Relations Ex 1.2 Q1

We have,

$R = \{(a,b):\ a-b \text{ is divisible by } 3;\ a,b, \in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a-a=0$

$\Rightarrow \quad a-a$ is divisible by 3

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a-b$ is divisible by 3

$\Rightarrow \quad a-b=3p \quad$ For some $p \in Z$

$\Rightarrow \quad b-a=3\times(-p)$

$\Rightarrow \quad b-a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a,b,c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a-b=3p$ and $b-c=3q$ For some $p,q \in Z$

$\Rightarrow \quad a-c=3(p+q)$

$\Rightarrow \quad a-c$ is divisible by 3

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is equivalence relation.

Relations Ex 1.2 Q2

We have,

$R = \{(a,b) : \ a - b \text{ is divisible by } 2;\ a, b, \in Z\}$

To prove: R is an equivalence relation

Proff:

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 2

$\Rightarrow \quad (a, a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a, b \in Z$ and $(a,b) \in R$

$\Rightarrow \quad a - b$ is divisible by 2

$\Rightarrow \quad a - b = 2p \quad$ For some $p \in Z$

$\Rightarrow \quad b - a = 2 \times (-p)$

$\Rightarrow \quad b - a \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $a, b, c \in Z$ and such that $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 2p$ and $b - c = q$ For some $p, q \in Z$

$\Rightarrow \quad a - c = 2(p + q)$

$\Rightarrow \quad a - c$ is divisible by 2

$\Rightarrow \quad (a, c) \in R$

$\Rightarrow \quad R$ is transitive

Relations Ex 1.2 Q3

We have,

$R = \{(a,b) : (a\text{-}b) \text{ is divisible by } 5\}$ on Z.

We want to prove that R is an equivalence relation on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0$

$\Rightarrow \quad a - a$ is divisible by 5.

$\therefore \quad (a,a) \in R$, so R is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a - b = 5P \quad$ For some $P \in Z$

$\Rightarrow \quad b - a = 5 \times (-P)$

$\Rightarrow \quad b - a$ is divisible by 5

$\Rightarrow \quad (b,a) \in R, \quad$ so R is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = 5p$ and $b - c = 5q$ For some $p, q \in Z$

$\Rightarrow \quad a - c = 5(p + q)$

$\Rightarrow \quad a - c$ is divisible by 5.

$\Rightarrow \quad R$ is transitive.

Thus, R being reflexive,symmetric and transitive on Z.

Hence, R is equivalence relation on Z

Relations Ex 1.2 Q4

$R = \{(a,b):$ a-b is divisible by n$\}$ on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow \quad a - a = 0 \times n$

$\Rightarrow \quad a - a$ is divisible by n

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(a,b) \in R$

$\Rightarrow \quad a - b = np$ For some $p \in Z$

$\Rightarrow \quad b - a = n(-p)$

$\Rightarrow \quad b - a$ is divisible by n

$\Rightarrow \quad (b,a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a - b = xp$ and $b - c = xq$ For some $p,q \in Z$

$\Rightarrow \quad a - c = n(p + q)$

$\Rightarrow \quad a - c$ is divisible by n

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Thus, R being reflexive, symmetric and transitive on Z.

Hence, R is an equivalence relation on Z

Relations Chapter 1 Ex 1.2 Q5

We have, Z be set of integers and

$R = \{(a,b):\ a,b \in Z$ and $a+b$ is even $\}$ be a relation on Z.

We want to prove that R is an equivalence relation on Z.

Now,

Reflexivity: Let $a \in Z$

$\Rightarrow$ $a+a$ is even $\begin{bmatrix} \text{if a is even} \Rightarrow a+a \text{ is even} \\ \text{if a is odd} \Rightarrow a+a \text{ is even} \end{bmatrix}$

$\Rightarrow$ $(a,a) \in R$

$\Rightarrow$ R is reflexive

Symmetric: Let $a,b \in Z$ and $(a,b) \in R$

$\Rightarrow$ $a+b$ is even

$\Rightarrow$ $b+a$ is even

$\Rightarrow$ $(b,a) \in R$,

$\Rightarrow$ R is symmetric

Transitivity: Let $(a,b) \in R$ and $(b,c) \in R$ For some $a,b,c \in Z$

$\Rightarrow$ $a+b$ is even and $b+c$ is even

$\Rightarrow$ $a+c$ is even $\begin{bmatrix} \text{if b is odd, then a and c must be odd} \Rightarrow a+c \text{ is even,} \\ \text{If b is even, then a and c must be even} \Rightarrow a+c \text{ is even} \end{bmatrix}$

$\Rightarrow$ $(a,c) \in R$

$\Rightarrow$ R is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q6

Let Z be set of integers

$R = \{(m,n) : m-n \text{ is divisible by } 13\}$ be a relation on Z.

Now,

Reflexivity: Let m $\in Z$

$\Rightarrow \quad m - m = 0$

$\Rightarrow \quad m - m$ is divisible by 13

$\Rightarrow \quad (m,m) \in R,$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let m,n $\in Z$ and $(m,n) \in R$

$\Rightarrow \quad m - n = 13.p$ For some $p \in Z$

$\Rightarrow \quad n - m = 13 \times (-p)$

$\Rightarrow \quad n - m$ is divisible by 13

$\Rightarrow \quad (n - m) \in R,$

so

$\Rightarrow \quad R$ is symmetric

Transitivity: Let $(m,n) \in R$ and $(n,q) \in R$ For some m,n,q $\in$ Z

$\Rightarrow \quad m - n = 13p$ and $n - q = 13s$ For some p,s $\in$ Z

$\Rightarrow \quad m - q = 13(p + s)$

$\Rightarrow \quad m - q$ is divisible by 13

$\Rightarrow \quad (m,q) \in R$

$\Rightarrow \quad R$ is transitive

Hence, R is an equivalence relation on Z

Relations Ex 1.2 Q7

$(x, y)\ R\ (u, v) \Leftrightarrow xv = yu$

TPT Reflexive $\because xy = yx$

$\therefore (x, y)\ R\ (x, y)$

TPT Symmetric Let $(x, y)\ R\ (u, v)$

TPT $(u, v)\ R\ (x, y)$

Given $xv = yu$

$\Rightarrow yu = xv$

$\Rightarrow uy = vx$

$\therefore (u, v)\ R\ (x, y)$

Transitive Let $(x, y)\ R\ (u, v)$ and $(u, v)\ R\ (p, q)$ ....(i)

TPT $(x, y)\ R\ (p, q)$

TPT $xq = yp$

from (1) $xv = yu$ & $uq = vp$

$xvuq = yuvp$

$xq = yp$

$\therefore$ R is transitive

since R is reflexive symmetric & transitive all means it is an equivalence relation

Relations Ex 1.2 Q8

We have, $A = \{x \in z: 0 \leq x \leq 12\}$ be a set and

$R = \{(a,b): a = b\}$ be a relation on A

Now,

Reflexivity: Let $a \in A$

$\Rightarrow \quad a = a$

$\Rightarrow \quad (a,a) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $a,b \in A$ and $(a,b) \in R$

$\Rightarrow \quad a = b$

$\Rightarrow \quad b = a$

$\Rightarrow \quad (b,a) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let a, b & $c \in A$

and Let $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow \quad a = b$ and $b = c$

$\Rightarrow \quad a = c$

$\Rightarrow \quad (a,c) \in R$

$\Rightarrow \quad R$ is transitive

Since R is being relfexive,symmetric and transitive, so

R is an equivalance relation.

Also, we need to find the set of all elements related to 1.

Since the relation is given by, R={(a,b):a=b}, and 1 is an element of A,

R={(1,1):1=1}

Thus, the set of all elements related to 1 is 1.

Relations Ex 1.2 Q9

(i) We have, L is the set of lines.

$R = \{(L_1, L_2):$ L_1 is parallel to $L_2\}$ be a relation on L

Now,

Reflexivity: Let $L_1 \in L$

Since a line is always parallel to itself.

$\therefore \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $L_1, L_2 \in L$ and $(L_1, L_2) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2

$\Rightarrow \quad L_2$ is parallel to L_1

$\Rightarrow \quad (L_1, L_2) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $L_1, L_2 and L_3 \in L$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$

$\Rightarrow \quad L_1$ is parallel to L_2 and L_2 is parallel to L_3

$\Rightarrow \quad L_1$ is parallel to L_3

$\Rightarrow \quad (L_1, L_3) \in R$

$\Rightarrow \quad R$ is transitive

Since, R is reflexive, symmetric and transitive, so R is an equivalence relation.

(ii) The set of lines parallel to the line $y = 2x + 4$ is

$y = 2x + c$ For all $c \in R$

Where R is the set of real numbers.

elations Ex 1.2 Q10

R = {(P_1, P_2): P_1 and P2 have same the number of sides}

R is reflexive since (P_1, P_1) ∈ R as the same polygon has the same number of sides with itself.

Let (P_1, P_2) ∈ R.

⇒ P_1 and P_2 have the same number of sides.

⇒ P_2 and P_1 have the same number of sides.

⇒ (P_2, P_1) ∈ R

∴R is symmetric.

Now,

Let (P_1, P_2), (P_2, P3) ∈ R.

⇒ P_1 and P_2 have the same number of sides. Also, P_2 and P3 have the same number of sides.

⇒ P_1 and P3 have the same number of sides.

⇒ (P_1, P3) ∈ R

∴R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Relations Ex 1.2 Q11

Let A be set of points on plane.

Let $R = \{(P, Q) : OP = OQ\}$ be a relation on A where O is the origin.

To prove R is an equivalence relation, we need to show that R is reflexive, symmetric and transitive on A.

Now,

Reflexivity: Let $p \in A$

$Since \quad OP = OP \Rightarrow \quad (P, P) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $(P, Q) \in R$ for $P, Q \in A$

$Then \quad OP = OQ$

$\Rightarrow \quad OQ = OP$

$\Rightarrow \quad (Q, P) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(P, Q) \in R$ and $(Q, S) \in R$

$\Rightarrow \quad OP = OQ$ and $OQ = OS$

$\Rightarrow \quad OP = OS$

$\Rightarrow \quad (P, S) \in R$

$\Rightarrow \quad R$ is transitive

Thus, R is an equivalence relation on A

elations Ex 1.2 Q12

Given A={1,2,3,4,5,6,7} and R={(a,b):both a and b are either odd or even number}

Therefore,

R={(1,1),(1,3),(1,5),(1,6),(3,3),(3,5),(3,7),(5,5),(5,7),(7,7),(7,5),(7,3),(5,3),(6,1),(5,1),(3,1),
(2,2),(2,4),(2,6),(4,4),(4,6),(6,6),(6,4),(6,2),(4,2)}

Form the relation R it is seen that R is symmetric, reflecive and transitive also. Therefore R is an equivalent relation.

From the relation R it is seen that {1,3,5,7} are related with each other only and {2,4,6} are related with each other

Relations Ex 1.2 Q13

$S = \{(a,b) : a^2 + b^2 = 1\}$

Now,

Reflexivity: Let $a = \frac{1}{2} \in R$

Then, $a^2 + a^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \neq 1$

$\Rightarrow \quad (a,a) \notin S$

$\Rightarrow \quad S$ is not reflexive

Hence, S in not an equivalenve relation on R

Relations Ex 1.2 Q14

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on $z \times z_0$

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$

$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cd = da$

$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow \quad ad = bc$ and $cf = de$

$\Rightarrow \quad \frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow \quad \frac{a}{b} = \frac{e}{f}$

$\Rightarrow \quad af = be$

We have, Z be set of integers and Z_0 be the set of non-zero integers.

$R = \{(a,b)(c,d) : ad = bc\}$ be a relation on Z and Z_0.

Now,

Reflexivity: $(a,b) \in Z \times Z_0$

$\Rightarrow \quad ab = ba$

$\Rightarrow \quad ((a,b),(a,b)) \in R$

$\Rightarrow \quad R$ is reflexive

Symmetric: Let $((a,b),(c,d)) \in R$

$\Rightarrow \quad ad = bc$

$\Rightarrow \quad cd = da$

$\Rightarrow \quad ((c,d),(a,b)) \in R$

$\Rightarrow \quad R$ is symmetric

$\Rightarrow$ R is symmetric

Transitive: Let $(a,b),(c,d) \in R$ and $(c,d),(e,f) \in R$

$\Rightarrow$ $ad = bc$ and $cf = de$

$\Rightarrow$ $\frac{a}{b} = \frac{c}{d}$ and $\frac{c}{d} = \frac{e}{f}$

$\Rightarrow$ $\frac{a}{b} = \frac{e}{f}$

$\Rightarrow$ $af = be$

$\Rightarrow$ $(a,b)(e,f) \in R$

$\Rightarrow$ R is transitive

Hence, R is an equivalence relation on $Z \times Z_0$

Relations Ex 1.2 Q15.

R and S are two symmetric relations on set A

(i) To prove: $R \cap S$ is symmetric

Let $(a,b) \in R \cap S$

$\Rightarrow$ $(a,b) \in R$ and $(a,b) \in S$

$\Rightarrow$ $(b,a) \in R$ and $(b,a) \in S$ $[\because R$ and S are symmetric$]$

$\Rightarrow$ $(b,a) \in R \cap S$

$\Rightarrow$ $R \cap S$ is symmetric

To prove: $R \cup S$ is symmetric.

Let $(a,b) \in R \cup S$

$\Rightarrow$ $(a,b) \in R$ or $(a,b) \in S$

$\Rightarrow$ $(b,a) \in R$ or $(b,a) \in S$ $[\because R$ and S are symmetric$]$

$\Rightarrow$ $(b,a) \in R \cup S$

$\Rightarrow$ $R \cup S$ is symmetric

(ii) R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.

This means that there is an $a \in R \cup S$ such that $(a,a) \notin R \cup S$

Since $a \in R \cup S$,

$\therefore$ $a \in R$ or $a \in S$

If $a \in R$, then $(a,a) \in R$ $[\because R$ is reflexive$]$

$\Rightarrow$ $(a,a) \in R \cup S$

Hence, $R \cup S$ is reflexive

Relations Ex 1.2 Q16.

We will prove this by means of an example.

Let $A = \{a, b, c\}$ be a set and

$R = \{(a, a)(b, b)(c, c)(a, b)(b, a)\}$ and

$S = \{(a, a)(b, b)(c, c)(b, c)(c, b)\}$ are two relations on A

Clearly R and S are transitive relation on A

Now, $R \cup S = \{(a, a)(b, b)(c, c)(a, b)(b, a)(b, c)(c, b)\}$

Here, $(a, b) \in R \cup S$ and $(b, c) \in R \cup S$

but $(a, c) \notin R \cup S$

$\therefore$ $R \cup S$ is not transitive