - 1 \right] \\
&\Rightarrow \frac{2xy}{2\sqrt{x^2+1}} + \sqrt{x^2+1} \frac{dy}{dx} = \frac{1}{\left(\sqrt{x^2+1} - x \right)} \left[\frac{2x}{2\sqrt{x^2+1}} - 1 \right] \\
&\Rightarrow \sqrt{x^2+1} \frac{dy}{dx} = \left[\frac{1}{\sqrt{x^2+1} - x} \right] \left[\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}} \right] - \frac{xy}{\sqrt{x^2+1}} \\
&\Rightarrow \sqrt{x^2+1} \frac{dy}{dx} = \frac{-1}{\sqrt{x^2+1}} - \frac{xy}{\sqrt{x^2+1}} \\
&\Rightarrow \sqrt{x^2+1} \frac{dy}{dx} = \frac{-(1+xy)}{\sqrt{x^2+1}} \\
&\Rightarrow (x^2+1) \frac{dy}{dx} = -(1+xy) \\
&\Rightarrow (x^2+1) \frac{dy}{dx} + 1 + xy = 0
\end{aligned}$$

25.

Solution :

We have, $\sin(xy) + \frac{y}{x} = x^2 - y^2$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx}(\sin xy) + \frac{d}{dx}\left(\frac{y}{x}\right) = \frac{d}{dx}(x^2) - \frac{d}{dx}(y^2)$$

$$\Rightarrow \cos(xy) \frac{d}{dx}(xy) + \left\{ \frac{x \frac{dy}{dx} - y \frac{d}{dx}(x)}{x^2} \right\} = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \cos(xy) \left\{ x \frac{dy}{dx} + y \frac{d}{dx}(x) \right\} + \left\{ \frac{x \frac{dy}{dx} - y(1)}{x^2} \right\} = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \cos(xy) \left\{ x \frac{dy}{dx} + y(1) \right\} + \frac{1}{x^2} \left(x \frac{dy}{dx} - y \right) = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow x \cos(xy) \frac{dy}{dx} + y \cos(xy) + \frac{1}{x} \frac{dy}{dx} - \frac{y}{x^2} = 2x - 2y \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} \left\{ x \cos(xy) + \frac{1}{x} + 2y \right\} = \frac{y}{x^2} - y \cos(xy) + 2x$$

$$\Rightarrow \frac{dy}{dx} \left\{ \frac{x^2 \cos(xy) + 1 + 2xy}{x} \right\} = \frac{1}{x^2} (y - x^2 y \cos(xy) + 2x^3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^3 + y - x^2 y \cos(xy)}{x(x^2 \cos(xy) + 1 + 2xy)}$$

26

Solution :

We have, $\tan(x+y) + \tan(x-y) = 1$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx} \tan(x+y) + \frac{d}{dx} \tan(x-y) = \frac{d}{dx}(1)$$

$$\Rightarrow \sec^2(x+y) \frac{d}{dx}(x+y) + \sec^2(x-y) \frac{d}{dx}(x-y) = 0$$

$$\Rightarrow \sec^2(x+y) \left[1 + \frac{dy}{dx} \right] + \sec^2(x-y) \left[1 - \frac{dy}{dx} \right] = 0$$

$$\Rightarrow \sec^2(x+y) \frac{dy}{dx} - \sec^2(x-y) \frac{dy}{dx} = -[\sec^2(x+y) + \sec^2(x-y)]$$

$$\Rightarrow \frac{dy}{dx} [\sec^2(x+y) - \sec^2(x-y)] = -[\sec^2(x+y) + \sec^2(x-y)]$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sec^2(x+y) + \sec^2(x-y)}{\sec^2(x-y) - \sec^2(x+y)}$$

27.

Solution :

$$e^x + e^y = e^{x+y}$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y} \left(1 + \frac{dy}{dx} \right)$$

$$\Rightarrow e^x + e^y \frac{dy}{dx} = e^{x+y} + e^{x+y} \frac{dy}{dx}$$

$$\Rightarrow e^y \frac{dy}{dx} - e^{x+y} \frac{dy}{dx} = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx} (e^y - e^{x+y}) = e^{x+y} - e^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^{x+y} - e^x}{e^y - e^{x+y}}$$

$$= \frac{e^x (e^y - 1)}{e^y (1 - e^x)}$$

$$= - \frac{e^x (e^y - 1)}{e^y (e^x - 1)}$$

28.

Solution :

We have, $\cos y = x \cos(a + y)$

Differentiating with respect to x , we get,

$$\frac{d}{dx} (\cos y) = \frac{d}{dx} \{x \cos(a + y)\}$$

$$\Rightarrow -\sin y \frac{dy}{dx} = \cos(a + y) \frac{d}{dx} (x) + x \frac{d}{dx} \cos(a + y)$$

$$\Rightarrow -\sin y \frac{dy}{dx} = \cos(a + y) + x [-\sin(a + y)] \frac{dy}{dx}$$

$$\Rightarrow [x \sin(a + y) - \sin y] \frac{dy}{dx} = \cos(a + y)$$

$$\Rightarrow \left[\frac{\cos y}{\cos(a + y)} \sin(a + y) - \sin y \right] \frac{dy}{dx} = \cos(a + y) \quad \left[\because \cos y = x \cos(a + y) \Rightarrow x = \frac{\cos y}{\cos(a + y)} \right]$$

$$\Rightarrow [\cos y \sin(a + y) - \sin y \cos(a + y)] \frac{dy}{dx} = \cos^2(a + y)$$

$$\Rightarrow \sin(a + y - y) \frac{dy}{dx} = \cos^2(a + y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a + y)}{\sin a}$$

29.

Solution :

$$\text{We have, } y = [\log_{\cos x} \sin x][\log_{\sin x} \cos x]^{-1} + \sin^{-1} \left(\frac{2x}{1+x^2} \right)$$

$$\Rightarrow y = [\log_{\cos x} \sin x][\log_{\cos x} \sin x] + \sin^{-1} \left(\frac{2x}{1+x^2} \right) \quad \left[\because \log_a b = (\log_b a)^{-1} \right]$$

$$\Rightarrow y = \left[\frac{\log \sin x}{\log \cos x} \right]^2 + \sin^{-1} \left(\frac{2x}{1+x^2} \right) \quad \left[\because \log_a b = \frac{\log b}{\log a} \right]$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left[\frac{\log \sin x}{\log \cos x} \right]^2 + \frac{d}{dx} \left\{ \sin^{-1} \left(\frac{2x}{1+x^2} \right) \right\}$$

$$\Rightarrow \frac{dy}{dx} = 2 \left[\frac{\log \sin x}{\log \cos x} \right] \frac{d}{dx} \left(\frac{\log \sin x}{\log \cos x} \right) + \frac{1}{\sqrt{1 - \left(\frac{2x}{1+x^2} \right)^2}} \times \frac{d}{dx} \left[\frac{2x}{1+x^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = 2 \left[\frac{\log \sin x}{\log \cos x} \right] \left[\frac{(\log \cos x) \frac{d}{dx} (\log \sin x) - \log \sin x \frac{d}{dx} (\log \cos x)}{(\log \cos x)^2} \right] + \left[\frac{(1+x^2)}{\sqrt{1+x^4-2x^2}} \right] \left[\frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = 2 \left[\frac{\log \sin x}{\log \cos x} \right] \left[\frac{\log \cos x \times \frac{1}{\sin x} \frac{d}{dx} (\sin x) - \log \sin x \times \frac{1}{\cos x} \frac{d}{dx} (\cos x)}{(\log \cos x)^2} \right] + \left[\frac{(1+x^2)}{\sqrt{1+x^4-2x^2}} \right] \left[\frac{(1+x^2)(2) - (2x)(2x)}{(1+x^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = 2 \left[\frac{\log \sin x}{\log \cos x} \right] \left[\frac{\log \cos x \times \left(\frac{\cos x}{\sin x} \right) + \log \sin x \times \left(\frac{\sin x}{\cos x} \right)}{(\log \cos x)^2} \right] + \left[\frac{1+x^2}{\sqrt{(1-x^2)^2}} \right] \left[\frac{2+2x^2-4x^2}{(1+x^2)^2} \right]$$

$$\Rightarrow \frac{dy}{dx} = 2 \frac{\log \sin x}{(\log \cos x)^3} (\cot x \log \cos x + \tan x \log \sin x) + \frac{2}{1+x^2}$$

$$\text{put } x = \frac{\pi}{4}$$

$$\Rightarrow \frac{dy}{dx} = 2 \left\{ \frac{\log \sin \frac{\pi}{4}}{\left(\log \cos \frac{\pi}{4}\right)^3} \right\} \left\{ \cot \frac{\pi}{4} \log \cos \frac{\pi}{4} + \tan \frac{\pi}{4} \log \sin \frac{\pi}{4} \right\} + 2 \left\{ \frac{1}{1 + \left(\frac{\pi}{4}\right)^2} \right\}$$

$$\Rightarrow \frac{dy}{dx} = 2 \left\{ \frac{1}{\left(\log \frac{1}{\sqrt{2}}\right)^2} \right\} \left\{ 1 \times \log \frac{1}{\sqrt{2}} + 1 \times \log \frac{1}{\sqrt{2}} \right\} + 2 \left(\frac{16}{16 + \pi^2} \right)$$

$$\Rightarrow \frac{dy}{dx} = 2 \times \frac{2 \log \left(\frac{1}{\sqrt{2}} \right)}{\left\{ \log \left(\frac{1}{\sqrt{2}} \right) \right\}^2} + \frac{32}{16 + \pi^2}$$

$$\Rightarrow \frac{dy}{dx} = 4 \frac{1}{\log \left(\frac{1}{\sqrt{2}} \right)} + \frac{32}{16 + \pi^2}$$

$$\Rightarrow \frac{dy}{dx} = 4 \frac{1}{-\frac{1}{2} \log 2} + \frac{32}{16 + \pi^2}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{8}{\log 2} + \frac{32}{16 + \pi^2}$$

$$\text{So, } \left(\frac{dy}{dx} \right)_{x=\frac{\pi}{4}} = 8 \left[\frac{4}{16 + \pi^2} - \frac{1}{\log 2} \right]$$

30.

Solution :

Here,

$$\sqrt{y+x} + \sqrt{y-x} = c$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx}(\sqrt{y+x}) + \frac{d}{dx}\sqrt{y-x} = \frac{d}{dx}(c)$$

$$\Rightarrow \frac{1}{2\sqrt{y+x}} \frac{d}{dx}(y+x) + \frac{1}{2\sqrt{y-x}} \frac{d}{dx}(y-x) = 0$$

$$\Rightarrow \frac{1}{2\sqrt{y+x}} \left(\frac{dy}{dx} + 1 \right) + \frac{1}{2\sqrt{y-x}} \left(\frac{dy}{dx} - 1 \right) = 0$$

$$\Rightarrow \frac{dy}{dx} \left(\frac{1}{2\sqrt{y+x}} \right) + \frac{dy}{dx} \left(\frac{1}{2\sqrt{y-x}} \right) = \frac{1}{2\sqrt{y-x}} - \frac{1}{2\sqrt{y+x}}$$

$$\Rightarrow \frac{dy}{dx} \times \frac{1}{2} \left[\frac{1}{\sqrt{y+x}} + \frac{1}{\sqrt{y-x}} \right] = \frac{1}{2} \left[\frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y-x}\sqrt{y+x}} \right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y+x} + \sqrt{y-x}} \times \frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y+x} - \sqrt{y-x}} \quad [\text{rationalizing the denominator}]$$

$$\Rightarrow \frac{dy}{dx} = \frac{(y+x) + (y-x) - 2\sqrt{y+x}\sqrt{y-x}}{y+x - y+x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y - 2\sqrt{y^2 - x^2}}{2x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y}{2x} - \frac{2\sqrt{y^2 - x^2}}{2x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2 - x^2}{x^2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$$